SET THEORY - II   LINEAR MODELS AND DECISIONS

$R \subset S \times S$ IS REFLEXIVE iff $x \in S \Rightarrow xRx$

$R \subset S \times S$ IS SYMMETRIC iff $x, y \in S \wedge xRy \Rightarrow yRx$

$R \subset S \times S$ IS ANTI-SYMMETRIC iff $x, y \in S \wedge xRy \wedge yRx \Rightarrow x = y$

$R \subset S \times S$ IS TRANSITIVE iff $x, y, z \in S \wedge xRy \wedge yRz \Rightarrow xRz$

$R \subset S \times S$ IS AN EQUIVALENCE RELATION iff R IS REFLEXIVE, SYMMETRIC, TRANSITIVE

EXAMPLE: "X IS THE SAME COLOR AS Y"

$R \subset S \times S$ IS A PARTIAL ORDERING iff R IS REFLEXIVE, ANTI-SYMMETRIC, TRANSITIVE

EXAMPLE: "X IS A SUBSET OF Y"

$R \subset S \times T$ IS A FUNCTION iff $x \in S \wedge y, z \in T \wedge xRy \wedge xRz \Rightarrow y = z$

$f: A \rightarrow B$ "f MAPS A INTO B" iff $x \in A \Rightarrow f(x) \in B$

NOTE: $f(x)$ MUST BE UNIQUELY DEFINED i.e., THERE IS ONLY ONE $f(x)$ FOR EACH X

$f: A \rightarrow B$ IS ONE-TO-ONE (AN INJECTION) iff $x, y \in A \wedge f(x) = f(y) \Rightarrow x = y$

i.e., DISTINCT ELEMENTS ARE MAPPED INTO DISTINCT ELEMENTS

$f: A \rightarrow B$ IS ONTO (A SURJECTION) iff $y \in B \Rightarrow \exists x \in A \ni f(x) = y$

i.e., EVERY ELEMENT OF B IS MAPPED INTO BY SOME ELEMENT OF A

$f: A \rightarrow B$ IS A ONE-TO-ONE CORRESPONDENCE (A BIJECTION) iff f IS 1-TO-1 AND ONTO

$f: A \rightarrow B$ A 1-TO-1 CORRESPONDENCE $\Rightarrow$ A AND B HAVE THE SAME NUMBER OF ELEMENTS

NOTE: IF B IS SMALLER THAN A, THEN f CAN'T BE 1-TO-1

IF B IS LARGER THAN A, THEN f CAN'T BE ONTO