27 SEP 77

LINEAR ALGEBRA X - LINEAR MODELS AND DECISIONS

PROJECTIONS

FOR ANY VECTOR X AND MATRIX F, Fx IS A VECTOR IN THE SPAN OF THE COLUMNS OF F

SUPPOSE WE WANT THAT VECTOR Fx TO BE THE PROJECTION OF SOME VECTOR A INTO THE SPAN OF F

THAT WOULD MEAN THAT THE DIFFERENCE VECTOR $A-Fx$ WOULD BE PERPENDICULAR TO THE SPAN OF F, THUS ORTHOGONAL TO THE COLUMNS OF F

$\therefore$ $F'(A-Fx)=0$ EXPRESSES THE CONDITION THAT Fx IS THE PROJECTION OF A INTO SPAN F

$$F'A - F'Fx = 0$$
$$F'Fx = F'A$$

NOW SUPPOSE THAT F IS OF FULL COLUMN RANK - A BASIS FOR SPAN OF F, THEN $\exists\,(F'F)^{-1}$

$$x = (F'F)^{-1}F'A$$
$$Fx = F(F'F)^{-1}F'A$$

FOR ANY FULL COLUMN RANK MATRIX F, $P=F(F'F)^{-1}F'$ IS THE PROJECTION INTO SPAN F

NOTE THAT $(F'F)^{-1}F'$ PROJECTS INTO SPAN F WITH COLUMNS OF F AS A BASIS,
WHILE $F(F'F)^{-1}F'$ PROJECTS INTO SPAN F WITH ORIGINAL BASIS FOR THE FULL VECTOR SPACE