LINEAR ALGEBRA VIII - LINEAR MODELS AND DECISIONS

IF $\|v\|=1$ AND $Mv=\alpha v$ FOR SOME SCALAR $\alpha \in F$, THEN v IS SAID TO BE AN EIGENVECTOR FOR M AND α IS THE EIGENVALUE ASSOCIATED WITH THAT VECTOR

A MATRIX WITH N COLUMNS WILL HAVE N EIGENVECTORS AND ASSOCIATED EIGENVALUES
SOME EIGENVALUES MAY EQUAL EACH OTHER

RANK M = MAXIMUM NUMBER OF LINEARLY INDEPENDENT COLUMNS
[= " " " " ROWS]

M IS OF FULL RANK iff RANK M = DIM M (NUMBER OF COLUMNS)

M IS SINGULAR iff IT HAS A ZERO EIGENVALUE iff IT IS NOT OF FULL RANK
NONSINGULAR OTHERWISE

$\underset{N\times N}{M^{-1}}$ EXISTS iff M IS NONSINGULAR iff M IS OF FULL RANK iff M HAS NO ZERO EIGENVALUES

DETERMINANT OF M = DET M = THE PRODUCT OF ALL OF M'S EIGENVALUES

$\underset{N\times N}{M^{-1}}$ EXISTS iff DET $M \neq 0$ (NONSINGULAR, FULL RANK, NO ZERO EIGENVALUES)

$\underline{0} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$ $\quad \underline{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ $\quad x'\underline{1} = \underline{1}'x = x_1 + \dots + x_N$

$AB \neq BA$ $\quad AB = O$ FOR $A \neq O$, $B \neq O$ POSSIBLE

$(A_1 \dots A_N)^{-1} = A_N^{-1} \dots A_1^{-1}$ IF $A_1, \dots, A_N$ HAVE INVERSES

$(A^{-1})^{-1} = A$

$(A_1 \dots A_N)' = A_N' \dots A_1'$

$(A')' = A$

$$A^{-1}A = AA^{-1} = I$$
$$\therefore (A^{-1}A)' = (AA^{-1})' = I' = I$$
$$\therefore A'A^{-1\prime} = A^{-1\prime}A' = I$$
$$\therefore A'^{-1} = A^{-1\prime}$$

M OF FULL RANK $\Leftrightarrow$ $M'M$ NONSINGULAR

$(A+B)C = AC + BC \qquad A(B+C) = AB + AC$