LINEAR ALGEBRA VII – LINEAR MODELS AND DECISIONS

NOTE $L: V_1 \to V_2$, FOR $\dim V_1 \neq \dim V_2$, INDUCES A NON-SQUARE MATRIX REPRESENTATION FOR L

e.g., V_2 A SUBSPACE OF V_1 AN IMPORTANT SPECIAL CASE

THE NUMBER OF ROWS AND COLUMNS OF A MATRIX ARE INDICATED AS $\underset{M\times N}{A}$ INDICATING THAT A HAS M ROWS AND N COLUMNS

$$\underset{M\times N}{A} + \underset{M\times N}{B} = \underset{M\times N}{C} \qquad \alpha \underset{M\times N}{A} = \underset{M\times N}{B} \qquad \underset{M\times N}{A}\ \underset{N\times P}{B} = \underset{M\times P}{C}$$

{NOTE SPECIAL CASE OF A MATRIX TIMES A VECTOR}

SPECIAL MATRIX PRODUCTS

HADAMARD PRODUCT

$$\underset{M\times N}{A} \times\times \underset{M\times N}{B} = (a_{ij}) \times\times (b_{ij}) = (a_{ij} \cdot b_{ij}) = (c_{ij}) = \underset{M\times N}{C}$$

KRONECKER (TENSOR) PRODUCT

$$\underset{M\times N}{A} \otimes \underset{P\times Q}{B} = \begin{pmatrix} a_{11}B & a_{12}B \ldots & a_{1N}B \\ \vdots & & \vdots \\ a_{M1}B & a_{M2}B \ldots & a_{MN}B \end{pmatrix} = \underset{MP\times NQ}{C}$$

MATRIX ALGEBRA

$A+B$, αA, AB ALREADY DEFINED

FOR $\underset{M\times M}{I} = \begin{pmatrix} 1 & 0 & \ldots & 0 \\ 0 & 1 & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix}$, $IA = A$ [AI IF A IS SQUARE] $\therefore$ I IS THE MATRIX IDENTITY OF DIM M

$A^{-1}A = AA^{-1} = I$ BY DEFINITION, $\therefore$ ONLY SQUARE MATRICES HAVE INVERSES

THE COMPUTATION OF A MATRIX INVERSE WILL BE LEFT UNSPECIFIED

TRANSPOSE

FOR $A' = B$, $b_{ij} = a_{ji}$ IN PARTICULAR $\begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_N \end{pmatrix}' = (\alpha_1 \ldots \alpha_N)$ NOTE $(A')' = A$

NOTE ALSO THAT FOR VECTORS X, Y, $X'Y$ = THE EUCLIDEAN INNER PRODUCT OF X WITH Y

THUS $AB = C = (c_{ij})$ WHERE c_{ij} = (ith ROW OF A)(jth COLUMN OF B) THE MATRIX PRODUCT OF A ROW VECTOR TIMES A COLUMN VECTOR, ALSO THE INNER PRODUCT

$A_{M\times N} \Rightarrow \underset{N\times M}{A'}$