LINEAR MODELS AND DECISIONS

$X \sim N(\mu, \sigma^2) \qquad aX \sim N(a\mu, a^2\sigma^2) \qquad X+a \sim N(\mu+a, \sigma^2) \qquad Y \sim N(\lambda, \xi^2) \qquad X+Y \sim N\left(\mu+\lambda, \sigma^2+\xi^2+2\,\mathrm{Cov}(X,Y)\right)$

$\dfrac{X-\mu}{\sqrt{\sigma^2}} \sim N(0,1) \qquad \phi_i \sim N(0,1) \qquad \sum_{i=1}^{N} \phi_i^2 \sim \chi^2_{(N)} \qquad \phi_i$'s UNCORRELATED

$E\left(\chi^2_{(N)}\right) = N \qquad V\left(\chi^2_{(N)}\right) = 2N \qquad E\left(\dfrac{\chi^2_{(N)}}{N}\right) = 1 \qquad V\left(\dfrac{\chi^2_{(N)}}{N}\right) = 2/N$

$$t_{(N)} = \frac{\phi}{\sqrt{\chi^2_{(N)}/N}} \qquad t_{(\infty)} = \phi$$

$$F_{(M,N)} = \frac{\chi^2_{(M)}/M}{\chi^2_{(N)}/N} \qquad F_{(1,N)} = \frac{\chi^2_{(1)}/1}{\chi^2_{(N)}/N} = \frac{\phi^2}{\chi^2_{(N)}/N} = t^2_{(N)}$$

$\bar{x} = \dfrac{1}{N}\sum_{i=1}^{N} x_i \qquad E(\bar{x}) = \dfrac{1}{N}\sum_{i=1}^{N} E(x_i) = \dfrac{1}{N} N\mu = \mu \qquad$ FOR x_i's $\sim N(\mu, \sigma^2)$

$V(\bar{x}) = \dfrac{1}{N^2}\sum_{i=1}^{N} V(x_i)$ [FOR x_i's INDEPENDENT] $= \dfrac{1}{N^2} N\sigma^2 = \sigma^2/N$

$S^2 = \dfrac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^2 = \dfrac{\sigma^2}{N-1}\sum_{i=1}^{N}\dfrac{(x_i-\bar{x})^2}{\sigma^2} = \dfrac{\sigma^2}{N-1}\sum_{i=1}^{N}\left(\dfrac{x_i-\bar{x}}{\sigma}\right)^2 = \dfrac{\sigma^2}{N-1}\sum_{i=1}^{N}\phi_i^2 \sim \dfrac{\sigma^2}{N-1}\chi^2_{(N-1)}$ [SINCE ONLY N−1 INDEPENDENT ϕ_i's]

$$E(S^2) = \frac{\sigma^2}{N-1} E\left(\chi^2_{(N-1)}\right) = \frac{\sigma^2}{N-1}(N-1) = \sigma^2$$

$$V(S^2) = \frac{(\sigma^2)^2}{(N-1)^2} V\left(\chi^2_{(N-1)}\right) = \frac{(\sigma^2)^2}{(N-1)^2}\,2(N-1) = \frac{2(\sigma^2)^2}{N-1}$$

$$\frac{\bar{x}-\mu}{\sqrt{S^2/N}} = \frac{\dfrac{\bar{x}-\mu}{\sqrt{\sigma^2}}}{\dfrac{\sqrt{S^2/N}}{\sqrt{\sigma^2}}} = \frac{\dfrac{\bar{x}-\mu}{\sqrt{\sigma^2}}}{\sqrt{S^2/N\sigma^2}} = \frac{\dfrac{\bar{x}-\mu}{\sqrt{\sigma^2}/\sqrt{N}}}{\sqrt{N}\sqrt{S^2/N\sigma^2}} = \frac{\dfrac{\bar{x}-\mu}{\sqrt{\sigma^2/N}}}{\sqrt{NS^2/N\sigma^2}} = \frac{\dfrac{\bar{x}-\mu}{\sqrt{\sigma^2/N}}}{\sqrt{S^2/\sigma^2}}$$

$$= \frac{\dfrac{\bar{x}-E(\bar{x})}{\sqrt{V(\bar{x})}}}{\sqrt{S^2/\sigma^2}} = \frac{\phi}{\sqrt{S^2/\sigma^2}} = \frac{\phi}{\sqrt{\dfrac{\sigma^2}{N-1}\chi^2_{(N-1)}/\sigma^2}} = \frac{\phi}{\sqrt{\dfrac{\sigma^2\chi^2_{(N-1)}}{\sigma^2(N-1)}}}$$

$$= \frac{\phi}{\sqrt{\chi^2_{(N-1)}/N-1}} = t_{(N-1)}$$