STATISTICAL DECISION THEORY - II LINEAR MODELS AND DECISIONS

BEST DECISION RULE = RULE THAT HAS THE SMALLEST RISK NO MATTER WHAT THE TRUE STATE OF NATURE

SITUATIONS IN WHICH A BEST DECISION RULE EXISTS ARE RARE AND UNINTERESTING

e.g., $d=\theta$ WILL BE BETTER THAN ANY OTHER d IF $\theta \in \Theta$ IS, IN FACT, THE STATE OF NATURE, BUT WILL BE WORSE THAN $d=\theta_1$ IF $\theta_1 \in \Theta$ IS THE STATE OF NATURE. $\therefore \nexists$ A 'UNIFORMLY BEST' $d \in D$

$D^* = \{\delta \mid \delta$ A RANDOMIZED DECISION RULE, i.e. $\delta : \mathfrak{X} \rightarrow$ SPACE OF PROBABILITY DISTRIBUTIONS OVER $D\}$

RANDOMIZED DECISION RULES ARE NOT THE KIND OF RULES GENERALLY DESIRED, BUT THEY FREQUENTLY MAKE THE MATHEMATICS SIMPLER, $\therefore$ MANY DEFINITIONS AND THEOREMS ARE STATED IN TERMS OF THEM

NOTE: $D \subset D^*$ $\qquad R(\theta,\delta) = E[R(\theta,Z)]$ Z RANDOM OVER D WITH DISTRIBUTION δ

IF $\nexists$ A BEST DECISION RULE, THEN WE WOULD LIKE ONE THAT IS OPTIMAL IN SOME SENSE

$\exists$ TWO GENERAL APPROACHES:

1) RESTRICT AVAILABLE RULES BY SOME CRITERIA OF OPTIMALITY, LOOK FOR UNIFORMLY BEST RISK WITHIN RESTRICTED CLASS

2) ORDER AVAILABLE RULES BY SOME PRINCIPLE OF OPTIMALITY, PICK BEST ACCORDING TO ORDERING

A NATURAL SEQUENCING:

δ_1 IS AS GOOD AS δ_2 iff $R(\theta,\delta_1) \leq R(\theta,\delta_2)\ \forall \theta \in \Theta$

δ_1 IS BETTER THAN δ_2 iff $R(\theta,\delta_1) \leq R(\theta,\delta_2)\ \forall \theta \in \Theta$ AND $\exists \theta_0 \in \Theta \ni R(\theta_0,\delta_1) < R(\theta_0,\delta_2)$

δ_1 IS EQUIVALENT TO δ_2 iff $R(\theta,\delta_1) = R(\theta,\delta_2)\ \forall \theta \in \Theta$

δ IS ADMISSABLE iff $\nexists\ \delta_0$ BETTER THAN δ $\qquad$ OTHERWISE INADMISSABLE

A NATURAL RESTRICTION:

$C \subset D^*$ IS (ESSENTIALLY) COMPLETE iff FOR EACH $\delta \in D^*$, $\delta \notin C$, $\exists\ \delta_0 \in C$ (AS GOOD AS) BETTER THAN δ

TH: C COMPLETE $\Rightarrow \{$ADMISSABLE RULES$\} \subset C$

TH: C ESSENTIALLY COMPLETE, $\delta \notin C$, δ ADMISSABLE $\Rightarrow \exists\ \delta' \in C$, δ' EQUIVALENT TO δ

C IS MINIMAL (ESSENTIAL) COMPLETE iff C IS (ESSENTIALLY) COMPLETE, AND NO PROPER SUBSET OF C IS (ESSENTIALLY) COMPLETE

TH: IF A MINIMAL COMPLETE CLASS EXISTS, IT $= \{$ADMISSABLE RULES$\}$

[NOTE THAT THESE DEFINITIONS DO NOT REQUIRE A 1-1 CORRESPONDENCE BETWEEN A AND Θ]