DECISION THEORY UNDER MIXED RISK AND UNCERTAINTY

LINEAR MODELS AND DECISIONS

DATA COLLECTION, EXPERIMENTATION (LUCE & RAIFFA)

POSSIBLE STATES OF NATURE $\{\theta_i\}$

POSSIBLE ACTIONS $\{A_i\}$

POSSIBLE CONSEQUENCES $\{C_i\}$ $C_{ij} = A_i(\theta_j)$

UTILITIES OF CONSEQUENCES $\{U_i\}$ $U_{ij} = U(C_{ij}) = U(A_i(\theta_j)) = U\langle A_i, \theta_j\rangle$

POSSIBLE DATA COLLECTION STRATEGIES (EXPERIMENTS) $\{E_i\}$

POSSIBLE OUTCOMES OF A GIVEN EXPERIMENT E_i $\{O_{ji}\}$

KNOWN PROBABILITY DISTRIBUTIONS $\{P_i(O_{ji} \mid \theta_k)\}$ FOR EACH E_i $[P(O_j \mid \theta_k, E_i)]$

POSSIBLE DECISION RULES $\{D_i\}$ WITH $A_{ijk} = D_i(O_{jk})$ - A GIVEN D_i POSSIBLY RESTRICTED TO PARTICULAR E_j'S

UTILITY OF AN $\langle E_i, D_j\rangle$ PAIR GIVEN θ_k IS

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\begin{aligned}
U\langle E_i, D_j\rangle &= \sum_m P_i(O_{mi} \mid \theta_k)\, U\langle D_j(O_{mi}), \theta_k\rangle \\
&= \sum_m P_i(D_j(O_{mi}) \mid \theta_k)\, U\langle D_j(O_{mi}), \theta_k\rangle \\
&= \sum_m P_i(A_{jmi} \mid \theta_k)\, U\langle A_{jmi}, \theta_k\rangle \\
&= \sum_m P_i(C_{jmik})\, U(C_{jmik})
\end{aligned}
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THE PROBLEM THUS REDUCES TO DECISION UNDER UNCERTAINTY, WITH $\langle E, D\rangle$ PAIRS AS CHOICES AND NATURE (S's) AS A NEUTRAL PLAYER

BAYESIAN INFERENCE ASSIGNS PRIOR PROBABILITIES OVER $\{\theta_i\}$ AND THUS REDUCES TO DECISION MAKING UNDER RISK