

LINEAR MODELS AND DECISIONS

$$\begin{aligned}
 1) \quad A'BXC'D &= Y \\
 A'BXC' &= YD^{-1} \\
 A'BX &= YD^{-1}C'^{-1} \\
 BX &= A'^{-1}YD^{-1}C'^{-1} \\
 X &= B^{-1}A'^{-1}YD^{-1}C'^{-1} = (A'B)^{-1}Y(C'D)^{-1}
 \end{aligned}$$

~~$$\begin{aligned}
 2) \quad A'BXC'D &= Y \\
 A'BX &= Y(C'D)^{-1} \\
 X &= (A'B)^{-1}Y(C'D)^{-1}
 \end{aligned}$$~~

CANNOT BE FURTHER REDUCED SINCE A, B, C, D NOT INDIVIDUALLY INVERTIBLE

$$3) \quad WW = X(X'X)^{-1}X'X(X'X)^{-1}X' = X(X'X)^{-1}IX' = X(X'X)^{-1}X' = W$$

$$4) \quad (I-W)W = W - WW = W - W = 0$$

$$5) \quad (I-W)(I-W) = I - W - W + WW = I - W - W + W = I - W$$

$$\begin{aligned}
 6) \quad W^* &= X^*(X^*X^*)^{-1}X^{*'} = XM \left[ (XM)'(XM) \right]^{-1} (XM)' = XM (M'X'XM)^{-1} M'X' \\
 &= XMM^{-1}(X'X)^{-1}M^{-1}M'X' = XI(X'X)^{-1}IX' = X(X'X)^{-1}X' = W
 \end{aligned}$$

$$7) \quad AA^{\#}A = (FR')R(R'R)^{-1}(F'F)^{-1}F'(FR') = F(R'R)(R'R)^{-1}(F'F)^{-1}(F'F)R' = FII R' = FR' = A$$

$$\begin{aligned}
 8) \quad A^{\#}AA^{\#} &= R(R'R)^{-1}(F'F)^{-1}F'(FR')R(R'R)^{-1}(F'F)^{-1}F' = R(R'R)^{-1}(F'F)^{-1}(F'F)(R'R)(R'R)^{-1}(F'F)^{-1}F' \\
 &= R(R'R)^{-1}II(F'F)^{-1}F' = R(R'R)^{-1}(F'F)^{-1}F' = A^{\#}
 \end{aligned}$$

$$\begin{aligned}
 9) \quad A'AX &= A'AA^{\#}Y = (FR')(FR')R(R'R)^{-1}(F'F)^{-1}F'Y = (RF')(FR')R(R'R)^{-1}(F'F)^{-1}F'Y \\
 &= R(F'F)(R'R)(R'R)^{-1}(F'F)^{-1}F'Y = R(F'F)I(F'F)^{-1}F'Y = R(F'F)(F'F)^{-1}F'Y \\
 &= RIF'Y = RF'Y = (FR')'Y = A'Y
 \end{aligned}$$

$$\begin{aligned}
 10) \quad \hat{E}'\hat{Q} &= (Y-\hat{Q})'\hat{Q} = (Y-AX)'AX = (Y-AA^{\#}Y)'AA^{\#}Y = (Y-Y'A^{\#}A)AA^{\#}Y \left[ = Y'(I-A^{\#}A)AA^{\#}Y \right] \\
 &= Y'AA^{\#}Y - Y'A^{\#}A'AA^{\#}Y = Y'(AA^{\#}-A^{\#}A'AA^{\#})Y \\
 &= Y' \left[ (FR')R(R'R)^{-1}(F'F)^{-1}F' - \left[ R(R'R)^{-1}(F'F)^{-1}F' \right]'(FR')(FR')R(R'R)^{-1}(F'F)^{-1}F' \right] Y \\
 &= Y' \left[ F(R'R)(R'R)^{-1}(F'F)^{-1}F' - F(F'F)^{-1}(R'R)^{-1}R'RF'FR'R(R'R)^{-1}(F'F)^{-1}F' \right] Y \\
 &= Y' \left[ FI(F'F)^{-1}F' - F(F'F)^{-1}I F'F I (F'F)^{-1}F' \right] Y \\
 &= Y' \left[ F(F'F)^{-1}F' - F(F'F)^{-1}(F'F)(F'F)^{-1}F' \right] Y = Y' \left[ F(F'F)^{-1}F' - F(F'F)^{-1}I F' \right] Y \\
 &= Y'(0)Y = 0
 \end{aligned}$$