LINEAR MODELS AND DECISIONS

15) (CONTINUED)

$$V(\hat{\varepsilon}) = E[\hat{\varepsilon}\hat{\varepsilon}'] - E(\hat{\varepsilon})E(\hat{\varepsilon})' = E[\hat{\varepsilon}\hat{\varepsilon}'] - 0 = E[\hat{\varepsilon}\hat{\varepsilon}']$$

$$= E[(y-\hat{y})(y-\hat{y})'] = E[yy' - \hat{y}y' - y\hat{y}' + \hat{y}\hat{y}'] = E[yy'] - E[\hat{y}y'] - E[y\hat{y}'] + E[\hat{y}\hat{y}']$$

$$= E[yy'] - E[X(X'X)^{-1}X'yy'] - E[yy'X(X'X)^{-1}X'] + E[X(X'X)^{-1}X'yy'X(X'X)^{-1}X']$$

$$= E[yy'] - X(X'X)^{-1}X'E[yy'] - E[yy']X(X'X)^{-1}X' + X(X'X)^{-1}X'E[yy']X(X'X)^{-1}X'$$

$$= X\beta\beta'X' + V(\varepsilon) - X(X'X)^{-1}X'[X\beta\beta'X' + V(\varepsilon)] - [X\beta\beta'X' + V(\varepsilon)]X(X'X)^{-1}X' + X(X'X)^{-1}X'[X\beta\beta'X' + V(\varepsilon)]X(X'X)^{-1}X'$$

$$= X\beta\beta'X' + V(\varepsilon) - X(X'X)^{-1}X'X\beta\beta'X' - X(X'X)^{-1}X'V(\varepsilon) - X\beta\beta'X'X(X'X)^{-1}X' - V(\varepsilon)X(X'X)^{-1}X' + X(X'X)^{-1}X'X\beta\beta'X'X(X'X)^{-1}X'$$
$$+ X(X'X)^{-1}X'V(\varepsilon)X(X'X)^{-1}X'$$

$$= X\beta\beta'X' + V(\varepsilon) - X\beta\beta'X' - X(X'X)^{-1}X'V(\varepsilon) - X\beta\beta'X' - V(\varepsilon)X(X'X)^{-1}X' + X\beta\beta'X' + X(X'X)^{-1}X'V(\varepsilon)X(X'X)^{-1}X'$$

$$= V(\varepsilon) - X(X'X)^{-1}X'V(\varepsilon) - V(\varepsilon)X(X'X)^{-1}X' + X(X'X)^{-1}X'V(\varepsilon)X(X'X)^{-1}X'$$

$$= \sigma^2 I - X(X'X)^{-1}X'\sigma^2 I - \sigma^2 I X(X'X)^{-1}X' + X(X'X)^{-1}X'\sigma^2 I X(X'X)^{-1}X'$$
$$= \sigma^2 I - \sigma^2 X(X'X)^{-1}X' - \sigma^2 X(X'X)^{-1}X' + \sigma^2 X(X'X)^{-1}X'X(X'X)^{-1}X'$$
$$= \sigma^2 I - \sigma^2 X(X'X)^{-1}X' - \sigma^2 X(X'X)^{-1}X' + \sigma^2 X(X'X)^{-1}X'$$
$$= \sigma^2 I - \sigma^2 X(X'X)^{-1}X'$$
$$= V(\varepsilon) - V(\hat{y})$$
$$= V(y) - V(\hat{y}) \qquad \therefore \quad V(y) = V(\hat{y}) + V(\hat{\varepsilon})$$

THIS COULD HAVE BEEN APPROACHED AS: (THOUGH ONLY WITH HINDSIGHT)

$$y = \hat{y} + \hat{\varepsilon} \quad \therefore \quad V(y) = V(\hat{y}) + V(\hat{\varepsilon}) + 2\,Cov(\hat{y}, \hat{\varepsilon})$$
$$= V(\hat{y}) + V(\hat{\varepsilon}) + 0$$
$$= V(\hat{y}) + V(\hat{\varepsilon})$$

WHERE $Cov(\hat{y}, \hat{\varepsilon}) = 0$ REQUIRES ITS OWN (SOMEWHAT SHORTER) DERIVATION