STATISTICAL DECISION THEORY - VIII LINEAR MODELS AND DECISIONS

OPTIMAL DECISION RULES ARE NOT NECESSARILY AVAILABLE, OR MAY BE DIFFICULT TO COMPUTE. DECISION RULES WHICH ARE NOT OPTIMAL, BUT ARE BETTER THAN PURE GUESSES, MIGHT BE CALLED 'REASONABLE'. A RULE MIGHT BE REASONABLE IN THE SENSE OF BEING QUICK AND EASY TO COMPUTE, OR IN THE SENSE OF BEING EXPLAINABLE AND DEFENSIBLE TO AN EMPLOYER, BUT THE MOST IMPORTANT CLASS OF REASONABLE RULES ARE THOSE JUSTIFIED BY LARGE SAMPLE PROPERTIES, THAT IS, BY DECISION RULES THAT HAVE NICE PROPERTIES IN THE LIMIT AS THE SAMPLE SIZE GOES TO ∞. NOTE THAT SUCH PROPERTIES DO NOT NECESSARILY CORRESPOND TO NICE PROPERTIES FOR SMALL SAMPLES.

AMONG THE LARGE SAMPLE JUSTIFIED METHODS, WE WILL CONSIDER TWO:

ESTIMATION — MAXIMUM LIKLIHOOD

LIKLIHOOD FUNCTION — THE LIKLIHOOD FUNCTION OF A RANDOM VECTOR $\underline{X}$ IS THE PROBABILITY DENSITY FUNCTION $f(\underline{x}|\underline{\theta})$ CONSIDERED AS A FUNCTION OF $\underline{\theta}$, $L(\underline{\theta}) = f(\underline{x}|\underline{\theta})$, THAT IS, $L(\theta)$ = THE PROBABILITY OF $\underline{X}$ GIVEN $\underline{\theta}$.

MAXIMUM LIKLIHOOD ESTIMATOR — $\hat{\underline{\theta}} = d(\underline{x})$ IS A MAXIMUM LIKLIHOOD ESTIMATOR OF $\underline{\theta}$ iff $L(\hat{\theta}) = \max_{\theta \in \Theta} L(\theta)$, i.e., IF IT MAXIMIZES THE LIKLIHOOD OF $\underline{\theta}$, OR THE PROBABILITY OF $\underline{X}$ GIVEN $\underline{\theta}$

HYPOTHESIS TESTING — LIKLIHOOD RATIO

THE GENERALIZED LIKLIHOOD RATIO IS

$$\lambda = \frac{\max_{\theta \in \Theta_0} L(\theta)}{\max_{\theta \in \Theta} L(\theta)}$$

i.e., THE RATIO OF THE MAXIMUM LIKLIHOOD ATTAINABLE IN Θ_0 (ACCORDING TO H_0) TO THE MAXIMUM LIKLIHOOD ATTAINABLE IN THE WHOLE PARAMETER SPACE Θ

$$= L(\hat{\theta}')/L(\hat{\theta})$$

WITH $\hat{\theta}$ = MAX LIKLIHOOD ESTIMATOR FOR Θ, $\hat{\theta}'$ = MAX LIKLIHOOD ESTIMATOR FOR Θ ASSUMING H_0

$0 \le \lambda \le 1$ WITH λ NEAR 1 CORRESPONDING TO LITTLE IMPROVEMENT IN LIKLIHOOD FROM REJECTING H_0, $\therefore$ LITTLE REASON TO REJECT H_0

REJECT H_0 iff THE SAMPLE VALUE OF λ SATISFIES $\lambda \le \lambda_0$ WITH $\int_0^{\lambda_0} g(\lambda)\,d\lambda = \alpha$ = PROB TYPE ONE ERROR, ASSUMING PROBABILITY DENSITY FUNCTION OF λ, $g(\lambda)$, IS KNOWN AND DOES NOT DEPEND ON UNKNOWN PARAMETERS

UNDER CERTAIN REGULARITY CONDITIONS ON THE DISTRIBUTION OF X (RATHER GENERAL CONDITIONS), WHEN H_0 TRUE

$$\lim_{n\to\infty} \left[\text{DISTRIBUTION}(-2\ln\lambda)\right] = \chi^2_{(t)}$$

WITH t = THE NUMBER OF CONSTRAINTS H_0 IMPOSES ON Θ = NUMBER OF PARAMETERS FIXED BY H_0 $[\theta_i = \theta_i^0]$ IF H_0 IS OF THAT FORM

$\therefore$ REJECT H_0 IF $-2\ln\lambda > \chi_0^2$ WHERE $P\{\chi^2 > \chi_0^2\} = \alpha$ = PROBABILITY OF TYPE I ERROR

FOR $\Theta = \Theta_1 \cup \Theta_2$, $\Theta_1 = \{\theta_1\}$, $\Theta_2 = \{\theta_2\}$, CALLED SIMPLE HYPOTHESES,

{LIKLIHOOD RATIO TESTS} = {BAYES TESTS} = {ADMISSABLE TESTS}

NON-SIMPLE (COMPOSITE) HYPOTHESES ARE NOT SO TRACTABLE

NOTE THAT IF THE DISTRIBUTION OF A MONOTONIC FUNCTION OF λ IS KNOWN, THEN THAT FUNCTION MAY BE USED INSTEAD OF λ AS A TEST STATISTIC