9 OCT 76-6

STATISTICAL DECISION THEORY - VI LINEAR MODELS AND DECISIONS

HYPOTHESIS TESTING

$A = \{A_0, A_1\}$

d A NONRANDOMIZED DECISION RULE $\Rightarrow d^{-1}(A_1) = \omega \subset \mathfrak{X}$ AND $d^{-1}(A_0) = \omega^c = \mathfrak{X} - \omega$

THAT IS, d IS COMPLETELY DETERMINED BY A REGION $\omega \subset \mathfrak{X}$, ω IS CALLED A TEST, OR A CRITICAL REGION

$P_\theta\{\omega\}$, THE POWER FUNCTION CORRESPONDING TO THE TEST ω, = THE PROBABILITY THAT X WILL FALL IN ω GIVEN θ

$$R(\theta,\omega) = (1 - P_\theta\{\omega\})L(\theta,A_0) + P_\theta\{\omega\}L(\theta,A_1)$$
$$= L(\theta,A_0) + P_\theta\{\omega\}[L(\theta,A_1) - L(\theta,A_0)]$$

CLASSICALLY, THE LOSS FUNCTION IS ASSUMED TO HAVE A SPECIAL FORM:

FOR SOME Θ_0, Θ_1 WITH $\Theta_0 \cap \Theta_1 = \phi$ AND $\Theta = \Theta_0 \cup \Theta_1$,

$$L(\theta,A_0) = \begin{cases} 1 & \text{IF } \theta \in \Theta_1 \\ 0 & \text{IF } \theta \in \Theta_0 \end{cases}$$
$$L(\theta,A_1) = \begin{cases} 1 & \text{IF } \theta \in \Theta_0 \\ 0 & \text{IF } \theta \in \Theta_1 \end{cases}$$

IN SUCH A FRAMEWORK, THE TERMINOLOGY HAS ARISEN OF CALLING $H_0: \theta \in \Theta_0$ THE NULL HYPOTHESIS AND $H_1: \theta \in \Theta_1$ THE ALTERNATIVE HYPOTHESIS. ONE OF THESE DISJOINT HYPOTHESES IS TRUE, WITH A LOSS OF ONE FOR AN INCORRECT SELECTION, AND A LOSS OF ZERO FOR A CORRECT CHOICE. A_0 IS CONSIDERED AS THE ACTION "ACCEPT H_0" AND A_1 THE ACTION "ACCEPT H_1" OR "REJECT H_0"

REJECT H_0 WHEN TRUE = TYPE I ERROR ACCEPT H_0 WHEN FALSE = TYPE II ERROR

IN SUCH CIRCUMSTANCES, RANDOMIZED DECISION RULES ARE MESSY, BUT THEY CAN BE REPLACED BY THE EQUIVALENT CLASS OF BEHAVIORAL DECISION RULES $\{\phi \mid \phi: \mathfrak{X} \to [0,1]$, WITH ACTION A_1 TAKEN WITH PROBABILITY $\phi(x)$ AND ACTION A_0 WITH PROBABILITY $(1-\phi(x))$, FOR $X = x \in \mathfrak{X}\}$

$\phi(x) = \begin{cases} 1 & \text{IF } x \in \omega \\ 0 & \text{IF } x \notin \omega \end{cases}$ CORRESPONDS TO A NONRANDOMIZED RULE

$$R(\theta,\phi) = L(\theta,A_0) + E_\theta[\phi(X)][L(\theta,A_1) - L(\theta,A_0)]$$

A TEST ϕ OF $H_0: \theta \in \Theta_0$ AGAINST $H_1: \theta \in \Theta_1$ IS SAID TO HAVE SIZE α IF $\sup_{\theta \in \Theta_0} E_\theta[\phi(X)] = \alpha$ [= PROB TYPE I ERROR]

[NOTE IMPLICIT AND CRUDE SUBJECTIVE CONSIDERATIONS OF LOSS FUNCTION IN CHOICE OF α]

A TEST ϕ_0 IS SAID TO BE UNIFORMLY MOST POWERFUL (UMP) OF SIZE α FOR TESTING $H_0: \theta \in \Theta_0$ AGAINST $H_1: \theta \in \Theta_1$ IF ϕ_0 IS OF SIZE α AND IF FOR ANY OTHER TEST ϕ OF SIZE AT MOST α

$$E_\theta[\phi_0(X)] \geq E_\theta[\phi(X)] \quad \text{FOR } \forall \theta \in \Theta_1$$ [i.e., MINIMUM β = PROB TYPE II ERROR]

[POWER OF A TEST = $1-\beta$]

THE NEYMAN-PEARSON LEMMA PROVIDES A WAY OF FINDING UMP TESTS IN THE SPECIAL CASE $\Theta = \{\theta_1, \theta_2\}$, AND IS SOMETIMES USEFUL FOR THE GENERAL CASE, BUT, GENERALLY, UMP TESTS EXIST ONLY IN SPECIAL CIRCUMSTANCES

ϕ A TEST OF SIZE α OF $H_0: \theta \in \Theta_0$ AGAINST $H_1: \theta \in \Theta_1$ IS UNBIASED IF $E_\theta[\phi(X)] \geq \alpha \ \forall \theta \in \Theta_1$

i.e., IF THE PROBABILITY OF REJECTING H_0 WHEN FALSE IS NEVER SMALLER THAN THE PROBABILITY OF REJECTING H_0 WHEN TRUE

UMP UNBIASED TESTS OFTEN DO EXIST [THAT IS, UMP AMONG UNBIASED TESTS]