STATISTICAL DECISION THEORY – V LINEAR MODELS AND DECISIONS

IT IS FREQUENTLY REASONABLE TO MAKE ASSUMPTIONS ABOUT THE FORM OF THE LOSS FUNCTION, AND OFTEN THIS IS SUFFICIENT TO SPECIFY A BEST DECISION RULE

FOR EXAMPLE, FOR THE PROBLEM OF ESTIMATING THE VALUE OF θ:

1) IF $L(\theta, A) = c(\theta - A)^2$ [SQUARED ERROR LOSS], THE BAYES DECISION RULE $\hat{\theta}$ = MEAN (OF POSTERIOR DISTRIBUTION OF θ GIVEN THE OBSERVATIONS)

2) IF $L(\theta, A) = c|\theta - A|$ [ABSOLUTE ERROR LOSS], THE BAYES DECISION RULE $\hat{\theta}$ = MEDIAN (")

3) IF $L(\theta, A) = \begin{cases} 0 \text{ IF } \theta = A \\ M \text{ IF } \theta \neq A \end{cases}$ [ALL OR NOTHING LOSS], THE BAYES DECISION RULE $\hat{\theta}$ = MODE (")

A SQUARED ERROR LOSS FUNCTION IS THE MOST COMMON ASSUMPTION

CORRESPONDINGLY, A DESIRABLE PROPERTY IN ESTIMATORS (DECISION RULES FOR ESTIMATION PROBLEMS) IS THAT IT HAVE MINIMUM MEAN SQUARE ERROR, i.e., THAT $E[(\hat{\theta} - \theta)^2]$ BE A MINIMUM FOR ALL ESTIMATORS OF θ

MINIMUM MEAN SQUARE ERROR ESTIMATORS, HOWEVER, RARELY EXIST, AND THUS ALTERNATIVE CRITERIA MUST NORMALLY BE USED. IT IS FREQUENTLY ASSUMED THAT A REASONABLE CONSTRAINT ON A CLASS OF DECISION RULES (ESTIMATORS) IS THAT THEY BE UNBIASED: $E(\hat{\theta}) = \theta$

AS AN APPROXIMATION TO MIN MEAN SQUARE ERROR, IT IS FREQUENTLY FURTHER ASKED THAT AN ESTIMATOR HAVE MINIMUM VARIANCE WITHIN THE CLASS OF UNBIASED ESTIMATORS:

1) $E(\hat{\theta}) = \theta$

2) $V(\hat{\theta}) \leq V(\theta^*)$, $\forall$ UNBIASED θ^*

$$E[(\hat{\theta} - \theta)^2] = \underbrace{(E(\hat{\theta}) - \theta)^2}_{\text{BIAS}} + V(\hat{\theta})$$

MVU (MIN VAR UNBIASED) ESTIMATORS FREQUENTLY EXIST, WHILE MIN MSE (MEAN SQ ERROR) ESTIMATORS RARELY DO, BUT MVU ESTIMATORS ARE NOT AS DESIRABLE AS MIN MSE ESTIMATORS, AND CAN, IN FACT, IN CERTAIN CIRCUMSTANCES LEAD TO ABSURD RESULTS

ANOTHER CONSTRAINT THAT CAN BE IMPOSED ON A CLASS OF DECISION RULES IS THAT THEY BE INVARIANT

THE BASIC IDEA IS THAT THERE MAY BE CERTAIN STRUCTURES IN θ AND IN A SUCH THAT THE LOSS FUNCTION IS CONSTANT ACROSS CORRESPONDING PARTS OF THESE STRUCTURES

IF SO, A DECISION RULE IS INVARIANT FOR THAT DECISION PROBLEM INSOFAR AS IT PRESERVES THAT STRUCTURE IN A. NOTE: $d: X \to A$ ∴ THE STRUCTURE IN θ MUST ALSO BE MAINTAINED IN P_θ, THE POSSIBLE PROBABILITY DISTRIBUTIONS OF X GIVEN ON θ

NOTE THAT WITH A SQUARED ERROR LOSS FUNCTION, UNBIASED ESTIMATORS BASED ON SUFFICIENT STATISTICS WILL AUTOMATICALLY HAVE LESS VARIANCE (FROM THEIR BEING AS GOOD AS ANY OTHER ESTIMATES) THAN ANY NOT BASED ON THE SUFFICIENT STATISTICS – IF THE PROBABILITY DENSITY OF THE SUFFICIENT STATISTIC IS [COMPLETE], THEN THERE IS ONLY ONE UNBIASED ESTIMATE BASED ON THE SUFFICIENT STATISTICS, AND ∴ IT IS MVU