

## STATISTICAL DECISION THEORY - IV LINEAR MODELS AND DECISIONS

THE INTERACTION OF AN ACTION WITH A STATE OF NATURE TO YIELD SOME CONSEQUENCE WITH A PARTICULAR COST (UTILITY) DEPENDS IN GENERAL ON THE NATURE OF THE WORLD - EXTREMELY DIFFICULT TO FORMALLY INCORPORATE INTO THE DECISION FRAMEWORK. THEREFORE,  $A$  IS FREQUENTLY RESTRICTED TO THOSE ACTIONS WHICH ARE CONSISTED BY A CHOICE OF A STATE (OR SET OF STATES) OF NATURE, LEAVING THE DECISION CONCERNING THE BEST ACTION GIVEN THAT STATE OF NATURE IN THE REALM OF SUBJECTIVE DECISION MAKING. I.E.,  $A$  IS IN 1 TO 1 CORRESPONDENCE WITH  $\theta$ ,  $\therefore d \in D \Rightarrow d: X \rightarrow \theta$

NOTE THAT THIS ELIMINATES SOME IMPORTANT KINDS OF ACTIONS (E.G., 'OVERALL BEST', BUT NEVER BEST FOR A PARTICULAR STATE OF NATURE), BUT THAT THIS CAN BE SOMETIMES AVOIDED BY DEFINING  $\theta$  SO AS TO CORRESPOND TO  $A$

$\theta$  IS GENERALLY ASSUMED TO HAVE A PARAMETERIZABLE STRUCTURE THAT IS, EACH  $\theta \in \theta$  IS ASSUMED TO BE SPECIFIABLE IN TERMS OF THE VALUES OF SOME SET OF PARAMETERS WHICH TAKE REAL VALUES -  $\theta$  IS THUS ASSUMED TO BE ISOMORPHIC TO A VECTOR SPACE OVER THE REALS, WITH VECTORS AS COLUMNS OF PARAMETER VALUES

[IN SOME CIRCUMSTANCES, THIS MIGHT BE JUST A 1-1 CORRESPONDENCE RATHER THAN AN ISOMORPHISM]

STATISTICAL DECISION PROBLEMS ARE CLASSIFIED ACCORDING TO THE SIZE OF  $A$ :

$|A|$  INFINITE - AN ESTIMATION PROBLEM

$2 < |A| < \infty$  - A MULTIPLE DECISION PROBLEM (INFREQUENTLY DEALT WITH)

$|A| = 2$  - A HYPOTHESIS TESTING PROBLEM

MANY CONCEPTS AND THEOREMS ARE SPECIFIC TO A PARTICULAR PROBLEM TYPE

### ESTIMATION - SUFFICIENT STATISTICS

FOR  $X$  A RANDOM VARIABLE (VECTOR) WITH DISTRIBUTION DEPENDENT ON A PARAMETER VECTOR  $\theta \in \theta$ , A REAL (OR VECTOR) VALUED FUNCTION  $T$  OF  $X$  IS SUFFICIENT FOR  $\theta$  IF THE DISTRIBUTION OF  $X|T=t$  IS INDEPENDENT OF  $\theta$  - I.E. IF  $T$  CONTAINS ALL THE INFORMATION IN  $X$  ABOUT  $\theta$

TH: LET  $X$  BE A RANDOM VARIABLE WHOSE PROBABILITY MASS (DENSITY) FUNCTION  $f(x|\theta)$  DEPENDS ON  $\theta \in \theta$ .

A FUNCTION  $T=t(x)$  IS SUFFICIENT FOR  $\theta$  IFF  $f(x|\theta)$  FACTORS INTO A PRODUCT OF A FUNCTION OF  $t(x)$  AND  $\theta$ , AND A FUNCTION OF  $x$  ALONE - I.E.  $f(x|\theta) = g(t(x), \theta)h(x)$

TH: (ROUGHLY)  $D_0 \subset D$ ,  $D_0$  BASED ON A SUFFICIENT STATISTIC  $T \Rightarrow D_0$  IS AN ESSENTIALLY COMPLETE CLASS (FOR THE RELEVANT GAME)

TH: ASSUME  $T$  SUFFICIENT FOR  $\theta$  (PLUS A FEW ADDITIONAL ASSUMPTIONS). FOR  $d(x)$  A NONRANDOMIZED DECISION RULE, THE NONRANDOMIZED DECISION RULE

$$\hat{\theta}(t) = E(d(x)|T=t)$$

BASED ON  $T$ , IS AS GOOD AS  $d$