

## STATISTICAL DECISION THEORY - III LINEAR MODELS AND DECISIONS

TWO FUNDAMENTAL PRINCIPLES: BAYES AND MINIMAX

BAYES:  $\{\gamma \text{ A PRIOR DISTRIBUTION ON } \Theta\} = \Theta^*$

$r(\gamma, \delta) = E[R(T, \delta)]$  = BAYES RISK OF A DECISION RULE  $\delta$  WITH RESPECT TO A PRIOR DISTRIBUTION  $\gamma$ ,  $T$  A RANDOM VARIABLE OVER  $\Theta$  WITH DISTRIBUTION  $\gamma$   
 DISTRIBUTION OF  $T|X$  IS THE POSTERIOR DISTRIBUTION GIVEN THE OBSERVATIONS

A DECISION RULE  $\delta_0$  IS BAYES WITH RESPECT TO THE PRIOR DISTRIBUTION  $\gamma \in \Theta^*$  IF

$$r(\gamma, \delta_0) = \inf_{\delta \in D^*} r(\gamma, \delta)$$

MINIMAX

$\delta_0$  IS MINIMAX IF  $\sup_{\theta \in \Theta} R(\theta, \delta_0) = \inf_{\delta \in D^*} \sup_{\theta \in \Theta} R(\theta, \delta)$

$\gamma_0 \in \Theta^*$  IS LEAST FAVORABLE IF  $\inf_{\delta} r(\gamma_0, \delta) = \sup_{\gamma} \inf_{\delta} r(\gamma, \delta)$

i.e.  $\gamma_0$  IS LEAST FAVORABLE IF IT CORRESPONDS TO NATURE DOING ITS WORST

TH: IF A BAYES RULE WITH RESPECT TO A PRIOR DISTRIBUTION  $\gamma$  EXISTS,  $\exists$  A NONRANDOMIZED BAYES RULE WITH RESPECT TO  $\gamma$

TH: A BAYES DECISION RULE MINIMIZES THE POSTERIOR CONDITIONAL EXPECTED LOSS, GIVEN THE OBSERVATIONS

TH: MINIMAX RULES ARE BAYES WITH RESPECT TO LEAST FAVORABLE DISTRIBUTIONS

TH: IF A BAYES RULE WITH RESPECT TO A GIVEN PRIOR  $\gamma$  IS UNIQUE UP TO EQUIVALENCE, THEN IT IS ADMISSIBLE

TH:  $\Theta$  FINITE,  $[a$  FINITE,  $P_{\theta}(x)$  FOR EACH  $\theta \in \Theta$  HAS NO POINT MASSES]  $\Rightarrow$  THE CLASS OF NONRANDOMIZED RULES IS ESSENTIALLY COMPLETE

TH:  $\delta$  ADMISSIBLE,  $\Theta$  FINITE  $\Rightarrow \delta$  IS BAYES WITH RESPECT TO SOME PRIOR  $\gamma$

TH: FOR A DECISION PROBLEM  $(\Theta, D, R)$ ,  $\Theta$  FINITE, [THE RISK SET  $S$  BOUNDED FROM BELOW AND CLOSED FROM BELOW]

$\Rightarrow$  THE CLASS OF ALL BAYES RULES IS COMPLETE AND THE ADMISSIBLE BAYES RULES FORM A MINIMAL COMPLETE CLASS

- CAN BE EXTENDED TO INFINITE  $\Theta$  WITH A FEW ADDITIONAL ASSUMPTIONS

IN GENERAL, SUCH QUESTIONS ARE ASKED AS:

- 1) WHEN DO MINIMAX RULES EXIST?
- 2) ARE MINIMAX RULES ADMISSIBLE?
- 3) WHEN DO BAYES RULES EXIST?
- 4) WHEN ARE BAYES RULES ADMISSIBLE?
- 5) ARE ALL ADMISSIBLE RULES ALSO BAYES RULES?
- 6) WHEN DOES A COLLECTION OF BAYES RULES FORM A COMPLETE CLASS?
- 7) WHAT SUBSET OF A CLASS OF BAYES RULES FORMS A MINIMAL COMPLETE CLASS?
- 8) WHEN DOES A MINIMAL COMPLETE CLASS EXIST?