28 AUG 76

FERGUSON

Θ STATES OF NATURE

A AVAILABLE ACTIONS

$L: \Theta \times A \to R$ LOSS FUNCTION

GAME (Θ, A, L)

X RANDOM VARIABLE (VECTOR) w/ DISTRIBUTION DEPENDENT ON STATE OF NATURE $\mathcal{X}$ SAMPLE SPACE OVER WHICH OBSERVED VALUE OF X IT IS DEFINED

$d: \mathcal{X} \to A$ DECISION RULE $D = \{d\}$

$R(\theta, d) = E_\theta[L(\theta, d(X))]$ RISK FUNCTION - EXPECTED VALUE, ASSUMING $\theta \in \Theta$, OF LOSS FOR d

STATISTICAL DECISION PROBLEM GAME (Θ, D, R)

$A = \{A_1, A_2\}$ HYPOTHESIS TESTING $A = R^{EAL}$ POINT ESTIMATION

NOTE: INITIAL SUBORDINATE ACTION OF CHOICE OR 'STATE OF NATURE' - PARAMETER
A = CHOICES AMONG Θ

PREFERENCES AMONG PROB ROULETTE DIST OVER GAME PAYOFFS (w/ CERTAIN REASONABLE ASSUMPTIONS) → EQUIVALENT UTILITY FUNCTIONS OVER PAYOFFS UP TO LINEAR TRANSFORMATION

PREFERENCES AMONG GAMES w/OUT KNOWLEDGE OF PROB DIST (w/ " " ") → EQUIVALENT SUBJECTIVE PROBABILITIES ASSIGNED TO PAYOFFS (HORSE RACE)

BEST DECISION RULE - SMALLEST RISK NO MATTER WHAT STATE OF NATURE - "SITUATIONS IN WHICH A BEST DECISION RULE EXISTS ARE RARE AND UNINTERESTING" $d_\theta = \theta$ IS BEST, IF θ IS TRUE STATE OF NATURE

'REASONABLE' DECISION RULES HYP TEST - LIKLIHOOD RATIO ESTIMATION - MAX LIKLIHOOD

RESTRICT AVAILABLE RULES BY SOME CRITERIA OF REASONABLENESS - PERHAPS w/IN RESTRICTED CLASS IS A 'BEST'

UNBIASEDNESS $E_\theta \hat{\theta} = \theta \ \forall \theta$ $\hat{\theta} = d(x)$ FOR SOME d [NOTE ASSIMILATION OF A TO Θ (A = CHOICE OF Θ) AND Θ TO R]

INVARIANCE IF DECISION PROBLEM IS INVARIANT TO CERTAIN OPERATIONS (e.g. LINEAR TRANS), RESTRICT RULES TO SIMILAR INVARIANCE

$\hat{\theta}(f(x)) = f\hat{\theta}(x)$ IS INVARIANT WITH RESPECT TO f

PRINCIPLE OF SELECTION/ORDERING OF DECISION RULES (REASONABLENESS)

δ INDICATES POSSIBILITY OF RANDOMIZED DECISION RULE

BAYES PRINCIPLE τ PRIOR PROB DIST ON Θ

$r(\tau, \delta) = E[R(T, \delta)]$ δ DEC RULE T R.V. OVER Θ w/ DIST τ

DIST T/X - POSTERIOR PROB DIST

DECISION RULE δ_0 IS BAYES (ε-BAYES) w/ RESPECT TO PRIOR τ IF $r(\tau, \delta_0) = \inf_{\delta \in D^*} r(\tau, \delta)$ $[+\varepsilon]$

MINIMAX δ_0 MINIMAX (ε-MINIMAX) IF

$\sup_{\theta \in \Theta} R(\theta, \delta_0) = \inf_{\delta \in D^*} \sup_{\theta \in \Theta} R(\theta, \delta)$ $[\varepsilon \dots + \varepsilon]$