STATISTICAL DECISION THEORY - II LINEAR MODELS AND DECISIONS

BEST DECISION RULE = RULE THAT HAS THE SMALLEST RISK NO MATTER WHAT THE TRUE STATE OF NATURE

SITUATIONS IN WHICH A BEST DECISION RULE EXISTS ARE RARE AND UNINTERESTING

e.g., $d = \theta$ WILL DO BETTER THAN ANY OTHER d IF $\theta \in \Theta$ IS, IN FACT, THE STATE OF NATURE, BUT, OBVIOUSLY, NOT OTHERWISE $\therefore \nexists$ A 'UNIFORMLY' BEST $d \in D$

IF $\nexists$ A BEST DECISION RULE, THEN WE WOULD LIKE ONE THAT IS OPTIMAL IN SOME SENSE

$\exists$ TWO GENERAL APPROACHES:

1) RESTRICT AVAILABLE RULES BY SOME CRITERIA OF OPTIMALITY, LOOK FOR UNIFORMLY BEST RISK WITHIN RESTRICTED CLASS

2) ORDER AVAILABLE RULES BY SOME PRINCIPLE OF OPTIMALITY, PICK BEST ACCORDING TO ORDERING (OBVIOUSLY, 1 AND 2 ARE COMBINABLE)

$D^* = \{\delta \mid \delta$ A RANDOMIZED DECISION RULE, i.e. δ = A PROBABILITY DISTRIBUTION OVER $D\}$

RANDOMIZED DECISION RULES ARE NOT THE KIND OF RULES GENERALLY DESIRED, BUT THEY FREQUENTLY MAKE THE MATHEMATICS SIMPLER AND ALLOW MORE EXACT RESULTS, $\therefore$ MANY DEFINITIONS AND THEOREMS ARE STATED IN TERMS OF THEM. NOTE: $D \subset D^*$

$R(\theta, \delta) = E[R(\theta, z)]$ z A RANDOM VARIABLE OVER D WITH DISTRIBUTION δ

SOME NATURAL CRITERIA:

δ_1 IS AS GOOD AS δ_2 iff $R(\theta, \delta_1) \leq R(\theta, \delta_2)\ \forall \theta \in \Theta$

δ_1 IS BETTER THAN δ_2 iff $R(\theta, \delta_1) \leq R(\theta, \delta_2)\ \forall \theta \in \Theta$ AND $\exists\, \theta_0 \in \Theta \ni R(\theta_0, \delta_1) < R(\theta_0, \delta_2)$

δ_1 IS EQUIVALENT TO δ_2 iff $R(\theta, \delta_1) = R(\theta, \delta_2)\ \forall \theta \in \Theta$

δ IS ADMISSABLE iff $\nexists\, \delta_0$ BETTER THAN δ OTHERWISE, INADMISSABLE

$C \subset D^*$ IS (ESSENTIALLY) COMPLETE iff FOR EACH $\delta \in D^*$, $\delta \notin C$, $\exists\, \delta_0 \in C$ (AS GOOD AS) BETTER THAN δ

TH: C COMPLETE $\Rightarrow$ {ADMISSABLE RULES} $\subset C$

TH: C ESSENTIALLY COMPLETE, $\delta \notin C$, δ ADMISSABLE $\Rightarrow \exists\, \delta' \in C$, δ' EQUIVALENT TO δ

C IS MINIMAL (ESSENTIAL) COMPLETE iff C IS (ESSENTIALLY) COMPLETE, AND NO PROPER SUBSET OF C IS (ESSENTIALLY) COMPLETE

TH: IF A MINIMAL COMPLETE CLASS EXISTS, IT = {ADMISSABLE RULES}