# SPECIAL ESTIMATION PROCEDURES LINEAR MODELS AND DECISIONS

(GOLDBERGER, THEIL, BLALOCK)

$Y = Z\beta + \varepsilon$ X INSTRUMENTS

| | $\Sigma = \sigma^2 I$ | $\Sigma \neq \sigma^2 I$ |
|---|---|---|
| $Cov(Z, \varepsilon) = 0$ | ΦLS | GLS |
| $Cov(Z, \varepsilon) \neq 0$ | 2SLS | 3SLS |

ΦLS

$$\hat{\beta} = (Z'Z)^{-1} Z'Y$$

GLS

$$\hat{\beta} = (Z'WW'Z)^{-1} Z'WW'Y \quad W = \text{WEIGHT MATRIX}$$

$$(Z'\hat{\Sigma}^{-1}Z)^{-1} Z'\hat{\Sigma}^{-1}Y$$

2SLS

$$\hat{\beta} = [Z'X(X'X)^{-1}X'Z]^{-1} Z'X(X'X)^{-1}X'Y$$

3SLS

$$\hat{\beta} = [Z'(I \otimes X)(\hat{\Sigma}^{-1} \otimes (X'X)^{-1})(I \otimes X)'Z]^{-1} Z'(I \otimes X)(\hat{\Sigma}^{-1} \otimes (X'X)^{-1})(I \otimes X)'Y$$

WITH $Y = Z\beta + \varepsilon$ FOR $\begin{pmatrix} Y_1 \\ \vdots \\ Y_M \end{pmatrix} = \begin{pmatrix} Z_1 & & 0 \\ & \ddots & \\ 0 & & Z_M \end{pmatrix} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_M \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_M \end{pmatrix}$

AND $\hat{\Sigma}^{-1}$ COMPUTED FROM 2SLS RESIDUALS