STATISTICAL DECISION THEORY - VIII LINEAR MODELS AND DECISIONS

ESTIMATION

SOME DESIRABLE LARGE SAMPLE PROPERTIES:

1) SIMPLE CONSISTENCY $\lim_{N\to\infty} P\{\theta-\varepsilon<\hat{\theta}_N<\theta+\varepsilon\}=1\ \forall\theta\varepsilon\Theta$

2) SQUARED ERROR CONSISTENCY $\lim_{N\to\infty} E\left[(\hat{\theta}_N-\theta)^2\right]=0\ \forall\theta\varepsilon\Theta$

3) SQUARED ERROR ASYMPTOTICALLY EFFICIENT SQ ER CONS AND $\nexists\,\theta^*$ SQ ER CONS $\ni \overline{\lim_{N\to\infty}}^{\text{UPPER LIMIT}} \dfrac{E\left[(\hat{\theta}_N-\theta)^2\right]}{E\left[(\theta_N^*-\theta)^2\right]}>1\ \forall\theta$ IN SOME OPEN INTERVAL

4) BEST ASYMPTOTICALLY NORMAL (BAN)

A) $\lim_{N\to\infty}\sqrt{N}(\hat{\theta}_N-\theta)\longrightarrow N(0,\sigma^2(\theta))$ ASYMPTOTICALLY NORMAL

B) $\lim_{N\to\infty} P\{|\hat{\theta}_N-\theta|>\varepsilon\}=0\ \forall\theta\varepsilon\Theta$

C) $\nexists\,\theta^*$ ASYMPTOTICALLY NORMAL $\ni \dfrac{\sigma^2(\theta)}{\sigma^{*2}(\theta)}>1\ \forall\theta$ IN SOME OPEN INTERVAL

UNDER GENERAL REGULARITY CONDITIONS ON THE DENSITY FUNCTION $f(x,\theta)$, MAXIMUM LIKLIHOOD ESTIMATORS ARE: [INCLUDING, IN PARTICULAR, FOR NORMAL DISTRIBUTIONS]

1) SIMPLE CONSISTENT

2) SQUARED ERROR CONSISTENT

3) ASYMPTOTICALLY EFFICIENT

4) BAN

5) A FUNCTION OF THE MINIMAL SUFFICIENT STATISTICS

6) INVARIANT; i.e., $U(\theta)$ A FUNCTION OF Θ WITH A SINGLE VALUED INVERSE $\Longrightarrow$ THE MAXIMUM LIKLIHOOD ESTIMATOR OF $U(\theta)$ IS $U(\hat{\theta})$, $\hat{\theta}$ THE MAX LIKLIHOOD ESTIMATOR OF θ

MAXIMUM LIKLIHOOD ESTIMATORS ARE NOT ALWAYS UNBIASED, BUT ARE MANY TIMES MODIFIABLE TO YIELD UNBIASEDNESS

e.g., MAX LIKLIHOOD ESTIMATOR OF $\sigma^2=\frac{1}{N}\sum_i(x_i-\bar{x})^2$ WHILE $\frac{1}{N-1}\sum_i(x_i-\bar{x})^2$ IS UNBIASED

MAXIMUM LIKLIHOOD ESTIMATORS FOR PARAMETERS OF $N(\mu,\sigma^2)$

$\bar{x}=\frac{1}{N}\sum x_i$ FOR μ $\hat{\sigma}^2=\frac{1}{N}\sum(x_i-\bar{x})^2$ FOR σ^2 (BIASED)

TESTS INVOLVING $N(\mu,\sigma^2)$ DERIVED BY LIKLIHOOD RATIO

$H_0:\mu=\mu_0$ σ^2 UNKNOWN t TEST

$N(\mu_1,\sigma_1^2)$ $N(\mu_2,\sigma_2^2)$ $\sigma_1^2=\sigma_2^2$ KNOWN $H_0:\mu_1=\mu_2$ t TEST

$H_0:\sigma^2=\sigma_0^2$ μ UNKNOWN χ^2

$H_0:\sigma_1^2=\sigma_2^2$ μ_1,μ_2 UNKNOWN F