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Hamilton cycles in Trivalent Cayley graphs

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Abstract

It is shown that the Trivalent Cayley graphs, TC_n, are *near recursive*. In particular, TC_n is a union of four copies of TC_{n-2} with additional well placed nodes. This allows one to recursively build the Hamilton cycle in TC_n.

Keywords: Computer architecture; Cayley graphs; Interconnection networks; Mapping; Parallel algorithms

1. Introduction

Interconnection networks form the backbone of parallel architectures and have therefore attracted wide attention from researchers in this area. The structural properties of these networks are often studied by abstracting them as graphs with nodes representing the processors and edges representing the physical communication channels between them. Desirable properties of these graphs include a small node degree, a small diameter, large number of nodes, symmetry and recursiveness.

It has been shown that networks modeled by Cayley graphs have the important property of symmetry and may also possess several of the other important characteristics [1]. However, the node degree of these graphs generally increases with the size of the graph. This is in contrast with the physical characteristics of real world processors which have a limited number of DMA channels and therefore a bounded set of links. A constant node degree graph has an advantage in that it can still be used to interconnect these processors to form an unbounded size of parallel architecture. Unfortunately the constant node degree graphs such as the de Bruijn, Shuffle Exchange and Generalized de Bruijn graphs, are highly asymmetrical [2-4,6]. On the other hand, symmetric graphs such as hypercubes have a node degree that increases with the graph size.

Recently, Trivalent Cayley graphs have been proposed [7]. They have node symmetry, a constant node degree of three, logarithmic diameter and maximal fault tolerance. However, the usefulness of an interconnection network is ultimately decided by its ability to support mappings of algorithm skeletons. We show here that the Trivalent Cayley graph TC_n supports a Hamilton cycle. Thus algorithms whose skeletons are cyclic can be mapped on TC_n with unit dilation. We also show that even though TC_n is not recursive, its four copies (augmented by a few properly placed nodes) may be merged to create TC_{n+2}.

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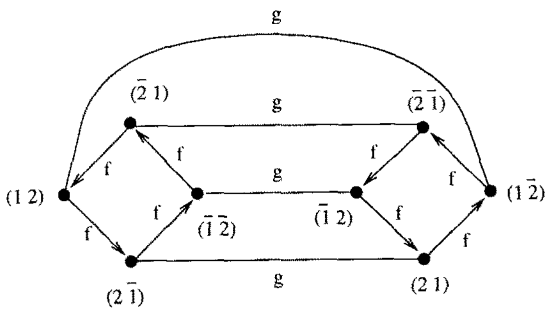

Fig. 1. Degree 2 Trivalent Cayley graph TC_2.

2. Preliminaries

Degree n Trivalent Cayley Graph TC_n has $N = n \times 2^n$ vertices labeled with a circular permutation of n symbols in lexicographic order where each symbol may be present in either uncomplemented or complemented form. We use the (ordered) set of n symbols $S = \{1, 2, 3, \ldots, n\}$, and denote an arbitrary node in TC_n as $(a_1^* a_2^* \ldots a_n^*)$ where a_i^* stands for either $\bar{a}_i$ or a_i with $a_i \in S$. All the permutations in TC_n form a group. The generators of this group are:

$$f(a_1 a_2 \ldots a_n) = (a_2 a_3 \ldots a_n \bar{a}_1),$$

$$f^{-1}(a_1 a_2 \ldots a_n) = (\bar{a}_n a_1 a_2 \ldots a_{n-1}) \quad \text{and}$$

$$g(a_1 a_2 \ldots a_n) = (a_1 a_2 \ldots \bar{a}_n).$$

Two nodes $u, v \in TC_n$ are connected if

$$v = f \cdot u, \text{ or } v = f^{-1} \cdot u \text{ or } v = g \cdot u.$$

The edges between u and fu or $f^{-1}u$ are called the f-edges and are drawn as directed edges marked with f or f^{-1}. The edge between u and gu is called a g-edge and is shown as an undirected edge between the two nodes. Since f^{-1} is the inverse of f and $g^{-1} = g$, and because $f \cdot v$, $f^{-1} \cdot v$ and $g \cdot v$ are all distinct, TC_n is a trivalent undirected graph. It is node symmetric and has a diameter $2n - 1$ [7].

Let I denote the identity permutation and $\bar{u}$ denote the permutation u with all its components complemented. Define the composition of generators as $(g_1 \cdot g_2)(u) = g_1(g_2(u))$. It is then easy to verify that $f^n(u) = \bar{u}$ and $f^{2n}(u) = u$ for every $u \in TC_n$. The $2n$ nodes $u, f(u), f^2(u), \ldots, f^{2n-1}(u)$ therefore form a cycle of length $2n$ called here as the f-cycle. Note that in any f-cycle, there is a unique node whose label has symbol 1 in the first position. We will refer to this node as the *base node* of the f-cycle. Further, by converting the uncomplemented symbols in the remaining $n-1$ positions of the base vector to 0 and complemented symbols to 1, we get an $(n-1)$-bit unique descriptor s of the cycle. Since $0 \leqslant s < 2^{n-1}$, TC_n may be partitioned into 2^{n-1} distinct f-cycles. Fig. 1 shows TC_2. One can easily identify the two f-cycles in this structure.

3. The near recursive property of TC_n

Let the (ordered) symbol set of TC_{n+2} be $S' = \{1, 2, \ldots, n-1, x, y, n\}$. Assume that $n \geqslant 3$. We now show how four copies of TC_n: TC_n^0, TC_n^1, TC_n^2 and TC_n^3 (with some additional well placed nodes) may be merged to form TC_{n+2}. Define mapping $\phi_0 : TC_n^0 \to TC_{n+2}$ as follows:

(a) Map the base nodes of TC_n^0 to the base nodes of TC_{n+2}:

$$\phi_0(1\,2^* \ldots (n-2)^*\,(n-1)\,n^*)$$
$$= (1\,2^* \ldots (n-2)^*\,(n-1)\,x\,y\,n^*),$$
$$\phi_0(1\,2^* \ldots (n-2)^*\,\overline{(n-1)}\,n^*)$$
$$= (1\,2^* \ldots (n-2)^*\,\overline{(n-1)}\,\bar{x}\,\bar{y}\,n^*).$$

(b) Map any other node $f^i(u)$ where u is base node as:

$$\phi_0(f^i(u)) = \begin{cases} f^i(\phi_0(u)) & \text{if } 0 \leqslant i < n, \\ f^{i+2}(\phi_0(u)) & \text{if } n \leqslant i < 2n. \end{cases}$$

The mapping ϕ_0 maps a cycle in TC_n into a cycle in TC_{n+2} as illustrated in Fig. 2. In this figure, u is a base node of TC_n and v, a base node of TC_{n+2}, is its image under mapping ϕ_0.

One may note that the cycle in TC_{n+2} has four additional nodes $f^n v$, $f^{n+1} v$, $f^{2n+1} v$ and $f^{2n+2} v$, that are not images of the nodes of TC_n. Because there are 2^{n-1} cycles of length $2n$ in TC_n, the mapping ϕ_0 along with these four additional nodes per cycle account for $2^{n-1}(2n+4) = 2^n(n+2)$, or exactly one fourth of the nodes of TC_{n+2}. We will call this set of nodes (comprising of 2^{n-1} cycles, each of length $2n+4$) as partition 0 of TC_{n+2}. We obtain the other partitions

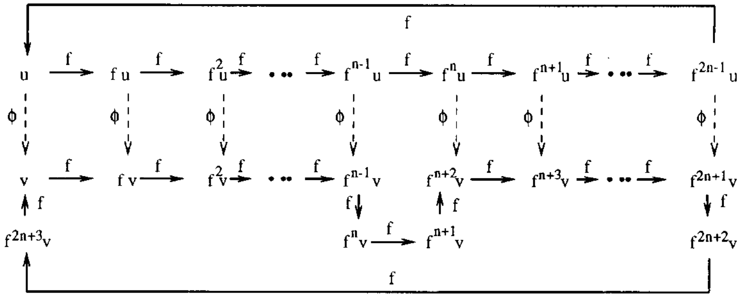

Fig. 2. The mapping ϕ of an f-cycle of TC_n (top) *into* an f-cycle of TC_{n+2} (bottom).

of TC_{n+2} by using mappings $\phi_i : TC_n^i \to TC_{n+2}$, for $i = 1, 2, 3$ which are similarly defined except for the images of the base nodes which are specified as:

$$\phi_1(1\,2^* \,\ldots\, (n-2)^*\,(n-1)\,n^*) = (1\,2^* \,\ldots\, (n-2)^*\,(n-1)\,x\,\bar{y}\,n^*),$$
$$\phi_1(1\,2^* \,\ldots\, (n-2)^*\,\overline{(n-1)}\,n^*) = (1\,2^* \,\ldots\, (n-2)^*\,\overline{(n-1)}\,\bar{x}\,y\,n^*),$$

$$\phi_2(1\,2^* \,\ldots\, (n-2)^*\,(n-1)\,n^*) = (1\,2^* \,\ldots\, (n-2)^*\,(n-1)\,\bar{x}\,y\,n^*),$$
$$\phi_2(1\,2^* \,\ldots\, (n-2)^*\,\overline{(n-1)}\,n^*) = (1\,2^* \,\ldots\, (n-2)^*\,\overline{(n-1)}\,x\,\bar{y}\,n^*),$$

$$\phi_3(1\,2^* \,\ldots\, (n-2)^*\,(n-1)\,n^*) = (1\,2^* \,\ldots\, (n-2)^*\,(n-1)\,\bar{x}\,\bar{y}\,n^*),$$
$$\phi_3(1\,2^* \,\ldots\, (n-2)^*\,\overline{(n-1)}\,n^*) = (1\,2^* \,\ldots\, (n-2)^*\,\overline{(n-1)}\,x\,y\,n^*).$$

Clearly, the images of these TC_n^i's are disjoint as the base nodes in the mappings are distinct. Thus, together, these four copies enumerate all the elements of TC_{n+2}. Because these mappings are similar, we will henceforth use symbol $\phi : TC_n \to TC_{n+2}$ to denote any of them. The following properties of ϕ are easy to verify using Fig. 2.

Theorem 1. (a) *ϕ maps exactly two f-edges of each f-cycle of TC_n (namely, $f^{n-1}u \to f^n u$, and $f^{2n-1}u \to u$) to f-edges of TC_{n+2} in the same partition with dilation 3.*

(b) *ϕ maps the rest of the f-edges of TC_n to the f-edges of TC_{n+2} in the same partition with unit dilation.*

(c) *ϕ maps two g-edges per f-cycle, (namely $f^{n-1}u \to gf^{n-1}u$ and $f^{2n-1}u \to gf^{2n-1}u$, where u is a base node) into g-edges of TC_{n+2}, but these g-edges span across complementary partitions, i.e., edges from the nodes in the image of TC_n^0 go to the nodes in the image of TC_n^3, those from the image of TC_n^1 go to the image of TC_n^2, and vice versa.*

(d) *ϕ maps the rest of the g-edges of TC_n to the g-edges of TC_{n+2} in the same partition with unit dilation.*

It should be noted that TC_{n+2} has other g-edges that are not images of any edges of TC_n. These edges from the four additional nodes in each f-cycle of TC_{n+2} link cycles in different partitions.

4. The Hamilton cycle

We enumerate the Hamilton cycles in TC_2, TC_3 and TC_4 and then use recursion to find Hamilton cycle in larger trivalent graphs.

Hamilton cycle in TC_2.

$$(1\,2) \xrightarrow{f} (2\,\bar{1}) \xrightarrow{f} (\bar{1}\,\bar{2}) \xrightarrow{f} (\bar{2}\,1) \xrightarrow{g}$$
$$(\bar{2}\,\bar{1}) \xrightarrow{f} (\bar{1}\,2) \xrightarrow{f} (2\,1) \xrightarrow{f} (1\,\bar{2}) \xrightarrow{g} (1\,2)$$

Hamilton cycle in TC_3.

$$(2\,3\,\bar{1}) \xrightarrow{f} (3\,\bar{1}\,\bar{2}) \xrightarrow{f} (\bar{1}\,\bar{2}\,\bar{3}) \xrightarrow{f} (\bar{2}\,\bar{3}\,1) \xrightarrow{f} (\bar{3}\,1\,2) \xrightarrow{f}$$
$$(1\,2\,3) \xrightarrow{g} (1\,2\,\bar{3}) \xrightarrow{f^{-1}} (3\,1\,2) \xrightarrow{f^{-1}} (\bar{2}\,3\,1) \xrightarrow{f^{-1}} (\bar{1}\,\bar{2}\,3) \xrightarrow{f^{-1}}$$
$$(\bar{3}\,\bar{1}\,\bar{2}) \xrightarrow{f^{-1}} (2\,\bar{3}\,\bar{1}) \xrightarrow{g} (2\,\bar{3}\,1) \xrightarrow{f} (\bar{3}\,1\,\bar{2}) \xrightarrow{f} (1\,\bar{2}\,3) \xrightarrow{f}$$
$$(\bar{2}\,3\,\bar{1}) \xrightarrow{f} (3\,\bar{1}\,2) \xrightarrow{f} (\bar{1}\,2\,\bar{3}) \xrightarrow{g} (\bar{1}\,2\,3) \xrightarrow{f^{-1}} (\bar{3}\,\bar{1}\,2) \xrightarrow{f^{-1}}$$
$$(\bar{2}\,\bar{3}\,\bar{1}) \xrightarrow{f^{-1}} (1\,\bar{2}\,\bar{3}) \xrightarrow{f^{-1}} (3\,1\,\bar{2}) \xrightarrow{f^{-1}} (2\,3\,1) \xrightarrow{g} (2\,3\,\bar{1})$$

Hamilton cycle in TC_4.

$$(2\,3\,4\,\bar{1}) \xrightarrow{f} (3\,4\,\bar{1}\,\bar{2}) \xrightarrow{f} (4\,\bar{1}\,\bar{2}\,\bar{3}) \xrightarrow{f} (\bar{1}\,\bar{2}\,\bar{3}\,\bar{4}) \xrightarrow{f}$$
$$(\bar{2}\,\bar{3}\,\bar{4}\,1) \xrightarrow{f} (\bar{3}\,\bar{4}\,1\,2) \xrightarrow{f} (\bar{4}\,1\,2\,3) \xrightarrow{f} (1\,2\,3\,4) \xrightarrow{g}$$
$$(1\,2\,3\,\bar{4}) \xrightarrow{f^{-1}} (4\,1\,2\,3) \xrightarrow{f^{-1}} (\bar{3}\,4\,1\,2) \xrightarrow{f^{-1}} (\bar{2}\,\bar{3}\,4\,1) \xrightarrow{f^{-1}}$$
$$(\bar{1}\,\bar{2}\,\bar{3}\,4) \xrightarrow{f^{-1}} (\bar{4}\,\bar{1}\,\bar{2}\,\bar{3}) \xrightarrow{f^{-1}} (3\,\bar{4}\,\bar{1}\,\bar{2}) \xrightarrow{f^{-1}} (2\,3\,\bar{4}\,\bar{1}) \xrightarrow{g}$$
$$(2\,3\,\bar{4}\,1) \xrightarrow{f^{-1}} (\bar{1}\,2\,3\,\bar{4}) \xrightarrow{f^{-1}} (4\,\bar{1}\,2\,3) \xrightarrow{f^{-1}} (\bar{3}\,4\,\bar{1}\,2) \xrightarrow{f^{-1}}$$
$$(\bar{2}\,\bar{3}\,4\,\bar{1}) \xrightarrow{f^{-1}} (1\,\bar{2}\,\bar{3}\,4) \xrightarrow{f^{-1}} (\bar{4}\,1\,2\,\bar{3}) \xrightarrow{f^{-1}} (3\,\bar{4}\,1\,\bar{2}) \xrightarrow{g}$$
$$(3\,\bar{4}\,1\,2) \xrightarrow{f} (\bar{4}\,1\,2\,\bar{3}) \xrightarrow{f} (1\,2\,\bar{3}\,4) \xrightarrow{f} (2\,\bar{3}\,4\,\bar{1}) \xrightarrow{f}$$
$$(\bar{3}\,4\,\bar{1}\,\bar{2}) \xrightarrow{f} (4\,\bar{1}\,\bar{2}\,3) \xrightarrow{f} (\bar{1}\,\bar{2}\,3\,\bar{4}) \xrightarrow{f} (\bar{2}\,3\,\bar{4}\,1) \xrightarrow{g}$$
$$(\bar{2}\,3\,\bar{4}\,\bar{1}) \xrightarrow{f} (3\,\bar{4}\,\bar{1}\,2) \xrightarrow{f} (\bar{4}\,\bar{1}\,2\,\bar{3}) \xrightarrow{f} (\bar{1}\,2\,\bar{3}\,4) \xrightarrow{f}$$
$$(2\,\bar{3}\,4\,1) \xrightarrow{f} (\bar{3}\,4\,1\,\bar{2}) \xrightarrow{f} (4\,1\,\bar{2}\,3) \xrightarrow{f} (1\,\bar{2}\,3\,\bar{4}) \xrightarrow{g}$$
$$(1\,\bar{2}\,3\,4) \xrightarrow{f^{-1}} (\bar{4}\,1\,\bar{2}\,3) \xrightarrow{f^{-1}} (\bar{3}\,\bar{4}\,1\,\bar{2}) \xrightarrow{f^{-1}} (2\,\bar{3}\,\bar{4}\,1) \xrightarrow{f^{-1}}$$
$$(\bar{1}\,2\,\bar{3}\,\bar{4}) \xrightarrow{f^{-1}} (4\,\bar{1}\,2\,\bar{3}) \xrightarrow{f^{-1}} (3\,4\,\bar{1}\,2) \xrightarrow{f^{-1}} (\bar{2}\,3\,4\,\bar{1}) \xrightarrow{g}$$
$$(\bar{2}\,3\,4\,1) \xrightarrow{f^{-1}} (\bar{1}\,\bar{2}\,3\,4) \xrightarrow{f^{-1}} (\bar{4}\,\bar{1}\,\bar{2}\,3) \xrightarrow{f^{-1}} (\bar{3}\,\bar{4}\,\bar{1}\,\bar{2}) \xrightarrow{f^{-1}}$$
$$(2\,\bar{3}\,\bar{4}\,\bar{1}) \xrightarrow{f^{-1}} (1\,2\,\bar{3}\,\bar{4}) \xrightarrow{f^{-1}} (4\,1\,2\,\bar{3}) \xrightarrow{f^{-1}} (3\,4\,1\,2) \xrightarrow{g}$$
$$(3\,4\,1\,\bar{2}) \xrightarrow{f} (4\,1\,\bar{2}\,\bar{3}) \xrightarrow{f} (1\,\bar{2}\,\bar{3}\,\bar{4}) \xrightarrow{f} (\bar{2}\,\bar{3}\,\bar{4}\,\bar{1}) \xrightarrow{f}$$
$$(\bar{3}\,\bar{4}\,\bar{1}\,2) \xrightarrow{f} (\bar{4}\,\bar{1}\,2\,3) \xrightarrow{f} (\bar{1}\,2\,3\,4) \xrightarrow{f} (2\,3\,4\,1) \xrightarrow{g}$$
$$(2\,3\,4\,\bar{1})$$

We now state and prove the central result of this paper.

Theorem 2. *It is possible to find a Hamilton cycle in TC_n, $n \geqslant 3$, such that for any base node $u \in TC_n$, it passes through both the f-edges at $f^{n-1}u$ as well as at f^{2n-1}.*

Proof. It is simple to verify that the Hamilton cycles of TC_3 and TC_4 enumerated above satisfy the specified property. We now demonstrate the construction of a Hamilton cycle in TC_{n+2} from a Hamilton cycle in TC_n which has the stated properties.

First, using function ϕ_0, map the Hamilton cycle in TC_n into TC_{n+2}. However, from the assumption, the Hamilton cycle in TC_n uses the edge between $f^{n-1}u$ and $f^n u$, and there is no single f-edge in TC_{n+2} corresponding to this (see Fig. 2). This edge in the Hamilton cycle of TC_n may be mapped to a path that includes two additional nodes $f\phi(f^{n-1}u)$ and $f^2\phi(f^{n-1}u)$ of TC_{n+2}. Note that these nodes were not images of any nodes of TC_n. Similarly, the Hamilton cycle edge in TC_n between $f^{2n-1}u$ and u may be replaced by a path that includes two more additional nodes $f\phi(f^{2n-1}u)$ and $f^2\phi(f^{2n-1}u)$ of TC_{n+2}.

Now since the Hamilton cycle in TC_n uses both f-edges at $f^{n-1}u$, it cannot pass through the g-edge at $f^{n-1}u$. Similarly, it does not use the g-edge at f^{2n-1}. From Theorem 1, all the other f-edges and g-edges of TC_n map into the same partition of TC_{n+2}. Counting the extra 4 nodes per cycle, the image of the Hamilton cycle in TC_n is a cycle of length $|TC_n| + 4 \cdot 2^{n-1}$. It covers all the nodes of the 0th partition of TC_{n+2}.

Repeating the procedure with mappings ϕ_1 ϕ_2 and ϕ_3, one finally obtains four disjoint cycles in TC_{n+2} (one per partition) that together cover all the nodes of TC_{n+2}.

On account of this construction, given any base node $v \in TC_{n+2}$, both the f-edges of each of the nodes $f^n v$, $f^{n+1}v$, $f^{2n+2}v$ and $f^{2n+3}v$, are present in these cycles.

Next, we design a length 8 cycle that would link the four cycles in TC_{n+2}. Let v be a base node in partition 0 with uncomplemented $(n-1)$. We begin this 8-cycle at $f^n v = (y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots \overline{(n-1)}\, \bar{x})$ and let it go through the following nodes:

$$(y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots \overline{(n-1)}\, \bar{x}) \xrightarrow{f^{-1}}$$
$$(x\, y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots \overline{(n-1)}) \xrightarrow{g}$$
$$(x\, y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots (n-1)) \xrightarrow{f}$$
$$(y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots (n-1)\, \bar{x}) \xrightarrow{g}$$
$$(y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots (n-1)\, x) \xrightarrow{f^{-1}}$$
$$(\bar{x}\, y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots (n-1)) \xrightarrow{g}$$
$$(\bar{x}\, y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots \overline{(n-1)}) \xrightarrow{f}$$
$$(y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots \overline{(n-1)}\, x) \xrightarrow{g}$$
$$(y\, n^*\, \bar{1}\, 2^*\, 3^* \ldots \overline{(n-1)}\, \bar{x})$$

Note that none of the g-edges in this 8-cycle are included in the cycles in the four partitions. The first pair of nodes in this 8-cycle are in partition 0. In fact, the edge between them, being an f-edge from $f^n v$, is in the cycle spanning this partition. We replace this edge by the alternate path between these two nodes that is suggested by the cycle spanning all the nodes of the partition. This is shown in Fig. 3. Similarly, the edge between the next pair of nodes in the 8-cycle is in the cycle spanning partition 3. We can therefore replace this edge by the path which would span all the nodes

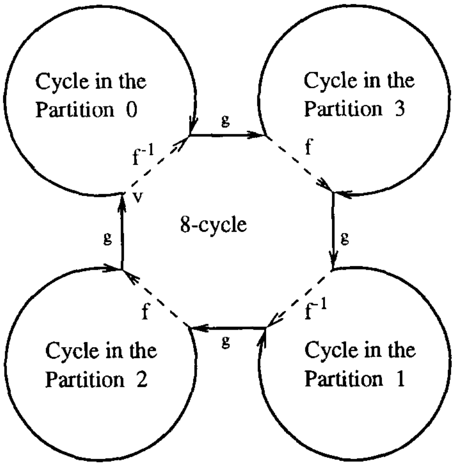

Fig. 3. Merging of the four cycles into the Hamilton cycle shown by dark lines.

in the partition. Similar replacements of the edges between the third and fourth pairs of nodes in the 8-cycle allow us to modify the 8-cycle to pass through all the nodes of the first and the second partition respectively. The resultant cycle therefore goes through all the nodes of TC_{n+2}. Further, since both the f-edges of each of $f^{n+1}v$ and $f^{2n+3}v$ are present in this cycle, this is the required Hamilton cycle in TC_{n+2}. □

Note that the property of *near recursion* can be generalized to merge 2^k copies of TC_n (with additional $2k$ nodes in each f-cycle) to produce TC_{n+k}. This process of merging is exactly identical to the one described here and the mappings ϕ_i, $i = 0, 1, \ldots, 2^k - 1$ exhibit properties similar to those given in Theorem 1. For example, any ϕ_i maps all the f-edges of TC_n to the f-edges of the same partition of TC_{n+k} with dilation 1 except for two which are mapped with a dilation of $k+1$. Similarly, all but two g-edges of TC_n are mapped within the same partition of TC_{n+k} and the other two are mapped across the partitions. The applicable level of recursion, k, is determined by the application. Thus while building the Hamilton cycle in TC_n one cannot partition TC_n into two copies of TC_{n-1} because one would need a length 4 cycle f^*gf^*g (where f^* stands for either f or f^{-1}), to merge the cycles in the two partitions. But such a length 4 cycle does not exist.

5. Conclusions

Recursion is a very desirable property of interconnection networks. It allows one to recursively extend mappings of parallel algorithms to larger architectures. Unfortunately, the Trivalent Cayley graphs are not recursive. But their *near recursiveness* property may help one to build larger mappings as has been demonstrated here to build a Hamilton cycle. Other popular network geometries such as Hypercubes support all even length cycles [5] and de Bruijn and generalized de Bruijn networks support cycles of all lengths [8]. The methods given here may be used to map cycles of other lengths and for developing mappings under faults on Trivalent Cayley graphs.

References

[1] S.B. Akers and B. Krishnamurthy, A group theoretic model for symmetric interconnection networks, *IEEE Trans. Comput.* **38** (1989) 555–566.
[2] M. Baumslag, An algebraic analysis of the connectivity of de Bruijn and shuffle exchange digraphs, *Discrete Appl. Math.* **61** (1995) 213–227.
[3] D.Z. Du and F.K. Hwang, Generalized de Bruijn digraphs, *Networks* **18** (1988) 27–38.
[4] D.Z. Du, D.F. Hsu, F.K. Hwang and X.M. Zhang, The Hamiltonian property of generalized de Bruijn digraphs, *J. Combin. Theory, Ser. B* **52** (1991) 1–8.
[5] F.T. Leighton, *Introduction to Parallel Algorithms and Architectures: Arrays, Trees, Hypercubes* (Morgan Kaufmann, San Mateo, CA, 1992).
[6] M.R. Samantham and D.K. Pradhan, The de Bruijn multiprocessor network: A versatile parallel processing and sorting network for VLSI, *IEEE Trans. Comput.* **38** (1989) 567–581.
[7] P. Vadapalli and P.K. Srimani, Trivalent Cayley graphs for interconnection network, *Inform. Process. Lett.* **54** (1995) 329–335.
[8] M.D. Wagh and J. Mo, Applications of finite fields to map parallel algorithms to dense graphs, Manuscript.