Supply and Demand Uncertainty in Multi-Echelon Supply Chains

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Outline

1. Introduction
   - Motivation
   - Literature Review
   - Roadmap
   - Methodology and Assumptions

2. Cost of Unreliability

3. Order Frequency

4. Inventory Placement

5. Supply Chain Structure

6. Cost of Reliability
Supply vs. Demand Uncertainty

- **Demand uncertainty (DU)**
  - Randomness in demand quantity, timing, product mix, etc.
  - Purview of SCM/OM for decades

- **Supply uncertainty (SU)**
  - Disruptions
  - Capacity/yield uncertainty
  - Lead-time uncertainty
  - etc.
  - Increased attention only recently
Are DU and SU the Same?

- Under both DU and SU, the main issue is the same:
  - Not enough supply to meet demand
  - May be irrelevant whether the mismatch came from DU or SU

- Mitigation strategies are similar:
  - Safety stock
  - Multiple sourcing
  - Improved forecasts
  - Demand management
  - Excess capacity
  - etc.

The good news: We know a lot about supply chains under DU
The bad news: The “conventional wisdom” from DU is often wrong under SU
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- Under both DU and SU, the main issue is the same:
  - Not enough supply to meet demand
  - May be irrelevant whether the mismatch came from DU or SU
- Mitigation strategies are similar:
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  - Improved forecasts
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  - Excess capacity
  - etc.
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Literature Review

- Classical inventory models + disruptions

- Classical inventory models + yield uncertainty

- Strategic questions
  - Tomlin (2006): optimal mitigation strategy
  - Tomlin and Snyder (2006): advanced warning
  - Lewis, Erera, and Whilte (2005): border closures
  - Chopra, Reinhardt, and Mohan (2005): “bundling” disruptions and yield uncertainty
Multi-Echelon Models

- Very few multi-echelon models with disruptions
  - Hopp and Yin (2006): Optimal placement and size of inventory and capacity buffers in assembly network
- Must study disruptions in multi-echelon setting
  - Disruptions are never local
  - Cascading effect
A Newsboy-Style Result

Theorem (Tomlin 2006)

In a single-stage base-stock system with deterministic demand and stochastic supply disruptions, the optimal base-stock level is given by

\[ S^* = d + dF^{-1}\left(\frac{p}{p+h}\right), \]

where \( d \) is the demand per period and \( F \) is the cdf of supply.

- \( F(x) = P(\text{we are in a disruption lasting } x \text{ periods or fewer}) \)
- Cycle/safety stock interpretation
- Similar (but less sharp) result given by Güllü et al. (1997)
SU vs. DU: Roadmap

1. The cost of unreliability
2. Order frequency
3. Inventory placement
   - Centralization vs. decentralization
   - Upstream vs. downstream
4. Supply chain structure
   - Hub-and-spoke vs. point-to-point
   - Supplier redundancy
   - Supplier flexibility
5. The cost of reliability
Methodology

- Some of our results are proved analytically
- Others we demonstrate using simulation
  - BaseStockSim software
  - Rough optimization of base-stock levels
Supply Chain Assumptions

- Multi-echelon SC
  - Each stage has processing function and output buffer:
    
    ![Diagram of a multi-echelon SC with stages 2 and 1 connected]
    
  - May represent physical location, processing activity, or SKU
- Backordered demand
- Costs $h, p$
- Processing (lead) time $T$
- Under DU, demands are $N(\mu, \sigma^2)$
- Under SU, disruption process follows 2-state Markov process
  - Disruption probability $\alpha$
  - Recovery probability $\beta$
The Cost of Unreliability

- Two stages: supplier and retailer
  - Supplier cannot hold inventory, is subject to disruptions
- Under either DU or SU, base-stock policy is optimal at retailer
- Suppose firm fails to plan for either type of uncertainty
  - i.e., it sets $S = \mu$

Key Question:
Is this a bigger mistake under DU or SU?
Level of Uncertainty

- We need a way to compare DU and SU fairly
- Let **level of uncertainty** = % of demands backordered when $S = \mu$

\[
LOU = 1 - \text{fill rate}
\]

- A DU process and an SU proce are **equivalent** if they have the same LOU
Under **DU**, fill rate is

\[ 1 - \frac{\sigma \mathcal{L}(z)}{\mu}, \]

where \( \mathcal{L}(z) \) is standard normal loss function and \( z = (S - \mu)/\sigma \).

Therefore

\[ LOU_{DU} = \frac{\sigma \mathcal{L}(0)}{\mu} \approx 0.3989 \frac{\sigma}{\mu}. \]
Level of Uncertainty, cont’d

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where \( \mathcal{L}(z) \) is standard normal loss function and \( z = (S - \mu)/\sigma \).

- Therefore

\[
LOU_{DU} = \frac{\sigma \mathcal{L}(0)}{\mu} \approx 0.3989 \frac{\sigma}{\mu}.
\]

- Under **SU**, fill rate = % of periods in which supplier is up

- Therefore

\[
LOU_{SU} = P(\text{supplier down}) = \frac{\alpha}{\alpha + \beta}.
\]
Simulation Experiment

- Varied LOU from 0.0 to 0.2 (by 0.01)
- For each LOU, find $\sigma$ and $\alpha$ that achieve it
  - (Keeping $\mu$ and $\beta$ fixed)

![Graph showing the relationship between Level of Uncertainty and Mean Cost per Period. The graph compares DU Cost and SU Cost.](image-url)
Insights

- More costly to fail to plan for SU than for DU
- Holds under a wide range of parameters
- Cost difference is greater when
  - Holding cost is smaller
  - Stockout cost is larger
  - Recovery probability is smaller
Two-stage supply chain
- $\mu = 20$, $p = 100$ at retailer
- $T = 1$ at supplier
- Under DU, $\sigma = 5$

Two possible cost structures:
1. $h = 2.85$ and $K = 0$
2. $h = 0.1$ and $K = 250$

Key Question:
Does firm prefer #1 (one-for-one ordering) or #2 (batch ordering)?
Order Frequency: **DU**

- **Option 1:** $h = 2.85$, $K = 0$
  - Base-stock policy is optimal, with
    
    $$S^* = \mu + \sigma \Phi^{-1} \left( \frac{p}{p + h} \right) \approx 30$$

  - $E[\text{cost}] \approx 32.8$

- **Option 2:**
  $$h = 0.1$$, $K = 250$

  $(s, S)$ policy is optimal with
  
  $$s^* \approx 31$$, $S^* \approx 349$

  $E[\text{cost}] \approx 32.8$

So the firm is indifferent between the two options under DU.
Order Frequency: DU

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So the firm is indifferent between the two options under DU.
Order Frequency: SU

- **Option 1**: $h = 2.85$, $K = 0$
  - Base-stock policy is optimal (Tomlin 2006), with
    \[
    S^* = \mu + \mu F^{-1} \left( \frac{p}{p + h} \right) \approx 60
    \]
  - $E[\text{cost}] \approx 497.7$
Order Frequency: SU

- **Option 1**: $h = 2.85, \ K = 0$
  - Base-stock policy is optimal (Tomlin 2006), with
    \[
    S^* = \mu + \mu F^{-1} \left( \frac{p}{p + h} \right) \approx 60
    \]
  - $E[\text{cost}] \approx 497.7$

- **Option 2**: $h = 0.1, \ K = 250$
  - Optimal policy not known (deterministic demand, stochastic disruptions, fixed cost)

**Lemma**

$s^*$ and $S^*$ are integer multiples of $\mu$.

- $s^* \approx 40, \ S^* \approx 340, \ E[\text{cost}] \approx 391.1$
- So the batch ordering policy is preferred
Insights

Why is batch policy preferred?
- If an order is disrupted, the impact is the same under either policy
- But the likelihood of a disruption affecting an order is smaller under batch policy
Batch policy is usually—though not always—preferred
- $s$ and $S$ may not be optimal
- Instances are generated so that batch and base-stock policies are equivalent under DU
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   • Upstream vs. Downstream

5 Supply Chain Structure

6 Cost of Reliability

7 Conclusions

Snyder and Shen (Lehigh and Berkeley)
Centralization vs. Decentralization

- One warehouse, multi-retailer (OWMR) system
- Cost of holding inventory is equal at the two echelons
- Lead times are negligible

Key Question:
Should we hold inventory at the warehouse or at the retailers?
OWMR under DU

- Let $C_D$, $C_C$ be cost under decentralized and centralized systems, resp.

**Theorem (Eppen 1979)**

*Under DU,*

- $E[C_D] \propto N$
- $E[C_C] \propto \sqrt{N}$

- Therefore, centralization is optimal
  - The *risk-pooling effect*
OWMR under SU

- Under **SU**: 
  - Disruptions affect inventory sites 
  - In decentralized system, a disruption affects one retailer 
  - In centralized system, a disruption affects the whole supply chain

**Theorem**

*Under SU,*

\[(a) \quad E[C_D] = E[C_C]\]

\[(b) \quad V[C_D] \propto N \quad V[C_C] \propto N^2\]

- Therefore **decentralization** is preferable
- We call this the **risk-diversification effect**
Implication for Facility Location

- Joint location–inventory model by Daskin et al. (2002) and Shen et al. (2003)
  - Considers DU via concave inventory costs in location model
  - Optimal # of facilities decreases because of risk-pooling effect (and inventory economies of scale)
- Reliability model by Snyder and Daskin (2005)
  - Considers SU in the form of facility failures
  - Optimal # of facilities increases—related to risk-diversification effect
- Model by Jeon et al. (working paper, 2006) balances these competing tendencies
Serial supply chain
Cost of holding inventory is non-increasing as we move downstream
Lead times are negligible

Key Question:
Should we hold inventory upstream or downstream?
Under DU, conventional wisdom says hold inventory upstream
- Holding costs increase as we move downstream
But under SU, downstream inventory may be preferable
- Protects against stockouts anywhere in the system
- Depends on relative holding costs
Savings Increases as Disruption Probability Increases

Snyder and Shen (Lehigh and Berkeley)
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   - Supplier Redundancy
   - Supplier Flexibility
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Hub-and-Spoke vs. Point-to-Point Systems

Hub-and-Spoke:

Point-to-Point:

Key Question:
Which type of network is preferred?
Under **DU**, hub-and-spoke systems are optimal

- Due to **risk-pooling effect**: fewer stocking locations
  \[ \Rightarrow \] smaller inventory requirement
Hub-and-Spoke vs. Point-to-Point Systems, cont’d

- Under **DU**, hub-and-spoke systems are optimal
  - Due to **risk-pooling effect**: fewer stocking locations
    \[ \Rightarrow \] smaller inventory requirement
- Under **SU**, point-to-point systems are optimal
  - Related to **risk-diversification effect**: more stocking locations
    \[ \Rightarrow \] reduced severity of disruptions
Simulation Results

![Simulation Results Diagram]

- **Mean Cost of P-P Network** vs. **Mean Cost of H-S Network**
- Snyder and Shen (Lehigh and Berkeley)
- INFORMS 2006
Supplier Redundancy

- Single retailer with one or more suppliers
- Suppliers are identical in terms of cost, capacity, reliability

Key Question:
What is the value of having backup suppliers?
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Supplier Redundancy under DU

- Under DU, second supplier provides value if capacities are tight
  - e.g., if capacity = $\mu + \sigma$
  - But value decreases quickly as capacity increases
  - Third, etc. suppliers provide little value
Value of Backup Suppliers: DU

\[ \mu = 20, \sigma = 5 \]
Supplier Redundancy under SU

- Under SU, second supplier provides great benefit
  - Fills in when primary supplier is disrupted
  - Also helps ramp back up after disruption
  - Even third+ supplier provides some benefit
Value of Backup Suppliers: SU

![Graph showing the value of backup suppliers with varying supplier capacity and demand uncertainty. The graph illustrates the percentage savings from the second supplier in different capacity ranges and demand uncertainty intervals.]
Supplier Flexibility

- Related concept: supplier flexibility
- Multiple suppliers, multiple retailers
  - How many suppliers per retailer?
- Closely related to process flexibility (Jordan and Graves 1995)
  - Bipartite network of jobs and workers
  - How much cross-training is required?
  - i.e., how dense should network be?
- Results are similar
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The Cost of Reliability

- Firms are accustomed to planning for DU
- Often reluctant to plan for SU if it requires large investment

Key Question

How much DU cost must be sacrificed to achieve a given level of reliability?
Firms are accustomed to planning for DU. Often reluctant to plan for SU if it requires large investment.

Key Question
How much DU cost must be sacrificed to achieve a given level of reliability?

The short answer: Not much
Tradeoff Curve

- Each point represents a solution (set of base-stock levels) for serial system
  - Left-most point is “optimal” solution considering DU only
  - Second point: 21% fewer stockouts, 2% more expensive
- “Steep” left-hand side of tradeoff curve is fairly typical
  - Especially for combinatorial problems
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Conclusions

- Planning for SU is critical
- Optimal strategy under SU is often exact opposite from that under DU
  - That’s not to say firms are doing everything wrong
  - But SU should be accounted for more than it is
  - Strategy chosen should account for both
- Many of these results are related to risk-diversification effect
  - Disruptions are less severe when eggs aren’t all in one basket
- Tradeoff between cost and reliability is often steep
  - Large improvements in reliability with small increases in cost
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Working paper available at
www.lehigh.edu/~lvs2/research.html

BaseStockSim software available at
www.lehigh.edu/~lvs2/software.html
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