The Effect of Supply Disruptions on Supply Chain Design Decisions

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Abstract

We study an integrated supply chain design problem that determines the locations of retailers and the assignments of customers to retailers in order to minimize the expected costs of location, transportation, and inventory. The system is subject to random supply disruptions that may occur at either the supplier or the retailers. Analytical and numerical studies reveal the effects of these disruptions on retailer locations and customer allocations. In addition, we demonstrate numerically that the cost savings from considering supply disruptions at the supply chain design phase (rather than at the tactical or operational phase) are usually significant.

1 Introduction

Integrated supply chain design problems make facility location and demand assignment decisions based on the locations and demand characteristics of fixed customers in order to minimize the total cost (or maximize the total profit), including location, inventory and transportation costs. They have been recently studied by many researchers, including Shen, Coullard and Daskin (2003), Daskin, Coullard, and Shen (2002), Shu, Teo, and Shen (2005), and Shen and Qi (2007). Shen (2007) provides a comprehensive literature review on integrated supply chain design problems. The existing research on integrated supply chain design problems assumes that the suppliers and facilities in supply chain networks are always available to serve their customers. However, supply chain disruptions are possible at any stage of a supply chain network, and failing to plan for them adequately may result in significant economic losses.

Recent examples of supplier/facility disruptions include:

- The west-coast port lockout in 2002 and the subsequent inventory shortages caused factories across a variety of industries to close and perishable cargo to rot. Economists estimated damage to the economy at $1 billion a day (Greenhouse, 2002).

- The logistical logjam due to the disruptions of numerous facilities after hurricanes Katrina and Rita caused a huge economic loss in 2005 (Barrionuevo and Deutsch, 2005). For example, 124 local Wal-Mart stores and two distribution centers were shut down, and fifteen stores remained closed half a month after Hurricane Katrina (Halkias, 2005).

We learn from these examples that supplier and facility disruptions should be both considered in integrated supply chain design problems so that the facilities' locations are determined with the possible losses caused by facility disruptions taken into consideration and the inventory decisions at the facilities are made to protect against supplier disruptions.

In summary, we believe that it is important for supply chain designers and managers to be able to address the following questions:

- What are the impacts of disruptions both at internal facilities and at those facilities' suppliers on the optimal facility location and demand-allocation decisions?
- Can significant cost savings be achieved in practice if we consider supply disruptions at the supply chain design phase?

We explore answers to these questions, borrowing ideas from inventory management problems with supply disruptions, such as the models of Parlar and Berkin (1991), Berk and Arreola-Risa (1994), Snyder (2005), Qi, Shen and Snyder (2009), and Tomlin (2006).

Specifically, we consider the following single-product problem: Customers are distributed throughout a certain region. We wish to open one or more retailers that are served directly from a supplier whose location is fixed. The retailers satisfy deterministic demands from the customers and place replenishment orders to the supplier. (This deterministic-demand assumption is relaxed in Section 7, in which we also show that relaxing this assumption has little, if any, impact on the solution.) We assume zero lead time for order processing at the supplier and retailers when non-disrupted. However, both the supplier and the retailers may be disrupted randomly:

- When a retailer is disrupted, it becomes unavailable, and no customer demand received during the disruption can be filled until the disruption has ended. In addition, any inventory on hand at the retailer is destroyed when the disruption occurs.
- If a retailer wishes to place an order when the supplier is disrupted, this order will not be filled until the supplier recovers from the disruption. Hence, a retailer may not be able to serve its customers even if it is available itself, since it may have no inventory on hand due to the delayed shipment from the disrupted supplier.

If a customer is assigned to a retailer but the retailer is disrupted or out of stock, the unmet demands are backlogged, at a cost. We assume that customers may not be temporarily reassigned to non-disrupted retailers if their own assigned retailer is disrupted or out of stock; that is, we do not consider dynamic sourcing. In addition, we allow some customers not to be served at all, even when no disruptions have occurred, if the cost of serving them is prohibitive. In this case, a lost-sales penalty is applied for each unit of unserved customer demand.

We formulate an integrated model to determine 1) how many retailers should be opened, and where to locate them; 2) which retailers should serve which customers; and 3) how often and how much to order at each retailer, so as to minimize the total location, working inventory (including ordering, holding and backorder costs), transportation, and lost-sales costs. Since customer demands are deterministic, the inventory at each retailer serves two main purposes: 1) to take advantage of economies of scale due to fixed costs, and 2) to protect against supplier disruptions.

We analyze this model to evaluate the impact of random supply disruptions at the supplier and retailers on the retailer location and customer demand allocation decisions. We use numerical experiments to verify the conclusions made in our analytical studies. Our results show that significant cost savings can often be achieved if we consider supply disruptions when making supply chain design decisions.

The remainder of this paper is organized as follows. In Section 2, we review the related literature. We then propose an integrated supply chain design model in Section 3 for the problem stated above. We analyze the model in Section 4 to evaluate the impact of supply disruptions on facility location and demand assignment decisions. We suggest a solution algorithm in Section 5 for the model, and conduct numerical experiments in Section 6 to further study the impact of supply disruptions and explore the conditions under which significant cost savings can be achieved by considering supply disruptions during the supply chain design phase. An extension to our original model is addressed in Section 7, in which we relax the deterministic-demand assumption made in Section 3.

We conclude our work in Section 8, and suggest some future research directions.

2 Literature Review

The study by Parlar and Berkin (1991) is among the earliest works that incorporate supply disruptions into classical inventory models. They consider a variant of the EOQ model in which supply is available during an interval of random length and then unavailable for another interval of random length. Their model assumes that the firm knows the availability status of the supplier and that it follows a zero-inventory ordering (ZIO) policy. Berk and Arreola-Risa (1994) show that Parlar and Berkin's original model is incorrect in two respects. Their corrected cost function cannot be minimized in closed form, nor is it known whether it is convex.

Snyder (2005) develops an effective approximation for the model introduced by Berk and Arreola-Risa (1994). His approximate cost function not only is convex but also yields a closed-form solution and behaves similarly to the classical EOQ cost function in several important ways. Heimann and Waage (2005) relax the ZIO assumption in Snyder's model and derive a closed-form approximate solution.

Parlar and Perry (1995) relax the two assumptions made in (1991). First, they consider the case in which the decision maker is not aware of the ON-OFF status of the supply before an order is placed. Second, their model allows the reorder point to be a decision variable. In addition to random supply disruptions, Gupta (1996) assumes that the customer demands are random, generated according to a Poisson process. He considers constant lead times, whereas Parlar (1997) introduces a more general model in which the lead time may be stochastic.

Qi, Shen and Snyder (2009) extend the works of Berk and Arreola-Risa (1994) and Snyder (2005) by considering random disruptions at two echelons—at the supplier (as in Berk and Arreola-Risa, 1994, and Snyder, 2005) and at the retailers. They conduct analytical and numerical studies to determine the impact of supply disruptions on the retailer's optimal inventory decisions. They also propose an effective approximation of their cost function that we embed into the objective function of our integrated model in the present paper. Qi, Shen and Snyder prove that their approximation is a concave and increasing function of the total demand the retailer faces, a property that we make use of in the present paper.

The above works assume there is only one supplier, and if that supplier is disrupted, the firm has no recourse. In contrast, Tomlin (2006) presents a dual-sourcing model in which orders may be placed with either a cheap but unreliable supplier or an expensive but reliable supplier. He considers a very general stochastic recovery process at the unreliable supplier. He evaluates the firm's optimal strategy under various realizations of the problem parameters.

Since this paper considers multiple retailers in an integrated supply chain design setting, our work is also closely related to the literature on integrated supply chain design. Shen, Coullard and Daskin (2003), Daskin, Coullard, and Shen (2002) study a joint location/inventory model in which location, shipment and nonlinear inventory costs are included in the same model. They develop an integrated approach to determine the number of distribution centers (DCs) to establish, the location of the DCs, the assignments of customers to DCs, and the magnitude of inventory to maintain at each DC. More general problems are studied by Shu, Teo, and Shen (2005), Shen and Qi (2007), and Snyder, Daskin, and Teo (2007). None of these integrated supply chain design problems consider random supply disruptions.

Our paper is also closely related to the literature on facility location with disruptions. Snyder and Daskin (2005) consider facility location models in which some facilities will fail with a given probability. Their models are based on two classical facility location models and assume that customers may be re-assigned to alternate DCs if their closest DC is disrupted. Their models minimize a weighted sum of the nominal cost (which is incurred when no disruptions occur) and the expected transportation cost accounting for disruptions. They do not consider inventory costs. Related models are studied by Berman, Krass, and Menezes (2007), Church and Scaparra (2007), and Scaparra and Church (2008). Snyder and Daskin (2007) compare models for reliable facility location under a variety of risk measures and operating strategies. Snyder, Scaparra, Daskin, and Church (2006) provide a tutorial and literature review for supply chain design models with disruptions.

Like our paper, Berman, Krass, and Menezes (2007), Church and Scaparra (2007), Scaparra and Church (2008), Snyder and Daskin (2005), Snyder and Daskin (2007), Snyder et al. (2006) consider facility location with disruptions. Our paper differs from the earlier literature in two main respects. First, we consider the cost of inventory at the facilities, optimizing the inventory levels to account for supplier disruptions. Second, we consider disruptions at both the supplier and at the retailers, whereas the earlier literature considers disruptions only at the retailers.

Finally, we mention two additional papers on supply chain design under supply uncertainty: those of Qi and Shen (2007) and Kim, Lu and Kvam (2005). Both papers consider yield uncertainty/product defects in supply chain design decisions for a three-echelon supply chain using ideas from the random yield literature. However, supply disruptions are not considered in these papers.

3 Model Formulation

3.1 Notation and Formulation

In this section, we formulate an integrated model for the problem stated in Section 1. The objective is to minimize the expected total annual cost including 1) the fixed cost to open retailers, 2) the working inventory cost (including ordering, holding and backorder costs) at the open retailers; 3) the transportation cost from retailers to customers; and 4) the lost-sales penalty cost of choosing not to serve some customers. (Although we use one year as the time horizon for our model, it can easily be adapted for other time units.)

We use the following notation throughout the paper:

I: index set of all customers

J: index set of candidate locations for retailers

D_i: annual demand of customer $i \in I$

f_j: annual fixed cost to open a retailer at location $j \in J$

π: penalty cost for not assigning a customer to any retailer, per unit of demand

$\mathcal{T}_j(\cdot)$: the working inventory cost at retailer $j \in J$ (This includes ordering, holding, and backorder costs at retailer j and is a function of the total demand assigned to retailer j. It is zero if no retailer is opened at location j. We examine $\mathcal{T}_j(\cdot)$ in greater detail in Section 3.2.)

$\hat{d}_{ij}$: unit cost to deliver items from retailer $j \in J$ to customer $i \in I$

There are two sets of decision variables:

X_j= 1 if a retailer is opened at site $j \in J$, 0 otherwise;

Y_{ij}= 1 if the demand from customer $i \in I$ is to be served by the retailer at site $j \in J$, 0 otherwise.

It is expedient to create a "dummy" retailer with index s; assigning a customer i to this retailer ($Y_{is} = 1$) represents not assigning the customer at all. We therefore formulate our problem as

$$\text{minimize} \quad \sum_{j \in J} \left\{ f_j X_j + \mathcal{T}_j\left(\sum_{i \in I} D_i Y_{ij} \right) + \sum_{i \in I} \hat{d}_{ij} D_i Y_{ij} \right\} + \pi \sum_{i \in I} D_i Y_{is} \tag{1}$$

$$\text{subject to} \quad \sum_{j \in J} Y_{ij} + Y_{is} = 1 \qquad \forall i \in I \tag{2}$$

$$Y_{ij} - X_j \leq 0 \qquad \forall i \in I, j \in J \tag{3}$$

$$Y_{ij} \in \{0, 1\} \qquad \forall i \in I, j \in J \cup \{s\} \tag{4}$$

$$X_j \in \{0, 1\} \qquad \forall j \in J \tag{5}$$

The first three terms in the objective function represent the annual fixed cost to open retailers, the working inventory cost at open retailers, and the retailer–customer transportation cost, respectively. (The transportation cost from the supplier to the retailers is included in $\mathcal{T}_j(\cdot)$, as discussed in Section 3.2.) The last term in the objective function represents the lost-sales cost for those customers not served by any retailer. The constraints of the above model are similar to those of other well known warehouse location problems, such as the one studied by Erlenkotter (1978). In particular, (2) requires each customer to be assigned to exactly one retailer or to the "dummy" retailer. Constraint (3) requires customers to be served only by open retailers. Constraints (4) and (5) are standard integrality constraints.

We define $D_j(Y) = \sum_{i \in I} D_i Y_{ij}$ to simplify the notation. Replacing Y_{is} in (1) with $1 - \sum_{j \in J} Y_{ij}$ according to (2) and omitting the constant term $\pi \sum_{i \in I} D_i$, the original problem may be rewritten as follows:

$$
\begin{aligned}
\textbf{(P)} : \text{minimize} \quad & \sum_{j \in J} \left\{ f_j X_j + \mathcal{T}_j(D_j(Y)) + \sum_{i \in I} (\hat{d}_{ij} - \pi) D_i Y_{ij} \right\} \\
\text{subject to} \quad & \sum_{j \in J} Y_{ij} \leq 1 \qquad \forall i \in I \\
& Y_{ij} - X_j \leq 0 \qquad \forall i \in I, j \in J \\
& Y_{ij} \in \{0, 1\} \qquad \forall i \in I, j \in J \\
& X_j \in \{0, 1\} \qquad \forall j \in J
\end{aligned}
$$

3.2 Formulation and Approximation of $\mathcal{T}_j(\cdot)$

We assume that each retailer uses the ZIO policy studied by Qi, Shen and Snyder (2009), depicted in Figure 1, to manage its inventory. Customer demands are constant and deterministic, as in the classical EOQ model (When we relax this assumption, and assume that the demands from customer $i \in I$ follow a Poisson process with rate D_i, then, according to the properties of Poisson processes, the demands each retailer faces also follow a Poisson process with rate $D_j(Y)$. We demonstrated in Qi, Shen and Snyder (2009) using simulation studies that $\mathcal{T}_j(\cdot)$, the working inventory cost at retailer j we formulate below, is robust to violations of this deterministic-demand assumption and produces nearly identical results when customer demands the retailer faces instead follow a Poisson process. We therefore believe that this deterministic customer demand assumption is reasonable for the problem we study in this paper. We further demonstrate this point in Section 7, in which we formulate an extended integrated model that relaxes the deterministic-demand assumption, and show that the solutions to the extended integrated model and the original integrated model are identical for all instances tested.). A retailer can only place orders and receive shipments from the supplier when it is not disrupted. Similarly, an order from a retailer can be filled only when the supplier is available. Any order from a retailer that arrives when the supplier is disrupted will not be filled until the supplier and the retailer are both available. Qi, Shen and Snyder consider two cases for handling unmet demands: lost sales and backorders with a penalty that does not depend on the duration of the backorder. We assume that stockouts at the retailers fall into the latter case.

We define

F_j: fixed ordering cost at retailer $j \in J$

a_j: per-unit ordering cost at retailer $j \in J$

h_j: per-unit annual holding cost at retailer $j \in J$

π_j: time-independent backorder cost per unit of unmet demand at retailer $j \in J$ (typically, $a_j < \pi_j$)

T_j: the inventory cycle length at retailer $j \in J$, equal to the duration between two consecutive shipments from the supplier to retailer j (a random variable)

Q_j: the inventory level at retailer $j \in J$ at the beginning of each inventory cycle; the order size from retailer j to the supplier is therefore Q_j plus any backlogged demand (Q_j is a decision variable;

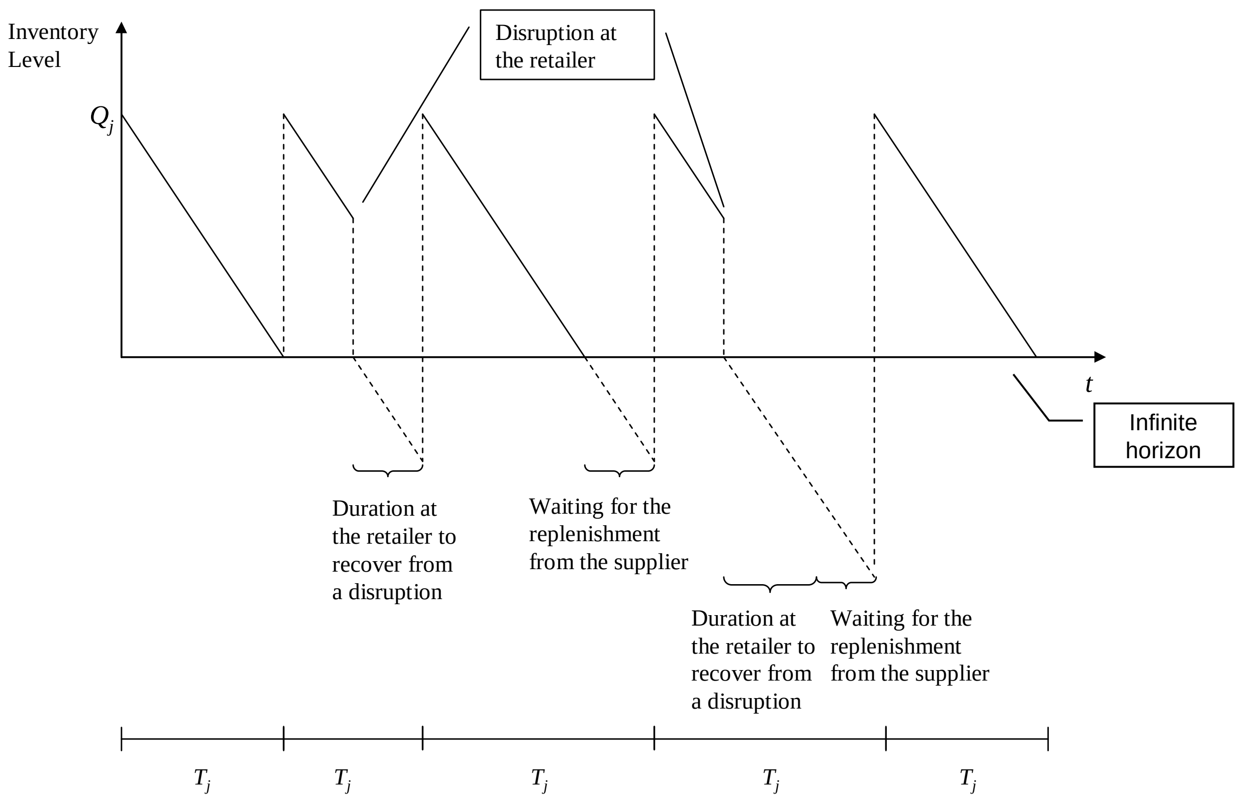


Figure 1: Inventory policy at a retailer.

however, we will express the optimal inventory cost in closed form without using Q_j explicitly in the objective function)

The retailers and supplier both experience ON (available) and OFF (disrupted) cycles of random durations. We assume that the durations of the ON and OFF cycles at the supplier follow independently and identically distributed (iid) exponential distributions with rates λ and ψ, respectively; and that the durations of the ON and OFF cycles at retailer $j \in J$ follow iid exponential distributions with rates α_j and β_j, respectively. To summarize:

λ: disruption rate at the supplier (times/year)

ψ: recovery rate at the supplier (times/year)

α_j: disruption rate at retailer $j \in J$ (times/year)

β_j: recovery rate at retailer $j \in J$ (times/year)

Following Qi, Shen, and Snyder (2009), we further assume that all inventory at a retailer is destroyed if the retailer is disrupted.

If $D_j(Y) > 0$, Qi, Shen, and Snyder (2009) prove that the expected annual working inventory cost (including ordering, holding, and backorder costs) at retailer j is given by:

$$\mathcal{I}_j(Q_j) \equiv \pi_j D_j(Y) + \frac{F_j + \left(a_j + \frac{h_j}{\alpha_j}\right) Q_j - \left(1 - e^{-\alpha_j \frac{Q_j}{D_j(Y)}}\right)\left[\frac{h_j D_j(Y)}{\alpha_j^2} + \frac{\pi_j D_j(Y)}{\alpha_j}\right]}{E[T_j]} \tag{6}$$

where

$$\begin{aligned}
E[T_j] &= \bar{A}_j \left[1 - e^{-(\alpha_j+\lambda+\psi)\frac{Q_j}{D_j(Y)}}\right] + \bar{B}_j \left[1 - e^{-\frac{\alpha_j Q_j}{D_j(Y)}}\right] \\
\bar{A}_j &= \frac{\lambda}{\beta_j \psi} \cdot \frac{\alpha_j + \beta_j}{\alpha_j + \lambda + \psi} \\
\bar{B}_j &= \frac{1}{\alpha_j} + \frac{1}{\beta_j}
\end{aligned}$$

Let $Q_j^* = \text{argmin}_Q\{\mathcal{I}_j(Q_j)\}$. Then $\mathcal{T}_j(D_j(Y))$, the optimal working inventory cost at retailer $j \in J$ in (1), is equal to $\mathcal{I}_j(Q_j^*)$ when $D_j(Y) > 0$ and to 0 otherwise.

Qi, Shen and Snyder (2009) suggest efficient solution algorithms to compute Q_j^* based on the cost function (6). Unfortunately, it is difficult to analyze **(P)** or to solve it using standard algorithms since $\mathcal{T}_j(D_j(Y))$ cannot be written in closed form without solving a separate non-linear optimization problem. Though numerical experiments by Qi, Shen, and Snyder (2009) suggest that $\mathcal{T}_j(D_j(Y))$ is concave when considered as a function of $D_j(Y)$, they do not prove this rigorously, nor does the exact cost function permit a closed-form expression for the optimal cost.

Instead, we use the following approximation for $\mathcal{T}_j(D_j(Y))$:

$$\hat{\mathcal{T}}_j(D_j(Y)) = \begin{cases} \pi_j D_j(Y) + \frac{F_j + \frac{(a_j-\pi_j)D_j(Y)}{\alpha_j} + \left(a_j + \frac{h_j}{\alpha_j}\right)\hat{Q}_j}{\bar{A}_j+\bar{B}_j} & , D_j(Y) > 0 \\ 0 & , D_j(Y) = 0 \end{cases} \tag{7}$$

and the following approximation for Q_j^*:

$$\hat{Q}_j = D_j(Y) \cdot \frac{-\bar{A}_j + \sqrt{\bar{A}_j^2 + \frac{2\alpha_j(\bar{A}_j+\bar{B}_j)\left[\frac{\alpha_j F_j \bar{B}_j}{D_j(Y)} + \bar{A}_j(\pi_j - a_j)\right]}{(\alpha_j a_j + h_j)}}}{(\bar{A}_j + \bar{B}_j)\alpha_j}, \tag{8}$$

both of which are proposed by Qi, Shen, and Snyder (2009).

$\hat{\mathcal{T}}_j(D_j(Y))$ has an important property, stated in the following theorem.

Theorem 1 (Qi, Shen and Snyder, 2009) $\hat{\mathcal{T}}_j(D_j(Y))$ *is a concave and increasing function of* $D_j(Y)$*, the total demand that retailer* j *serves.*

We replace $\mathcal{T}_j(\cdot)$ in the objective function of **(P)** using its approximation $\hat{\mathcal{T}}_j(\cdot)$. The objective function of **(P)** is thus approximated by:

$$\sum_{j\in J}\left\{f_j X_j + \hat{\mathcal{T}}_j(D_j(Y)) + \sum_{i\in I}(\hat{d}_{ij} - \pi)D_i Y_{ij}\right\} \tag{9}$$

We use $(\hat{\mathbf{P}})$ to denote the problem that minimizes (9) subject to the same constraints as **(P)**. In the following sections, we analytically and numerically study $(\hat{\mathbf{P}})$.

4 Model Analysis

It follows from (8) that

$$\hat{Q}_j = \frac{-\bar{A}_j}{(\bar{A}_j + \bar{B}_j)\alpha_j} \cdot D_j(Y)$$

$$+\sqrt{\left[\frac{\bar{A}_j^2}{(\bar{A}_j+\bar{B}_j)^2\alpha_j^2}+\frac{2\bar{A}_j(\pi_j-a_j)}{(\alpha_j a_j+h_j)(\bar{A}_j+\bar{B}_j)\alpha_j}\right]D_j^2(Y)+\frac{2F_j\bar{B}_jD_j(Y)}{(\alpha_j a_j+h_j)(\bar{A}_j+\bar{B}_j)}}$$
$$=-\bar{C}_j\cdot D_j(Y)+\sqrt{\left(\bar{C}_j^2+2\bar{C}_jK_j\right)D_j^2(Y)+\frac{2F_j(1-\alpha_j\bar{C}_j)D_j(Y)}{\alpha_j a_j+h_j}},$$

where we define

$$\bar{C}_j=\frac{\bar{A}_j}{(\bar{A}_j+\bar{B}_j)\alpha_j}=\frac{\lambda}{(\psi+\alpha_j)(\psi+\lambda)}$$
$$K_j=\frac{\pi_j-a_j}{\alpha_j a_j+h_j}$$

to simplify the notation.

Furthermore, we have

$$\frac{\partial}{\partial D_j(Y)}\hat{Q}_j=-\bar{C}_j+\frac{\left(\bar{C}_j^2+2\bar{C}_jK_j\right)D_j(Y)+\frac{F_j(1-\alpha_j\bar{C}_j)}{\alpha_j a_j+h_j}}{\sqrt{\left(\bar{C}_j^2+2\bar{C}_jK_j\right)D_j^2(Y)+\frac{2F_j(1-\alpha_j\bar{C}_j)D_j(Y)}{\alpha_j a_j+h_j}}} \tag{10}$$

$$\frac{\partial^2}{\partial D_j^2(Y)}\hat{Q}_j=-\frac{\left[\frac{F_j(1-\alpha_j\bar{C}_j)}{\alpha_j a_j+h_j}\right]^2}{\left[\sqrt{\left(\bar{C}_j^2+2\bar{C}_jK_j\right)D_j^2(Y)+\frac{2F_j(1-\alpha_j\bar{C}_j)D_j(Y)}{\alpha_j a_j+h_j}}\right]^3}$$
$$=-\frac{\left[\frac{F_j}{\alpha_j a_j+h_j}\right]^2\sqrt{1-\alpha_j\bar{C}_j}}{\left[\sqrt{\frac{\bar{C}_j}{1-\alpha_j\bar{C}_j}\cdot(\bar{C}_j+2K_j)\,D_j^2(Y)+\frac{2F_jD_j(Y)}{\alpha_j a_j+h_j}}\right]^3}<0 \tag{11}$$

Let

$$L_j=\pi_j-\frac{\alpha_j a_j+h_j}{\alpha_j(\bar{A}_j+\bar{B}_j)}\cdot(\bar{C}_j+K_j-\sqrt{\bar{C}_j^2+2\bar{C}_j\cdot K_j}). \tag{12}$$

Lemma 1 *L_j provides a lower bound on the marginal working inventory cost at retailer $j\in J$ regardless of the demand already assigned to this retailer.*

Proof It follows from (10) and (11) that

$$\frac{\partial}{\partial D_j(Y)}\hat{Q}_j>-\bar{C}_j+\sqrt{\bar{C}_j^2+2\bar{C}_jK_j}\qquad\text{for } D_j(Y)>0.$$

Therefore, the following inequality can be derived from (7):

$$\begin{aligned}\frac{\partial}{\partial D_j(Y)}\hat{\mathcal{T}}_j(D_j(Y)) &> \pi_j + \frac{\frac{a_j-\pi_j}{\alpha_j} + (a_j + \frac{h_j}{\alpha_j})(-\bar{C}_j + \sqrt{\bar{C}_j^2 + 2\bar{C}_j \cdot K_j})}{\bar{A}_j + \bar{B}_j} \\ &= \pi_j - \frac{\alpha_j a_j + h_j}{\alpha_j(\bar{A}_j + \bar{B}_j)} \cdot (\bar{C}_j + K_j - \sqrt{\bar{C}_j^2 + 2\bar{C}_j \cdot K_j}) \\ &= L_j,\end{aligned}$$

as desired. ■

The following proposition provides a necessary condition for a given customer to be served by a given retailer in the optimal solution to ($\hat{\mathbf{P}}$). It follows from Lemma 1 and the fact that a customer should not be served by a retailer if the sum of the retailer's working inventory cost and the transportation cost is larger than the lost-sales penalty for not serving this customer. We omit a formal proof.

Proposition 1 *If customer $i \in I$ is served by retailer $j \in J$ in an optimal solution to ($\hat{\mathbf{P}}$), then $\pi > L_j + \hat{d}_{ij}$.*

We can rewrite L_j as

$$\begin{aligned}L_j &= \pi_j - \frac{\alpha_j a_j + h_j}{\alpha_j(\bar{A}_j + \bar{B}_j)} \cdot \left(K_j - \frac{2\bar{C}_j \cdot K_j}{\bar{C}_j + \sqrt{\bar{C}_j^2 + 2\bar{C}_j \cdot K_j}}\right) \\ &= \pi_j - \frac{\pi_j - a_j}{\alpha_j(\bar{A}_j + \bar{B}_j)} \cdot \left(1 - \frac{2\bar{C}_j}{\bar{C}_j + \sqrt{\bar{C}_j^2 + 2\bar{C}_j \cdot K_j}}\right) \\ &= \pi_j - \frac{\pi_j - a_j}{\alpha_j(\bar{A}_j + \bar{B}_j)} \cdot \left(1 - \frac{2}{1 + \sqrt{1 + \frac{2K_j}{\bar{C}_j}}}\right).\end{aligned}$$

Therefore, L_j is an increasing function of $\bar{A}_j$, $\bar{B}_j$ and $\bar{C}_j$. On the other hand, it is easy to see that $\bar{A}_j$ and $\bar{C}_j$ are both increasing functions of λ and decreasing functions of ψ, and that $\bar{A}_j$ and $\bar{B}_j$

are both decreasing functions of β_j. Therefore, L_j is an increasing function of λ and a decreasing function of ψ and β_j. Hence, Proposition 1 implies:

- When the supplier is more likely to be disrupted, or the recovery processes at the supplier or retailers are slower, fewer customers should be served by each open retailer, and the optimal solution will involve more customers not served by any retailer.
- Retailers are more likely to be opened at locations with quick recoveries, and customers are more likely to be served by retailers at these locations.

These conclusions conform with our intuition that to improve the service level or reduce the extra operational costs caused by disruptions, reliable suppliers are preferred, and retailers should be opened in low risk areas.

We are not able to analyze the impact of α_j on the supply chain design decisions because of the complexity of L_j as a function of α_j. We numerically study these effects in Section 6.2.

5 Solution Algorithm

Theorem 1 allows us to apply the algorithm proposed by Daskin, Coullard, and Shen (2002) to solve Problem $\hat{\mathrm{P}}$. The detailed solution algorithm, a Lagrangian relaxation approach embedded in branch and bound, is as follows:

Step I: Finding a Lower Bound

Relaxing the first constraint in $(\hat{\mathbf{P}})$ with Lagrange multipliers ω, we obtain the following Lagrangian dual problem:

$$\begin{aligned}
\max_{\omega} \ \min_{\mathbf{X},\mathbf{Y}} \quad & \sum_{j\in J}\left\{ f_j X_j + \hat{T}_j(D_j(Y)) + \sum_{i\in I}(\hat{d}_{ij}-\pi)D_i Y_{ij}\right\} + \sum_{i\in I}\omega_i\left(\sum_{j\in J}Y_{ij}-1\right) \\
= \quad & \sum_{j\in J}\left\{ f_j X_j + \hat{T}_j(D_j(Y)) + \sum_{i\in I}\left[(\hat{d}_{ij}-\pi)D_i+\omega_i\right]Y_{ij}\right\} - \sum_{i\in I}\omega_i
\end{aligned}$$

$$\begin{aligned} \text{subject to} \quad & Y_{ij} - X_j \le 0 && \forall i \in I, j \in J \\ & Y_{ij} \in \{0,1\} && \forall i \in I, j \in J \\ & X_j \in \{0,1\} && \forall j \in J \\ & \omega_i \ge 0 && \forall i \in I \end{aligned}$$

The optimal objective value of the Lagrangian dual problem provides a lower bound on the optimal objective value of $(\hat{\mathbf{P}})$. We use the standard subgradient optimization procedure discussed by Fisher (1981) to seek the optimal Lagrange multipliers. In each iteration of the Lagrangian procedure, ω_i is fixed for each $i \in I$. The resulting problem decomposes by j, and therefore, we need to solve the following subproblem for each candidate location $j \in J$:

$$\begin{aligned} (\mathbf{SP}_j) \qquad \tilde{V}_j \equiv \min \;\; & \hat{\mathcal{T}}_j\left(\sum_{i\in I} D_i Z_i\right) + \sum_{i \in I}\left[(\hat{d}_{ij} - \pi)D_i + \omega_i\right] Z_i \\ \text{subject to} \quad & Z_i \in \{0,1\} \qquad \forall i \in I \end{aligned}$$

We use Z_i^*, $i \in I$, to denote the optimal solution to $(\mathbf{SP}_j)$ for a given j. Then for each $j \in J$, in the optimal solution to the Lagrangian dual problem, $X_j = 1$ and $Y_{ij} = Z_i^*$ for all $i \in I$ if $f_j + \tilde{V}_j \le 0$, and $X_j = Y_{ij} = 0$ otherwise.

(The algorithm proposed by Daskin, Coullard and Shen, 2002 sets $X_j = 1$ for the j that minimizes $f_j + \tilde{V}_j$ if $f_j + \tilde{V}_j > 0$ for all $j \in J$, since they have an implicit constraint that $\sum_{j \in J} X_j \ge 1$. Our model does not have such an implicit constraint since one could open no facilities and assign every customer to the dummy facility s. Therefore, in our algorithm $X_j = 0$ for all j if $f_j + \tilde{V}_j > 0$.)

Shen, Coullard and Daskin (2003) propose an efficient and exact solution algorithm with complexity $O(|I| \log |I|)$ for a subproblem that is structurally identical to $(\mathbf{SP}_j)$, provided that $\hat{\mathcal{T}}_j(\cdot)$ is concave. Modified to our problem, their algorithm is as follows:

1. Define $I^- = \{i \in I : (\hat{d}_{ij} - \pi)D_i + \omega_i < 0\}$.

2. Sort the elements in I^- in increasing order of $[(\hat{d}_{ij} - \pi)D_i + \omega_i]/D_i$, and denote the resulting elements by 1^-, 2^-... n^-, respectively, where $n = |I^-|$.

3. Find the value of m $(1 \leq m \leq n)$ that minimizes

$$\hat{\mathcal{T}}_j \left(\sum_{i=1}^{m} D_{i^-} Z_{i^-} \right) + \sum_{i=1}^{m} \left[(\hat{d}_{i^- j} - \pi) D_{i^-} + \omega_{i^-} \right] Z_{i^-}.$$

4. Then an optimal solution to subproblem $(\mathbf{SP}_j)$ is given by $Z_{1^-} = Z_{2^-} = ... = Z_{m^-} = 1$ and $Z_i = 0$ for all other $i \in I$.

Step II: Finding an Upper Bound

In each iteration of the Lagrangian procedure, we derive a feasible solution to Problem $\hat{\mathrm{P}}$ based on the current Lagrangian solution by assigning each customer to one and only one retailer.

Note that assigning customers optimally to a fixed set of open retailers is not trivial in our problem, as it is in most linear facility location problems. Although the customer-assignment problem is polynomially solvable for a fixed number of open retailers (it is a special case of the [PTP(k)] problem discussed by Tuy et al., 1996), exact algorithms for this problem, such as the one proposed by Tuy et al., are impractical when the number of retailers is reasonably large. Therefore, we employ the following heuristic procedure, adapted from Shen, Coullard and Daskin (2003), to assign customers to retailers:

1. For each customer $i \in I$, let J_i be the set of potential retailer sites $(J_i \subseteq J)$ that customer i is assigned to in the Lagrangian solution. If $J_i \neq \emptyset$, we assign customer i to the retailer in $J_i \cup \{s\}$ that results in the least increase in cost. If $J_i = \emptyset$, we assign this customer to the dummy retailer s.

2. We close all open retailers that no longer serve any customers after performing step 1.

If the objective value of $(\hat{\mathbf{P}})$ under the resulting feasible solution is less than the current upper bound, we take the objective value of the new solution as the new upper bound, and further improve it using Step III. (One may choose to perform Step III even if the new solution is not better than the best known solution. However, we found that this strategy does not significantly enhance the algorithm's performance and in many cases makes the algorithm slower.)

Step III: Customer Reassignment

1. For each $i \in I$, we search the other open retailers (including the dummy retailer s) to see whether the cost would decrease if we assigned customer i instead to that retailer. We perform the best improving swap found.

2. We close all open retailers that no longer serve any customers after performing step 1.

We then recalculate the objective value of $(\hat{\mathbf{P}})$ with the new feasible solution obtained in this step, and update the upper bound if necessary.

Step IV: Variable Fixing

At the end of each iteration of the Lagrangian procedure, we employ the following two rules to see whether some of the X_j variables can be fixed. (See Shu, Teo, and Shen, 2005 for detailed proofs of the validity of these rules.)

Let LB and UB be the current lower and upper bounds on the optimal objective value, respectively.

Rule 1: If no retailer is opened at candidate location $j \in J$ in the Lagrangian solution obtained in Step I, and if $LB + f_j + \tilde{V}_j > UB$, then no retailer will be opened at location j in any optimal solution to $(\hat{\mathbf{P}})$, so we fix $X_j = 0$.

Rule 2: If a retailer is opened at candidate location $j \in J$ in the Lagrangian solution obtained in Step I, and if $LB - (f_j + \tilde{V}_j) > UB$, then a retailer will be opened at location j in every optimal solution to $(\hat{\mathbf{P}})$, so we fix $X_j = 1$.

If the lower and upper bounds are sufficiently close (please refer to Table 1), or if all candidate locations are fixed with the above two rules, we terminate the Lagrangian procedure. In either of these cases, the solution corresponding to the upper bound is a (near-)optimal solution to ($\hat{\mathbf{P}}$). We also stop the Lagrangian procedure based on certain conditions regarding the current Lagrangian settings (please refer to Table 1); in this case, we conduct branch and bound, branching on the unfixed location variables (X_j, $j \in J$).

Proposition 1 can also be used to filter out solutions that are impossible to make the algorithm more efficient. Based on this proposition, customer $i \in I$ should not be served by retailer $j \in J$ in the optimal solution if $\pi \leq L_j + \hat{d}_{ij}$.

6 Numerical Experiments

In this section, we report the results of our computational experiments to verify the conclusions drawn in Section 4. We also study the benefit of considering supply disruptions during the supply chain design phase.

We conduct computational experiments on the 88-node and 150-node data sets described by Daskin (1995). The weight factors associated with the transportation and inventory costs we used in our experiments on the 88-node data set are 0.005 and 0.1, respectively, and those we used in experiments on the 150-node data set are 0.0008 and 0.01, respectively. We fix the per-unit penalty cost for not serving customers, π, which is not included in the original data sets, to be 25. As in Daskin, Coullard and Shen (2002), we fix the fixed ordering cost, unit ordering cost and unit holding cost at retailer $j \in J$ (F_j, a_j and h_j) to be 10, 5, and 1, respectively.

Table 1 lists the parameters we used for the Lagrangian relaxation procedure in our computational experiments. Please refer to Fisher (1981) and Daskin, Coullard, and Shen (2002) for more details on the interpretation of these parameters. With these parameters, the algorithm solved the problems within 10 seconds for most instances associated with the 88-node data set and within 60 seconds for most instances associated with the 150-node data set. We do not provide a detailed report of the computational performance of the algorithm, since its efficiency has already been documented by Daskin, Coullard and Shen (2002), Shen and Qi (2007), Snyder, Daskin, and Teo (2004) and others.

Table 1: Parameters for Lagrangian relaxation

Parameter	**Value**
Initial value of the scalar used to define the step size	2
Minimum value of the scalar	1×10^{-10}
Maximum number of iterations before halving the scalar	12
Minimum gap between the upper bound and the lower bound	0.1
Initial Lagrange multiplier value	0

6.1 The Effect of Disruptions at the Supplier on Location Decisions

In this section, we test how the presence of supplier disruptions affects the optimal location decisions. For both the 88-node and 150-node data sets, we uniformly drew the disruption rate α_j from $[0.5, 2]$ and the recovery rate β_j from $[18, 30]$ for all $j \in J$. Similarly, we uniformly generated the time-independent backorder cost π_j from $[8, 16]$. For the randomly generated instance, we varied the disruption rate λ and the recovery rate ψ at the supplier. Table 2 indicates how the optimal solution changes as the values of λ and ψ vary.

For each pair of λ and ψ, and for each data set, Table 2 lists the objective value (in the column labeled "Total Cost") of $(\hat{\mathbf{P}})$ corresponding to the solution computed by the algorithm described in Section 5. The column labeled "Number of Opened Retailers" reports the number of candidate locations opened in that solution. Recall that when we formulated problem **(P)** in Section 3, we dropped the constant term $\pi \sum_{i\in I} D_i$ from the objective function, but this term is included in the total costs in Table 2, as well as all subsequent tables and figures.

Figures 2 and 3 plot the results in Table 2 graphically, illustrating the relationships between:

Table 2: The effect of the availability of the supplier on the optimal location decisions.

λ	ψ	# of Nodes	Total Cost	# of Opened Retailers	# of Nodes	Total Cost	# of Opened Retailers
0	12	88	276767.08	14	150	333286.37	30
0.01	12		277286.85	15		333834.36	30
0.05	12		278954.51	15		335607.10	30
0.1	12		280601.75	15		337391.01	30
0.5	12		288702.56	16		346615.31	31
1	12		294839.21	16		353874.28	29
4	12		312382.05	14		374168.50	27
8	12		322345.30	13		385723.81	27
12	12		327646.43	13		392088.10	26
1	6	88	313632.55	14	150	375981.89	28
1	12		294839.21	16		353874.28	29
1	18		287901.07	16		345635.92	31
1	24		284397.19	16		341599.14	30
1	48		279499.94	15		336151.49	30
1	96		277599.60	15		334151.26	30
1	10000		276767.16	14		333286.46	30

- the optimal total cost and the supplier disruption rate,
- the optimal number of opened retailers and the supplier disruption rate,
- the optimal total cost and the supplier recovery rate,
- and the optimal number of opened retailers and the supplier recovery rate.

Table 2 and Figures 2 and 3 indicate that the number of opened retailers increases as the availability of the supplier increases (i.e., λ decreases or ψ increases) and then becomes non-increasing as the availability of the supplier further increases. In other words, if the availability of the supplier increases, the number of opened retailers may increase at the very beginning; however, once this number decreases, it will never increase again.

An informal explanation of the above observation is as follows.

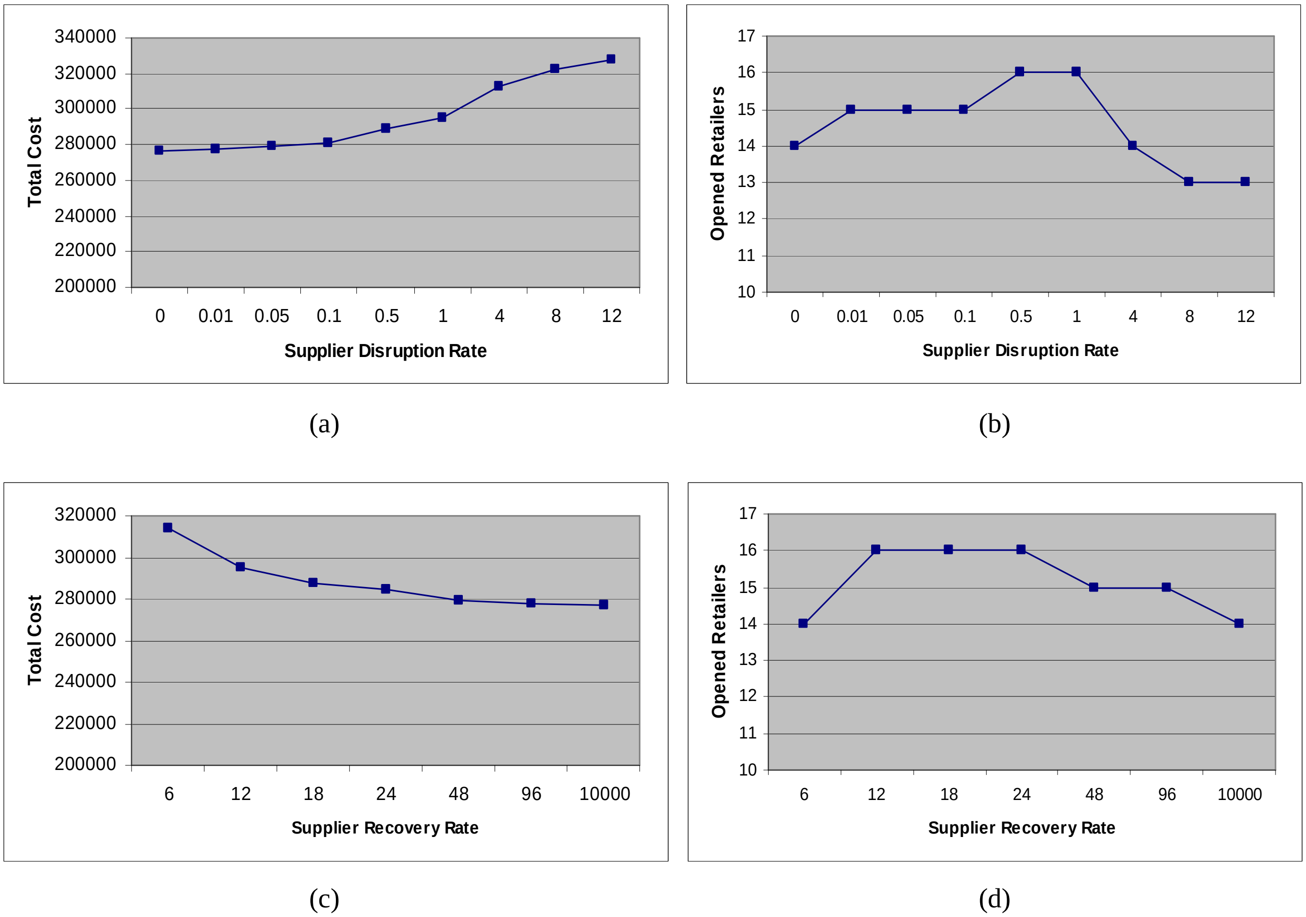
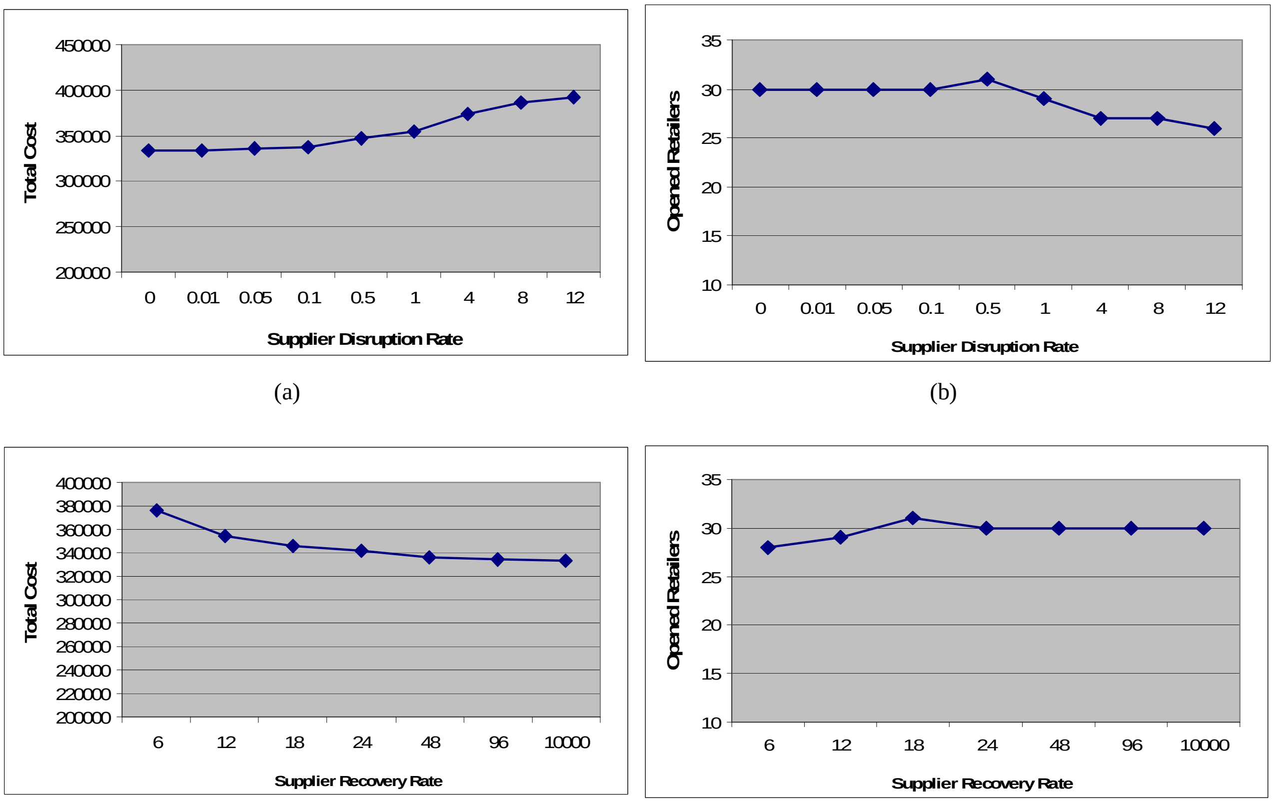


Figure 2: The effect of the availability of the supplier on the optimal location decisions for the 88-node data set.

When the availability of the supplier increases, the working inventory cost at each opened retailer becomes smaller and relatively unimportant. Therefore, the number of opened retailers goes up (similar conclusions can be found in Daskin, Coullard and Shen, 2002). However, if the availability of the supplier further increases, based on (11) and the fact that $\bar{C}_j = \frac{\lambda}{(\psi+\alpha_j)(\psi+\lambda)}$ is an increasing function of λ and a decreasing function of ψ, $\frac{\partial^2}{\partial D_j^2(Y)}\hat{Q}_j(D_j(Y))$ would be further decreasing. It, therefore, follows from (7) that $\frac{\partial^2}{\partial D_j^2(Y)}\hat{\mathcal{T}}_j(D_j(Y))$ becomes much smaller for given $D_j(Y)$ when the availability of the supplier further increases, which makes $\hat{\mathcal{T}}_j(D_j(Y))$ more concave as a function of $D_j(Y)$ based on Theorem 1. Thus, the consolidation of customer demand becomes a more attractive strategy at this time.



(c) (d)

Figure 3: The effect of the availability of the supplier on the optimal location decisions for the 150-node data set.

6.2 The Effect of Disruptions at the Retailers on Location Decisions

We next study the effect of retailer disruptions on the optimal retailer location decisions.

In Section 4, when we studied the properties of our model, we were not able to analytically determine the impact of the retailer disruption rate on the location and demand-allocation decisions. Our extensive numerical experiments show that if a retailer is more likely to be disrupted, fewer customers will be assigned to this retailer in the optimal solution; if a retailer is disrupted too often, then it will be closed, and the customers originally served by the retailer will be assigned to other open retailers or to a newly opened retailer. This result is intuitive, so we do not report our computational results here.

The next set of experiments evaluates how the optimal total cost and number of opened retailers change as the retailer disruption parameters change. These experiments use the same data sets used in the experiments in Section 6.1. We multiplied the α_j values by the scalars listed in Table 3, and multiplied the β_j values by the scalars listed in Table 4. The optimal cost and number of retailers opened are listed in Tables 3 and 4.

Table 3: The effect of the disruption rates at the retailers on the optimal location decisions.

α_j Multiplier	# of Nodes	Total Cost	# of Opened Retailers	# of Nodes	Total Cost	# of Opened Retailers
0.125	88	274269.06	21	150	329239.02	52
0.25		279395.85	21		335101.83	44
0.5		285619.70	16		342850.67	35
1		294839.21	16		353874.28	29
2		308045.24	15		367953.17	25
4		324901.20	11		387160.34	20
8		343593.78	10		410860.28	17

Table 4: The effect of the recovery rates at the retailers on the optimal location decisions.

β_j Multiplier	# of Nodes	Total Cost	# of Opened Retailers	# of Nodes	Total Cost	# of Opened Retailers
0.0625	88	342488.08	9	150	406058.75	18
0.125		325529.82	12		385028.78	21
0.25		311952.99	13		369741.73	27
0.5		301344.58	16		359901.35	27
1		294839.21	16		353874.28	29
2		291164.67	16		350049.93	32
4		289212.20	16		347920.20	33
8		288187.61	16		346799.72	34

From Tables 3 and 4, we can tell that if the candidate retailers are more likely to be disrupted, or if their recovery processes are slower, then fewer retailers are opened. This conforms with our conclusions made from Proposition 1.

6.3 The Benefit of Considering Supply Disruptions in the Supply Chain Design Phase

In this section we compare the performance of the following two supply chain design methods:

- Integrated Approach: consider supply disruptions when we make all supply chain design decisions including location, demand-assignment and inventory decisions, as we do in this paper; in other words, design supply chain networks according to the optimal solution to ($\hat{\mathbf{P}}$).
- Sequential Approach: make location and demand-assignment decisions using the supply chain design model introduced by Daskin, Coullard, and Shen (2002) without taking supply disruptions into consideration; then at the operational phase, make inventory decisions at opened retailers using the inventory model proposed by Qi, Shen and Snyder (2009), which considers supply disruptions.

The integrated approach considers supply disruptions in the supply chain design phase, while the sequential approach considers supply disruptions only in the operational phase. By comparing these two methods numerically, we demonstrate the benefit of our integrated model.

We used the same instances as in Section 6.1. For each pair of λ and ψ, and for each data set (88-node and 150-node), we apply the integrated and sequential approaches to derive the total costs, which we denote by TC_D and TC_S, respectively, and then calculate the cost increase for the sequential approach. As in Tables 2-4, when we calculate the values of TC_S and TC_D in the tables below, we add the term $\pi \sum_{i \in I} D_i$ to ensure that TC_S and TC_D represent the actual total costs.

Table 5 lists the costs and cost differences for the baseline data set. It shows that the benefit of using the integrated approach can be significant—up to 25%—and that the benefit increases as the supplier disruption rate increases or recovery rate decreases.

Next, we increased the disruption rates α_j at all candidate locations by multiplying them each by 2 (in Table 6) and 4 (in Table 7). By comparing these two tables with Table 5, we see that the benefit of using the integrated approach becomes pronounced as the retailer disruption rates

Table 5: The benefit of considering supply disruptions in the design phase for the baseline data set.

λ	ψ	# of Nodes	TC_D	TC_S	Cost Difference (%)
0	12	88	276767.08	292094.41	5.54
0.01	12		277286.85	292630.31	5.53
0.05	12		278954.51	294517.56	5.58
0.1	12		280601.75	296525.63	5.67
0.5	12		288702.56	307385.38	6.47
1	12		294839.21	316125.91	7.22
4	12		312382.05	342178.31	9.54
8	12		322345.30	357498.06	10.91
12	12		327646.43	365962.28	11.69
1	6	88	313632.55	341480.47	8.88
1	12		294839.21	316125.91	7.22
1	24		284397.19	301865.50	6.14
1	48		279499.94	295384.25	5.68
1	96		277599.60	293044.00	5.56
1	10000		276767.16	292094.50	5.54
0	12	150	333286.37	377301.31	13.21
0.01	12		333834.36	378084.41	13.26
0.05	12		335607.10	380783.22	13.46
0.1	12		337391.01	383628.66	13.70
0.5	12		346615.31	399278.69	15.19
1	12		353874.28	412264.09	16.50
4	12		374168.50	452270.03	20.87
8	12		385723.81	476354.66	23.50
12	12		392088.10	489781.72	24.92
1	6	150	375981.89	449949.28	19.67
1	12		353874.28	412264.09	16.50
1	24		341599.14	391487.81	14.60
1	48		336151.49	382188.00	13.70
1	96		334151.26	378763.50	13.35
1	10000		333286.46	377301.50	13.21

increase, especially if the sequential approach "unluckily" suggests opening retailers at locations with large disruption rates.

Finally, we decreased the recovery rates β_j at all candidate locations by multiplying them each by 0.5 (in Table 8) and 0.25 (in Table 9). We can see from these two tables that when the recovery rates at these candidate locations are small, the advantage from using the integrated approach is significant.

Figures 4 and 5 illustrate the results in Tables 5-9 graphically. For both the 88-node and 150-node data sets, Figures 4 and 5 show the relationships between the cost savings from considering supply disruptions in the design phase and the disruption and recovery rates at the retailers and the supplier.

We conclude that significant cost savings may be realized if disruptions are considered during the supply chain design phase, especially if the supplier or candidate retailers are often unavailable (e.g., the disruption rate is at least once per year, and the recovery rate is at most 24 times per year).

7 Stochastic Demand

In Section 3, we assume that the demand is deterministic when we formulate the working inventory cost at open retailers. We now relax this assumption and numerically compare the solutions to the stochastic-demand model and the original model to see how this assumption affects supply chain design decisions.

When the demand from customer i ($i \in I$) follows a Poisson process with rate D_i, the annual demand of this customer follows a Poisson distribution with parameter D_i, whose well known approximation is a normal distribution with both mean and variance equal to D_i. In addition, since we assume that demands from customers follow independent Poisson processes, the normal distributions associated with customer demands in our problem are all independent. Therefore, the

Table 6: The benefit of considering supply disruptions in the design phase when α_j is multiplied by 2 for all $j \in J$.

λ	ψ	# of Nodes	TC_D	TC_S	Cost Difference (%)
0	12	88	286594.36	315849.03	10.21
0.01	12		287226.59	316436.59	10.17
0.05	12		289255.37	318519.09	10.12
0.1	12		291240.88	320752.06	10.13
0.5	12		300799.04	333001.00	10.71
1	12		308045.24	342946.13	11.33
4	12		328353.67	372510.94	13.45
8	12		339192.12	389679.69	14.88
12	12		344416.35	399068.97	15.87
1	6	88	328807.45	368996.44	12.22
1	12		308045.24	342946.13	11.33
1	24		295910.60	327289.22	10.60
1	48		290037.38	319813.22	10.27
1	96		287663.23	317016.06	10.20
1	10000		286594.47	315849.16	10.21
0	12	150	343892.11	422257.22	22.79
0.01	12		344568.91	423050.28	22.78
0.05	12		346781.88	425853.91	22.80
0.1	12		348982.29	428871.75	22.89
0.5	12		359749.16	445875.78	23.94
1	12		367953.17	460178.28	25.06
4	12		391814.12	504301.88	28.71
8	12		405215.35	530608.44	30.94
12	12		412466.36	545138.06	32.17
1	6	150	392870.70	497223.59	26.56
1	12		367953.17	460178.28	25.06
1	24		354103.51	438269.69	23.77
1	48		347531.56	427932.16	23.13
1	96		344987.87	423988.81	22.90
1	10000		343892.22	422257.38	22.79

Table 7: The benefit of considering supply disruptions in the design phase when α_j is multiplied by 4 for all $j \in J$.

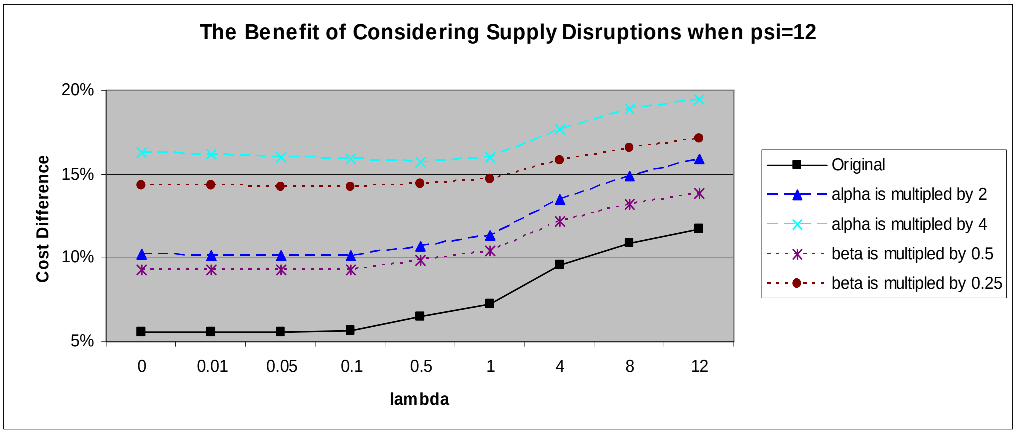
λ	ψ	# of Nodes	TC_D	TC_S	Cost Difference (%)
0	12	88	300643.97	349583.88	16.28
0.01	12		301348.46	350153.75	16.20
0.05	12		303628.21	352190.53	15.99
0.1	12		305857.29	354395.09	15.87
0.5	12		316807.59	366712.66	15.75
1	12		324901.20	376842.47	15.99
4	12		345745.36	406933.22	17.70
8	12		356900.87	424181.66	18.85
12	12		362928.64	433510.63	19.45
1	6	88	345535.10	400484.13	15.90
1	12		324901.20	376842.47	15.99
1	24		311433.43	361667.47	16.13
1	48		304647.89	353958.66	16.19
1	96		301897.47	350915.13	16.24
1	10000		300644.11	349584.03	16.28
0	12	150	358520.40	483347.41	34.82
0.01	12		359311.43	484024.72	34.71
0.05	12		361898.49	486494.69	34.43
0.1	12		364477.84	489228.38	34.23
0.5	12		377327.47	507188.22	34.42
1	12		387160.34	519899.78	34.29
4	12		414438.52	561427.75	35.47
8	12		429563.63	586568.63	36.55
12	12		437729.13	600325.44	37.15
1	6	150	414170.09	560268.13	35.27
1	12		387160.34	519899.78	34.29
1	24		370780.98	499115.84	34.61
1	48		362932.85	489179.38	34.79
1	96		359860.40	485187.81	34.83
1	10000		358520.54	483347.63	34.82

Table 8: The benefit of considering supply disruptions in the design phase when β_j is multiplied by 0.5 for all $j \in J$.

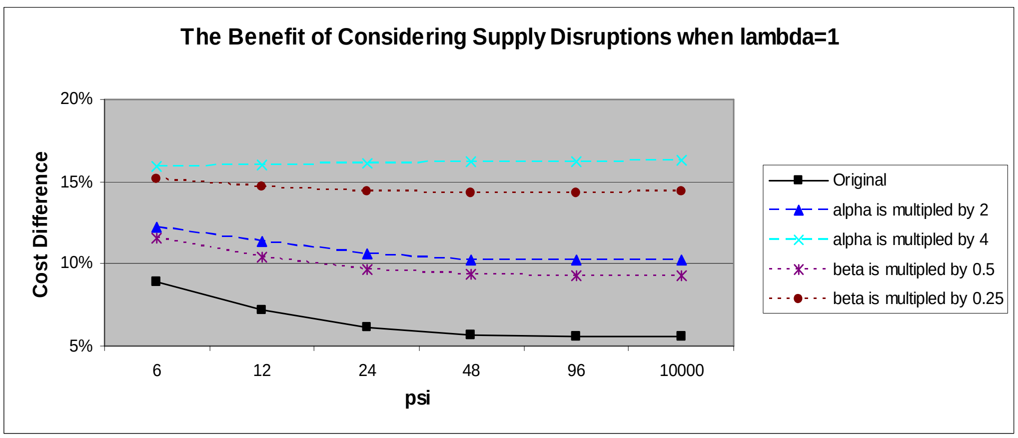
λ	ψ	# of Nodes	TC_D	TC_S	Cost Difference (%)
0	12	88	284472.82	310831.09	9.27
0.01	12		284950.46	311322.28	9.25
0.05	12		286505.22	313048.94	9.26
0.1	12		288043.99	314882.63	9.32
0.5	12		295581.49	324778.13	9.88
1	12		301344.58	332734.13	10.42
4	12		317831.69	356442.81	12.15
8	12		327181.71	370388.22	13.21
12	12		332123.32	378095.53	13.84
1	6	88	319069.47	355875.09	11.54
1	12		301344.58	332734.13	10.42
1	24		291579.30	319740.22	9.66
1	48		287007.03	313834.19	9.35
1	96		285238.02	311698.75	9.28
1	10000		284472.90	310831.16	9.27
0	12	150	340625.69	411817.72	20.90
0.01	12		341150.55	412505.94	20.92
0.05	12		342873.81	414886.06	21.00
0.1	12		344598.22	417404.22	21.13
0.5	12		353280.11	431308.16	22.09
1	12		359901.35	442870.50	23.05
4	12		379081.51	478535.66	26.24
8	12		390286.03	500022.28	28.12
12	12		396311.78	512004.84	29.19
1	6	150	380916.19	476566.25	25.11
1	12		359901.35	442870.50	23.05
1	24		348594.86	424363.69	21.74
1	48		343389.99	416119.50	21.18
1	96		341447.79	413100.34	20.98
1	10000		340625.78	411817.81	20.90

Table 9: The benefit of considering supply disruptions in the design phase when β_j is multiplied by 0.25 for all $j \in J$.

λ	ψ	# of Nodes	TC_D	TC_S	Cost Difference (%)
0	12	88	296541.50	339100.41	14.35
0.01	12		296995.36	339524.34	14.32
0.05	12		298447.02	341010.03	14.26
0.1	12		299853.47	342583.19	14.25
0.5	12		306739.44	351044.78	14.44
1	12		311952.99	357836.94	14.71
4	12		326593.57	378071.00	15.76
8	12		334618.96	389977.97	16.54
12	12		338679.62	396561.84	17.09
1	6	88	327864.01	377679.63	15.19
1	12		311952.99	357836.94	14.71
1	24		303050.70	346725.56	14.41
1	48		298887.02	341675.16	14.32
1	96		297256.63	339845.53	14.33
1	10000		296541.57	339100.50	14.35
0	12	150	351862.75	461380.53	31.13
0.01	12		352362.73	461938.63	31.10
0.05	12		353988.02	463878.53	31.04
0.1	12		355596.69	465940.75	31.03
0.5	12		363570.20	477392.63	31.31
1	12		369741.73	486947.25	31.70
4	12		387746.64	516470.59	33.20
8	12		398031.32	534276.13	34.23
12	12		403690.46	544210.75	34.81
1	6	150	389399.81	514970.25	32.25
1	12		369741.73	486947.25	31.70
1	24		359234.14	471644.44	31.29
1	48		354456.45	464875.81	31.15
1	96		352639.66	462417.38	31.13
1	10000		351862.83	461380.59	31.13



(a)

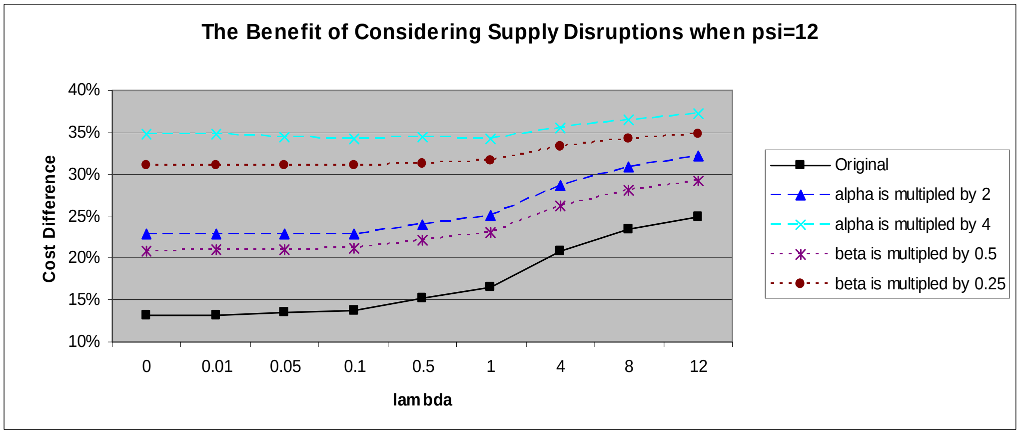


(b)

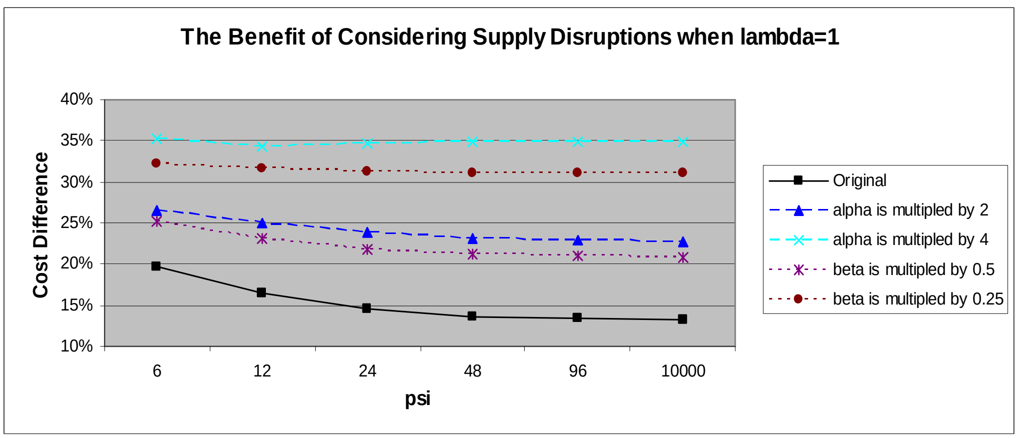
Figure 4: The benefit of considering supply disruptions in the design phase for the 88-node data set.

total customer demand retailer $j \in J$ faces is normally distributed with mean and variance both equal to $D_j(Y)$.

To formulate the safety inventory cost at open retailers to protect against customer demand uncertainty, we first need to derive the mean and variance of the lead time for inventory replenishment at each retailer. We use μ_{L_j} and $\sigma^2_{L_j}$ $(j \in J)$ to represent these two parameters associated with the lead time L_j at retailer j, respectively. For our problem, which uses a ZIO policy to manage the retailer inventory, the lead time is just the duration that the retailer does not have inventory on hand. When both the supplier and the retailers may be disrupted, it is very difficult to derive



(a)



(b)

Figure 5: The benefit of considering supply disruptions in the design phase for the 150-node data set.

an expression for $\sigma^2_{L_j}$. Therefore, in this section we consider only supplier disruptions; we assume that retailers cannot be disrupted.

When retailer j is non-disruptive, it follows from the Two-State Chain analysis in Ross (1996), p. 243 that the probability that the supplier is OFF when retailer j places an order to it is

$$\phi_j \equiv E\left[\frac{\lambda}{\lambda+\psi}\left(1-e^{-(\lambda+\psi)t}\right)\right],$$

where t is the time from the beginning of an inventory cycle until the retailer uses up its on-hand inventory. Since the demands the retailer faces follow a Poisson process with rate $D_j(Y)$, $t \sim Gamma(Q_j^*, D_j(Y))$. Therefore,

$$\phi_j = \frac{\lambda}{\lambda+\psi}\left\{1 - E\left[e^{-(\lambda+\psi)t}\right]\right\} = \frac{\lambda}{\lambda+\psi}\left[1 - \left(\frac{D_j(Y)}{\lambda+\psi+D_j(Y)}\right)^{Q_j^*}\right] = \frac{\lambda}{\lambda+\psi}\left[1 - e^{-\frac{(\lambda+\psi)Q_j^*}{D_j(Y)}\frac{D_j(Y)}{\lambda+\psi}\ln\left(\frac{\lambda+\psi}{D_j(Y)}+1\right)}\right]$$

(We still use Q_j^* derived from (6), which is formulated under the deterministic customer demand assumption, as the replenishment quantity at retailer j, because Qi, Shen and Snyder, 2009 demonstrate that Q_j^* derived from (6) is still a good inventory replenishment decision even when customer demands instead follow a Poisson process.)

In practice, $D_j(Y)$ is usually much bigger than $\lambda+\psi$, so $\frac{D_j(Y)}{\lambda+\psi}\ln\left(\frac{\lambda+\psi}{D_j(Y)}+1\right)$ is very close to 1. We therefore ignore it in the expression for ϕ_j, and use

$$\phi_j = \frac{\lambda}{\lambda+\psi}\left[1 - e^{-\frac{(\lambda+\psi)Q_j^*}{D_j(Y)}}\right].$$

Since the supplier's recovery process is assumed to be exponentially distributed with rate ψ, we can derive

$$\mu_{L_j} = \phi_j \cdot \frac{1}{\psi}$$
$$\sigma_{L_j}^2 = E[L_j^2] - \mu_{L_j}^2 = (1-\phi_j)\cdot 0 + \phi_j \cdot \frac{2}{\psi^2} - \phi_j^2 \cdot \frac{1}{\psi^2} = \phi_j \cdot \frac{2}{\psi^2} - \phi_j^2 \cdot \frac{1}{\psi^2}.$$

Therefore, according to Silver and Peterson (1985), the variance of the lead-time demand retailer $j \in J$ faces is

$$\mu_{L_j} D_j(Y) + \sigma_{L_j}^2 D_j^2(Y),$$

and the working inventory cost for holding safety stock required to ensure that stockouts occur at retailer j with probability α or less is

$$hz_\alpha\sqrt{\mu_{L_j} D_j(Y) + \sigma^2_{L_j} D^2_j(Y)},$$

where z_α is the standard normal deviate such that $P(z \leq z_\alpha) = \alpha$.

Based on Qi, Shen and Snyder (2009), we use $\hat{Q}_j$ (see (8)) to approximate Q^*_j. Then, the safety stock cost can be approximated by

$$hz_\alpha\sqrt{\left(\hat{\phi}_j \cdot \frac{1}{\psi}\right) D_j(Y) + \left(\hat{\phi}_j \cdot \frac{2}{\psi^2} - \hat{\phi}^2_j \cdot \frac{1}{\psi^2}\right) D^2_j(Y)} = hz_\alpha\sqrt{\hat{\phi}_j \cdot \frac{D_j(Y)}{\psi}\left(1 + \frac{2D_j(Y)}{\psi} - \hat{\phi}_j \cdot \frac{D_j(Y)}{\psi}\right)},$$

where

$$\hat{\phi}_j \equiv \frac{\lambda}{\lambda + \psi}\left[1 - e^{-\frac{(\lambda+\psi)\hat{Q}_j}{D_j(Y)}}\right]$$

Following the discussions by Shen, Coullard, and Daskin (2003), Daskin, Coullard, and Shen (2002), and Shu, Teo, and Shen (2005), who embed the classical EOQ model into a location-inventory model without disruptions, we add the above safety stock term into (9) to derive an extension of ($\hat{\mathbf{P}}$), which is a location-inventory model that considers both stochastic customer demand and random disruptions at the supplier. We refer to this extended model as ($\hat{\mathbf{P}}'$). ($\hat{\mathbf{P}}'$) has the following objective function:

$$\sum_{j \in J}\left\{f_j X_j + \hat{T}_j(D_j(Y)) + hz_\alpha\sqrt{\hat{\phi}_j \cdot \frac{D_j(Y)}{\psi}\left(1 + \frac{2D_j(Y)}{\psi} - \hat{\phi}_j \cdot \frac{D_j(Y)}{\psi}\right)} + \sum_{i \in I}(\hat{d}_{ij} - \pi) D_i Y_{ij}\right\}$$

and is subject to the same constraints as ($\hat{\mathbf{P}}$). To apply the solution algorithm proposed in Section 5 for this extended problem, we need to show that the safety stock cost term above is a concave function of $D_j(Y)$, which is proved as follows.

We define $f_j(D) \equiv \hat{\phi}_j \cdot \frac{D_j(Y)}{\psi}$, so we only need to show that $\sqrt{f_j(D)\left[1 + \frac{2D_j(Y)}{\psi} - f_j(D)\right]}$ is concave. Its first derivative is

$$\frac{f_j'(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]+f_j(D)\left[\frac{2}{\psi}-f_j'(D)\right]}{2\sqrt{f_j(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]}}$$

and its second derivative is

$$\frac{f_j''(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]+2f_j'(D)\left[\frac{2}{\psi}-f_j'(D)\right]-f_j(D)f_j''(D)}{2\sqrt{f_j(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]}}$$
$$-\frac{\left\{f_j'(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]+f_j(D)\left[\frac{2}{\psi}-f_j'(D)\right]\right\}^2}{4\left\{f_j(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]\right\}^{\frac{3}{2}}}$$
$$=\frac{2f_j(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]f_j''(D)\left[1+\frac{2D_j(Y)}{\psi}-2f_j(D)\right]-\left\{f_j'(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]-f_j(D)\left[\frac{2}{\psi}-f_j'(D)\right]\right\}^2}{4\left\{f_j(D)\left[1+\frac{2D_j(Y)}{\psi}-f_j(D)\right]\right\}^{\frac{3}{2}}}$$

Interested readers can easily show that $f_j''(D)\leq 0$. In addition, since $\hat{\phi}_j \leq 1$, $\frac{D_j(Y)}{\psi} \geq f_j(D)$. Therefore, the above second derivative is non-positive, and hence the safety stock cost term in the objective function of $(\hat{\mathbf{P}}')$ is a concave function of $D_j(Y)$.

In the experiments Table 10 reports, we used the same 88-node data set used in the experiments in Section 6.1 and fixed $z_\alpha = 1.96$. For each pair of λ and ψ, we solve both $(\hat{\mathbf{P}})$ and $(\hat{\mathbf{P}}')$, which considers the safety stock for demand uncertainty, using the algorithm proposed in Section 5, and report the associated total costs and number of opened retailers in Table 10. We also compare the resulting solutions to $(\hat{\mathbf{P}})$ and $(\hat{\mathbf{P}}')$ for each pair of λ and ψ to see the percentage of opened retailers that are identical in the two solutions, and report the percentages in the last column of Table 10.

We make the following observations based on Table 10:

- For every instance tested, the solutions to $(\hat{\mathbf{P}})$ and $(\hat{\mathbf{P}}')$ are identical. (Of course, the costs of these solutions are different under the two models since the objective functions themselves are different.) This is consistent with the observations made by Qi, Shen, and Snyder (2009).

Table 10: Comparisons of the solutions to $(\hat{\mathbf{P}})$ and $(\hat{\mathbf{P}}')$.

λ	ψ	Total Cost of $(\hat{\mathbf{P}})$	# of Opened Retailers in $(\hat{\mathbf{P}})$	Total Cost of $(\hat{\mathbf{P}}')$	# of Opened Retailers in $(\hat{\mathbf{P}}')$	% of Opened Retailers Identical in Two Solutions
0.001	12	254291.58	23	254300.80	23	100%
0.1	12	255149.00	24	255242.95	24	100%
0.5	12	256984.61	24	257190.01	24	100%
1	12	258390.75	24	258672.91	24	100%
4	12	262554.22	24	263039.54	24	100%
1	6	263205.98	24	263961.10	24	100%
1	12	258390.75	24	258672.91	24	100%
1	24	255961.88	24	256064.75	24	100%
1	48	254869.66	24	254906.70	24	100%
1	10000	254278.76	23	254278.78	23	100%

This observation shows that the deterministic-demand assumption is reasonable for our supply chain design problem.

- Supplier disruptions only have a minor impact on supply chain design decisions when the retailers are reliable: the total cost and number of opened retailers do not change much as the availability of the supplier varies in Table 10. This is also consistent with the observations made by Qi, Shen, and Snyder (2009).

- The numbers of opened retailers reported in Table 10 when random supplier disruptions exist are equal to or slightly bigger than those (23 opened retailers) calculated by Daskin, Coullard, and Shen (2002) on the same data set under the assumption that the supplier is always available. By opening more retailers, the solution spreads the risk of supply disruptions out and reduces the impact of any single disruption. This is related to the "risk-diversification effect" studied by Schmitt, Snyder, and Shen (2008).

8 Conclusions

We consider an integrated supply chain design problem in which the supplier and retailers are disrupted randomly. We formulate a nonlinear integer programming model for this problem. An effective approximation of the objective function of this model is used to make the model easier to analyze and to solve using a common solution algorithm.

Our analysis leads us to conclude that:

- when the supplier is disrupted more often, or the recovery processes at the supplier or retailer candidates become slower, it is optimal to serve fewer customers at each opened retailer;
- retailers are more likely to be opened at locations with quick recoveries, and customers are more likely to be served by retailers with higher recovery rates.

In addition, we conduct numerical experiments to verify our analytical conclusions, and numerically show that significant cost savings can usually be achieved if we consider supply disruptions during the supply chain design phase. Our numerical experiments suggest that when the supplier and retailer candidates are not extremely reliable (for each location, the disruption rate is no smaller than once per year, and the recovery rate is no larger than 24 per year), the cost savings from considering supply disruptions are pronounced.

We further discuss an extension in which we relax the deterministic-demand assumption in the original model. Our numerical experiments show that the solutions to the extended model are identical to the solutions to the original model, which means that the original model is robust to the violations of this deterministic-demand assumption. This observation is consistent with the conclusions made by Qi, Shen and Snyder (2009).

Our research can be extended in the following two respects:

- Dynamic sourcing is a topic of our ongoing research—a customer can be temporarily served by other retailer(s) when its assigned retailer is disrupted or out of stock.

- This paper assumes deterministic yields at the supplier and retailers. Random yield will be considered in our future research.

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