Expert Advice for Amateurs*

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Abstract

I analyze strategic information transmission between a biased, perfectly informed expert and a partially informed decision maker. The decision maker can tell whether the state is "high" or "low," and her definition of "high" and "low" is a private information constituting her type. The expert responds to his attenuated influence on the informed decision maker by providing less informative advice. For some types of decision maker, the negative strategic effect outweighs the benefit of information - being informed makes them worse off. The decision maker's information is, however, beneficial when welfare is evaluated before the realization of her type and when the expert's bias is moderate.

Keywords: Amateur; Cheap talk; Expert; Informed decision maker

JEL classification: C72; D82; D83

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"A little Learning is a dang'rous Thing;
Drink deep, or taste not the Pierian spring."
- Alexander Pope

1 Introduction

With a great deal of information only a few clicks away, the boundary between experts and novices blurs. Once the privilege of experts, specialized knowledge is now widely available on encyclopedic websites and through search engines. In the medical arena, for example, websites such as *www.webmd.com* have rendered patients much more informed and sophisticated than their counterparts a decade ago. We can perhaps jump a step ahead by saying that novices no longer exist today, and *amateurs* - those who know but do not know enough to dispense with the help from experts - have emerged to fill the void.

Is evolving from novices into amateurs a welfare improvement for patients or other decision makers? Advocates for consumer education would respond affirmatively; the underlying proposition of consumer education is that more information leads to better decision.[1] Yet it is a well-known result in information economics that more information is not necessarily better. In, for example, the classic lemons model of Akerlof (1970), information, when asymmetrically distributed, can eliminate trades that are otherwise mutually beneficial. In this paper, I examine the effects of decision makers' information in strategic information transmission (Crawford and Sobel, 1982), a setting that captures the interactions between experts and decision makers. The two questions that I explore are : 1) how a biased expert responds to a decision maker who is (partially) informed; and 2) whether in the strategic environment becoming amateurs improves welfare.

I start with Crawford and Sobel's (1982) model (the "CS model"). An expert (he), after privately observing the state of the world distributed uniformly on $[0, 1]$, sends a message (advice) to a decision maker (she). "Talk is cheap" - the message itself does not directly affect payoffs. Subsequent to receiving a message, the decision maker takes an action that affects the payoff of both. Interests are misaligned; while the decision maker wants to take an action that matches the state, the expert's most preferred action is higher than the state by a fixed bias parameter.

The novelty of my model - which I call the *amateur model* - lies in the decision maker being an "amateur" who is partially informed. The decision maker does not directly observe the state,

[1] Ben Bernanke was quoted on the Fed Education website: "In today's complex financial markets, financial education is central to helping consumers make better decisions for themselves and their families." As one of its missions, the Bureau of Consumer Protection "empowers consumers with free information to help them exercise their rights and spot and avoid fraud and deception"; they believe "education is the first line of defense against fraud and deception; it can help you make well-informed decisions before you spend your money."

but she can tell whether it is "high" or "low": she is informed about in which interval of a binary partition of $[0, 1]$ lies the true state. Her definition of "high" and "low" - the cutoff point in the partition or the *threshold* - is a private information constituting her type. The realization of the threshold privately determines the decision maker's partitional information structure, and the common prior is that the threshold is uniformly distributed on $[0, 1]$.

The way amateurs access and use information, which I attempt to capture with the above setup, may be illustrated with patient use of online information.[2] Fox (2006) reports that eight out of ten Internet users in the United States, accounting for some 113 million adults, have searched for health information on the Internet. These users may have access to only a limited source of information. Even when they have at their disposal the same information available to the professionals (e.g., by using the *Google Scholar*), as amateurs they typically lack the ability to interpret the information and sort out the relevant from the irrelevant. As one doctor puts it, "There's so much information (as well as misinformation) in medicine - and, yes, a lot of it can be Googled - that one major responsibility of an expert is to know what to ignore."[3] Given these extrinsic and intrinsic constraints, all a patient can get out of the websites may amount only to a rough idea as to whether her condition calls for serious attention ("high") or not ("low"). And with different sources of information and individual interpretations, it is plausible that even for the same underlying condition different individuals may arrive at different conclusions.

Fox (2006) reports that only one-third of her survey respondents mentioned their online findings to the doctors; a doctor facing a "Googler-patient" is likely to find himself offering advice in the midst of some private information on the patient's part. Suppose the doctor reports a diagnosis that is biased toward inducing the patient to take more intense and expensive treatments than necessary.[4] A patient who believes that her condition calls for serious attention may consider the biased diagnosis as a confirmation of her own findings and proceed with an expensive treatment. Otherwise, she may request for a less expensive option or even seek a second opinion.[5] Even for the same piece of advice, once decision makers have information of

[2]Cheap-talk models are applied to study information transmission in political science (Gilligan and Thomas, 1989, and Krishna and Morgan, 2001b) and finance (Benabou and Laroque, 1992, and Morgan and Stocken, 2003). The questions explored in this paper are relevant to these arenas as well. For example, financial information is now widely available online; investors no longer rely exclusively on the information provided by the management or investment advisors.

[3]This controversial *Time* magazine article, "When the Patient Is a Googler," is written by an orthopedist who reports his unpleasant experience with a "Googler-patient" whom he describe as "brainsucker." The doctor eventually decided not to treat the patient.

[4]The supplier-induced demand hypothesis in health economics (Evans, 1974) posits that doctors recommend more health care purchases than patients would buy if they had the same information. This coincides with the bias of the expert in the model.

[5]Thirty percent of the respondents in Fox (2006) indicate that online information led them to ask further questions to their doctors or seek a second opinion. In an article on *salon.com*, "Is There a Doctor in the Mouse?," the pediatrician-author mentions that some parents refused to vaccinate their children after being exposed to stories on autism websites about the dangers of vaccinating children.

their own, it is inevitable that different responses will ensue.

To illustrate how in the model different information partitions of the decision maker can give rise to different interpretations of advice, suppose there are two types of decision maker: A and B. A's partition allows her to distinguish between states below and above $\frac{1}{3}$; she considers any states lying in $[0, \frac{1}{3})$ as "low" and any states in $[\frac{1}{3}, 1]$ as "high." On the other hand, with a threshold at $\frac{2}{3}$, B considers any states lying in $[0, \frac{2}{3})$ as "low" and any states in $[\frac{2}{3}, 1]$ as "high." Suppose the true state is $\frac{1}{2}$, and the expert provides a vague advice that the state lies in $[\frac{1}{4}, \frac{7}{12}]$. In light of her information that the state is "high," i.e., it lies in $[\frac{1}{3}, 1]$, A will interpret the advice to mean that the state lies in $[\frac{1}{3}, \frac{7}{12}]$. Since B's information - that the state is "low," lying in $[0, \frac{2}{3})$ - is even vaguer than the advice, she will take the advice as it is. To A, the advice is a *complementary advice* - it adds on but does not supersede her information. The same advice is, however, a *substituting advice* to B - it substitutes what she knows. Should the expert provide a complete vague advice that the state can be anywhere in $[0, 1]$, both A and B will ignore it as they both know better; such advice is a *redundant advice* to them.[6] Finally, suppose the expert deceives with an advice $[\frac{7}{8}, 1]$ which does not contain $\frac{1}{2}$. While the advice will be considered by A as substituting, B can detect that it is a *false advice* because it contradicts what she knows.

Compared across the partitional equilibria of the amateur model and the CS model, in which the expert partitions $[0, 1]$ into a finite number of intervals (steps) and reveals which contains the true state, the expert in the amateur model provides less informative advice: for a fixed bias parameter, the maximum numbers of steps are weakly lower in the amateur model; when the numbers of steps are the same in the two models, the partitions in the amateur model are less even. An expert with upward bias is willing to provide information only when he is allowed to exaggerate in his advice strategy, manifesting as the steps becoming longer for higher states. In the amateur model, the influences of the expert on the decision maker's actions are attenuated because the decision maker has information of her own. A particular exaggerating advice strategy is no longer as effective in inducing the expert's preferred actions - and thus in keeping him to "talk informatively." In order to be willing to provide information, the expert in the amateur model needs to exaggerate more by extending the steps for higher states, leading to less even partitions or even elimination of steps in the lower end.

Without considering the expert's strategic response, as a decision problem under uncertainty more information is always beneficial. Taking into account the negative strategic effect, however, there exists a positive measure of information structures under which the amateur is strictly worse off compared to the novice - the uninformed decision maker in the CS model. The evolution from novices into amateurs does not benefit every (type of) decision maker in the strategic

[6] This "babbling advice" will also be ignored by an uninformed decision maker. While I use it as an example of redundant advice, there is non-babbling advice in the amateur model that can nevertheless be redundant.

environment. In partitional equilibria with two steps, the amateur is better off only when her own information structure has a relatively high threshold. A high threshold means that the amateur's information is biased, not the most desirable when she is making decision on her own. But when her choice of actions also depends on what the expert tells her, this bias in her own information helps "neutralize" the bias in the information provided by the expert. This allows the amateur to achieve a higher *ex-ante* payoff than would the novice, even though the novice is getting more informative advice. The amateur has access to an information structure, but before the realization of the threshold she is endowed with an opportunity to access one of the information structures. While the amateur may be worse off post the realization of this "lottery" over information structures, she is always better off in the two-step equilibria with the opportunity to participate in the "lottery."

The exposition of the rest of the paper is as follows. Section 2 describes the model. Section 3 analyzes the partitional equilibria of the amateur model and compare them with those of the CS model. Section 4 analyzes the decision maker's welfare. To the best of my knowledge, only a handful of papers have explicitly considered informed decision maker (receiver) in cheap talk models, and they are Seidmann (1990), Watson (1996), Olszewski (2004), Chen (2005, 2009) and Ishida and Shimizu (2010). To better compare these papers with the present paper, I postpone reviewing them and other related literature to Section 5 after the analysis is developed. Section 6 concludes. Proofs of the results in the main text are relegated to Appendix A. Appendix B (not intended for publication) contains analysis of off-equilibrium beliefs under false advice and how they relate to the existence of equilibria. It also contains calculation for the example.

2 The Model

2.1 Basic Set-up

There are two players, an expert (e) and a decision maker (d). They are in a principal-agent relationship, with the decision maker being the principal whose decision may depend on the information provided by the expert. The expert is perfectly informed about the state of the world, θ, which is commonly known to be uniformly distributed on $\Theta = [0, 1]$; throughout the paper, I shall interchangeably refer to θ as the "state" or the "type of the expert." After privately observing θ, the expert sends a cheap-talk message (advice) $m \in M$ to the decision maker, who then takes an action $a \in \mathbb{R}$. There is no restriction on M except that it should be sufficiently large; any infinite set will more than suffice.

I assume that payoffs adopt the following form:

$$U^e(a, \theta, b) = -(a - (\theta + b))^2,$$
$$U^d(a, \theta) = -(a - \theta)^2,$$

where $b > 0$ is a commonly known parameter measuring the misaligned interests between the parties. Together with the uniform state, the payoff functions constitute the *uniform-quadratic model* introduced by Crawford and Sobel (1982). A specification adopted by a majority of subsequent work especially in the strand of applications, the model offers tractability as well as a benchmark to compare the results in this paper with those in the literature.

Under the quadratic payoffs, the *ideal action* of the expert when he observes θ is $a^e(\theta, b) = \theta + b$. On the other hand, in a complete information regime in which the decision maker could observe θ perfectly, her ideal action would be $a^d(\theta) = \theta$. Note that the ideal actions are strictly increasing in θ. With $a^e(\theta, b) - a^d(\theta) = b > 0$, the misaligned interests between the parties manifest specifically as: for a given state, the expert prefers the decision maker to take an action that is higher than what the decision maker would prefer were she able to observe the state. Hereafter, b will be referred to as the expert's bias.

Interpretation. In the doctor-patient example, the state of the world may refer to a diagnosis of a certain disease, with a higher θ representing a more serious diagnosis. The doctor privately observes the true diagnosis and delivers his reported diagnosis, m, to the patient. The reported diagnosis induces the patient to take certain action, a, which can be interpreted as a treatment or procedure choice.[7] A higher action corresponds to a more intense treatment. Consistent with the supplier-induced demand hypothesis in health economics (Evans, 1974), the model therefore says that the biased doctor prefers treatments that are more intense - and thus in general more expensive - than are ideal from the patient's perspective. Alternatively, we can interpret a as the patient's lifestyle choice such as hours of exercises per week. With a stricter standard on what count as healthy habits, the doctor prefers a more disciplined lifestyle than is deemed optimal by the patient. In this case, b captures the doctor's "paternalistic bias." There are many diseases that do not have an objective diagnosis, most notably various emotional problems. Likewise, recommendations related to lifestyle choice are likely to suffer from the lack of objective evaluations. These are reflected in the doctor's reported diagnosis being "cheap talk" that cannot be verified *ex-post*.

[7] Kőszegi (2004, 2006) adopts these interpretations in his papers in which he uses a cheap-talk model and a model with verifiable messages to analyze the emotional aspects of doctor advice.

2.2 Partially Informed Decision Maker and Timing of the Game

The decision maker privately observes a *threshold*, t, commonly known to be uniformly distributed on $T = [0, 1]$, independently of θ. A threshold realized in the interior of T divides $\Theta = [0, 1]$ into two non-degenerate intervals, one low $t_l = [0, t)$ and one high $t_h = [t, 1]$. After observing t, the decision maker further receives a private signal, s, indicating to her in which interval the true θ lies. The common-knowledge property of the signal is as follows:

$$s(\theta|t) = \begin{cases} l, & \text{if } \theta < t, \\ h, & \text{if } \theta \geq t. \end{cases}$$

I call the decision maker an *amateur* decision maker; she knows less than an expert but almost always more than an uninformed, *novice* decision maker. The set of thresholds T coupled with the set of signals $\{l, h\}$ defines the type space of the amateur, $T \times \{l, h\}$, with generic element t_s. I call t_h collectively as the *high-interval* types and t_l the *low-interval* types.

The game begins with nature drawing the threshold t, followed by the realization of the state θ and the simultaneous generation of the signal s. The players then update their beliefs according to what they privately observe. The decision maker updates her beliefs about θ after privately observing t and s. The expert updates his beliefs about what the decision maker may know about θ after privately observing it.[8] After that, the players interact: the expert communicates with the decision maker by sending her a message m. Based on any information that may be contained in the message, the decision maker further updates her beliefs. The game ends with the decision maker's taking an action a and the distribution of payoffs.

There could be three meanings attached to what the decision maker's "information" refers to, depending on in which stage of the game we answer the question. I sharpen the terminology that will be adopted from now on. The decision maker's *information* will refer exclusively to what she knows about θ after she observes t and s. Thus, information in this sense corresponds to her interval types. After she observes t but before she receives s (i.e., before the realization of θ), her "information" should be interpreted as *information structures*. Finally, before t is realized, the correct interpretation of "information" is the opportunity to access one of the information structures characterized by a $t \in T$. I shall refer to this "lottery" over information structures as *information opportunity*.

Interpretation. The "Googler-patient," though not completely ignorant about the disease in question, is unable to distinguish among all the potential diagnoses $\theta \in [0, t)$ and lumps

[8] Given that the signal *property* is a common knowledge, that the signal *realization* is private to the decision maker hinges on the threshold being a private information. If the expert could observe t, then upon observing θ he would know which signal, l or h, the decision maker has received.

them together under the category of "no serious attention needed." Likewise, she lumps all the potential diagnoses $\theta \in [t, 1]$ under the category of "serious attention needed." In this context, t has an intuitive, somewhat behavioral interpretation: a high t means that an underlying diagnosis has to be very serious in order to be considered as serious - a patient with a high t is a carefree kind; on the other hand, a low t means that a diagnosis will be considered as serious even if it is not so serious - a patient with a low t is then a hypochondriac.[9]

2.3 Strategies and Equilibrium Definition

A behavior strategy of the expert, $\sigma : \Theta \to \Delta M$, specifies the distribution of message he sends for each $\theta \in \Theta$. A pure strategy of the decision maker, $\rho : M \times T \times \{l, h\} \to \mathbb{R}$, specifies for each combination of received message and interval type an action she takes.[10] I denote $\Theta_\sigma(m)$ to be the set of θ for which the expert sends message m with positive probability under σ, i.e., $\Theta_\sigma(m) = \{\theta \in \Theta : \sigma(m|\theta) > 0\}$. When there are messages in M that are not used under σ, I adopt the convention that $\Theta_\sigma(\cdot)$ is an empty set for those messages.

The decision maker's (post-communication) belief function, $\mu : M \times T \times \{l, h\} \to \Delta\Theta$, specifies for each combination of received message and interval type a density over Θ. Upon observing m' on the equilibrium path, an interval type t_s updates her beliefs using Bayes's rule:

$$\frac{\sigma(m'|\theta)\phi(\theta|t_s)}{\int_0^1 \sigma(m'|\theta')\phi(\theta'|t_s)d\theta'}, \tag{1}$$

where

$$\phi(\theta|t_h) = \begin{cases} 0, & \text{for } \theta \in [0, t), \\ \frac{1}{1-t}, & \text{for } \theta \in [t, 1], \end{cases} \quad \text{and} \quad \phi(\theta|t_l) = \begin{cases} \frac{1}{t}, & \text{for } \theta \in [0, t), \\ 0, & \text{for } \theta \in [t, 1]. \end{cases}$$

As illustrated in the Introduction, there are four types of advice that an interval type can receive: substituting advice, complementary advice, redundant advice, and false advice.[11] The first three types, including the useless redundant advice, arise as a result of the application of Bayes's rule via (1). False advice is in a kind of its own. It is a used but nevertheless out-of-equilibrium message, in which Bayes's rule cannot be used in updating beliefs. Note that $\Theta_\sigma(m) \cap t_s$ is the information set t_s finds herself in after receiving m, and Bayes's rule can be applied only when it is non-empty, i.e., when the advice does not contradict what the decision

[9] White and Horvitz (2009) provide the first systematic study of "cyberchondria." They document that the conclusions that users of medical websites drawn could depend on factors such as the distribution of medical content viewed and a user's predisposition to escalate or seek more reasonable medical explanations.

[10] That $U_{11}^d(\cdot) < 0$ guarantees that only pure strategy will be adopted by the decision maker.

[11] Given that messages have no intrinsic meaning in cheap talk, advice also has no intrinsic type; the type of an advice is determined only with respect to how it is interpreted, i.e., who is receiving it.

maker knows.[12]

The concept of perfect Bayesian equilibrium places no restriction on the decision maker's beliefs in the event that a false advice (when $\Theta_\sigma(m) \cap t_s = \emptyset$ but $\Theta_\sigma(m) \neq \emptyset$) or an unused message (when $\Theta_\sigma(m) = \emptyset$) is received. In general, any density function supported on Θ can be a candidate for off-equilibrium beliefs. In the equilibrium definition that follows, however, I shall put a restriction that the off-equilibrium beliefs of a type-t_s decision maker are supported on t_s. It is thus ruled out at the level of definition that a type-t_s decision maker would contradict herself (by putting positive density on $\Theta \setminus t_s$) after being deceived or surprised.[13]

The expert's expected payoff before a message is sent takes into account all possible values of t in $[0,1]$ and the corresponding interval types. Suppose an expert of type θ sends message m'. Using the interval types as the unit of analysis, his conditional expected payoff from sending m' if the decision maker has received signal s is $\int_0^1 U^e(\rho(m', t_s), \theta, b)\gamma(t_s|\theta)dt$, where[14]

$$\gamma(t_h|\theta) = \begin{cases} \frac{1}{\theta}, & \text{for } t \in [0,\theta], \\ 0, & \text{for } t \in (\theta, 1], \end{cases} \quad \text{and} \quad \gamma(t_l|\theta) = \begin{cases} 0, & \text{for } t \in [0,\theta], \\ \frac{1}{1-\theta}, & \text{for } t \in (\theta, 1]. \end{cases}$$

Weighting the conditional expected payoffs by their probabilities of occurrence, we arrive at the expert's unconditional expected payoff, his objective function when choosing what strategy to adopt:

$$V^e(m', \theta, b) = \int_0^\theta U^e(\rho(m', t_h), \theta, b)dt + \int_\theta^1 U^e(\rho(m', t_l), \theta, b)dt.$$

And here comes the equilibrium definition used in this paper:

Definition 1 (**Perfect Bayesian Equilibrium**). *A perfect Bayesian equilibrium of the amateur model is a pair of strategies (σ, ρ) and a set of beliefs μ such that*

1. *the expert maximizes his expected payoff given the decision maker's strategy: for all $\theta \in \Theta$, if $m \in \text{supp}[\sigma(\cdot|\theta)]$, then*

$$m \in \underset{m' \in M}{\text{argmax}}\, V^e(m', \theta, b),$$

[12] An empty information set will never arise in the CS model when all messages are used in equilibrium. See Appendix B.1 for a discussion about the difference between the CS model and the amateur model in this regard.

[13] Apart from being a natural restriction to make in this context, this also shares the spirit of equilibria with "support restrictions," which requires that the support of beliefs at an information set be a subset of that at the preceding information sets (see, e.g., Madrigal et al., 1987).

[14] From the vantage point of a type-θ expert, the probabilities that the decision maker has received h and l are, respectively, θ and $1-\theta$. Since a decision maker who has received h must have $t \in [0,\theta]$, the expert attaches positive probability density to facing a high-interval type t_h only for $t \in [0,\theta]$. Similar reasoning applies in deriving his beliefs about t_l.

2. *the decision maker maximizes her expected payoff given her beliefs: for all* $m \in M$ *and all* $t_s \in T \times \{l, h\}$,

$$\rho(m, t_s) = \underset{a'}{\operatorname{argmax}} \int_0^1 U^d(a', \theta)\mu(\theta|m, t_s)d\theta, \quad \textit{and}$$

3. *the decision maker updates her beliefs using Bayes's rule whenever possible, taking into account the expert's strategy and her interval type: for all* $m \in M$ *and all* $t_s \in T \times \{l, h\}$,

$$\mu(\theta|m, t_s) = \begin{cases} \frac{\sigma(m|\theta)\phi(\theta|t_s)}{\int_0^1 \sigma(m|\theta')\phi(\theta'|t_s)d\theta'}, & \textit{if } \Theta_\sigma(m) \cap t_s \neq \emptyset, \\ \psi(\theta|t_s), & \textit{if } \Theta_\sigma(m) \cap t_s = \emptyset, \end{cases}$$

where $\psi(\theta|t_s)$ *is any density supported on* t_s.

3 Equilibrium Analysis

In the CS model, all equilibria are *partitional*: the expert reveals in which of a finite number of intervals of $[0, 1]$ the true θ lies. In the amateur model, given that off-equilibrium beliefs emerge under false advice which are absent in the CS model, it is difficult to characterize all equilibria under all possible off-equilibrium beliefs.[15] The objective of this section is not a complete characterization of equilibria but to examine the effects of the decision maker's information in the same class of equilibria of the CS model; in the following, I shall analyze the partitional equilibria of the amateur model and compare them with those of the CS model.

In an N-step (partitional) equilibrium, the expert partitions the state space $[0, 1]$ into a finite number of N intervals $\{I_i\}_{i=1}^N = \{(\theta_{i-1}, \theta_i]\}_{i=1}^N$, where $\theta_0 = 0$ and $\theta_N = 1$. With respect to the uses of messages, it is without loss of generality to consider that the expert partitions the message space M into N distinct and exhaustive sets, M_i, $i = 1, \ldots, N$, and, if θ is realized in I_i, he randomizes uniformly over messages in M_i.[16] When $N \geq 2$, the equilibrium is informative, and the expert is said to provide informative advice. The subset of the elements of the partition, θ_i, $i = 1, \ldots, N - 1$, are called *boundary types*, who, in equilibrium, must be indifferent between sending messages in M_i and M_{i+1}.[17] In the CS model, to sustain a two-step equilibrium -

[15] Refer to Appendix B.2 for a discussion of the issues.

[16] Same as the CS model, insofar as equilibrium outcomes are concerned, we can without loss of generality assume that all messages are used (Appendix B.1). The correspondence between the index of the message sets and that of the intervals are for expositional convenience only; any shuffles among the message sets also constitute an equilibrium.

[17] This *indifferent condition*, while being necessary and sufficient for the existence of partitional equilibria in the CS model, is no longer sufficient in the amateur model; off-equilibrium beliefs have to be considered. Appendix B.2 contains the specification of off-equilibrium beliefs that is adopted for the analysis in this section. For example in two-step equilibria, such specification prescribes that the amateur adheres to her pre-communication beliefs

commonly considered the least informative equilibrium - in which there is one boundary type θ_1^{cs}, the expert's bias b has to be less than $\frac{1}{4}$. The requirement on bias is more restrictive in the amateur model:

Proposition 1. *In the amateur model, a two-step equilibrium exists if and only if $b < \frac{1}{6}$.*

Further comparing the two models with respect to two-step equilibria gives the following observation:

Corollary 1. *For $b \in [\frac{1}{6}, \frac{1}{4})$, the expert provides two-step advice to the novice but babbles to the amateur. For $b \in (\frac{1}{12}, \frac{1}{6})$, in which the most informative partitional equilibria in both models are of two steps, the boundary type in the amateur model $\theta_1(b) < \theta_1^{cs}(b)$ by the amount of b.*

The expert is providing an endogenous partitional information structure to the decision maker. The informativeness of a partitional information structure can be evaluated in the context of the model using its contribution to the decision maker's *ex-ante* payoff.[18] For $b \in (\frac{1}{12}, \frac{1}{6})$, the endogenous two-step partition is less even in the amateur model. To the extent that a less even partition contributes less to the decision maker's *ex-ante* payoff, the expert is said to provide less informative advice to the amateur. And the situation aggravates as the expert becomes more biased; had interests been perfectly aligned, the boundary types in the two models would have coincided.

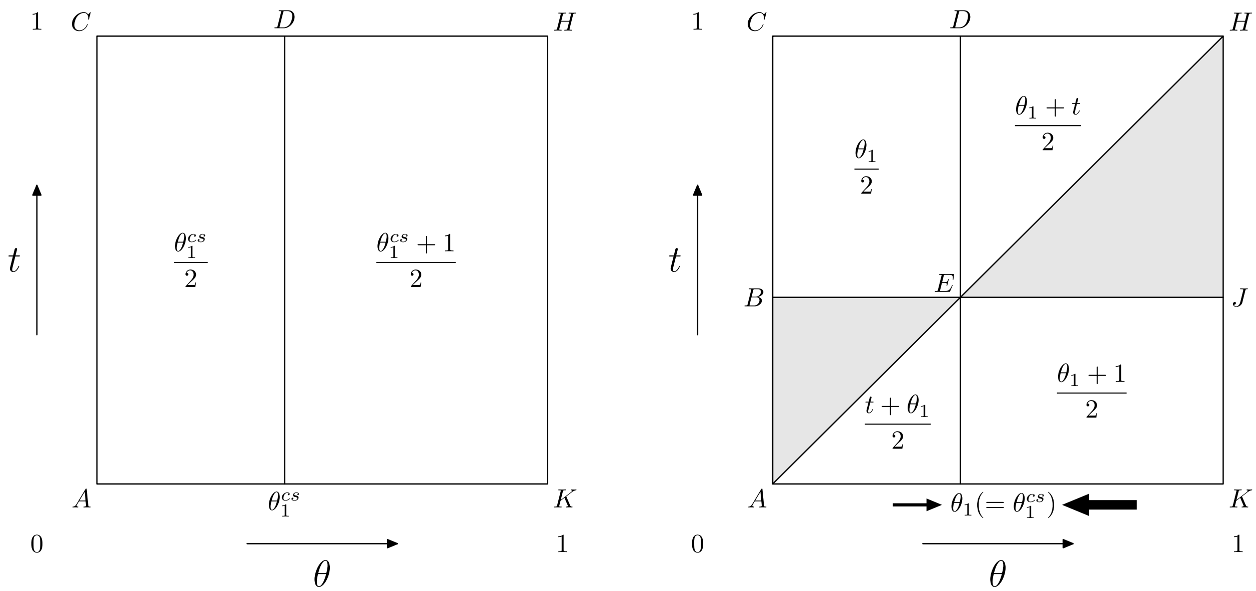
The expert's upward bias, coupled with an attenuated influence on the amateur, drives his response. The rationale can be elaborated with the aid of Figure 1. Consider first the CS model (Figure 1a). We can imagine that the novice also has a continuum of threshold types but has no access to the signal s. Without the signal, t by itself does not provide any information, and all types of novice behave the same. Suppose the expert sends m_1 for $\theta \in [0, \theta_1^{cs}]$ and m_2 for $\theta \in (\theta_1^{cs}, 1]$.[19] The novice's best responses - the *induced actions* - are $\frac{\theta_1^{cs}}{2}$ for m_1 and $\frac{\theta_1^{cs}+1}{2}$ for m_2. Given $b > 0$, in order for the expert to be willing to provide information - that θ_1^{cs} is an equilibrium boundary type who is indifferent between actions $\frac{\theta_1^{cs}}{2}$ and $\frac{\theta_1^{cs}+1}{2}$ - he has to be allowed to exaggerate, i.e., it must be the case that, as in Figure 1a, $\theta_1^{cs} < \frac{1}{2}$.

Suppose in the amateur model, the expert uses the same strategy by setting the boundary type $\theta_1 = \theta_1^{cs}$ (Figure 1b). By definition, there are two sets of interval types that θ_1 will never face, namely, t_l with $t < \theta_1$ and t_h with $t \geq \theta_1$ (ABE and EHJ), and they are taken out of consideration. When θ_1 sends m_2, some of the relevant interval types take, as in the CS model,

$\phi(\theta|t_s)$ when a false advice is received.

[18] There can be two ways to evaluate the informativeness of a partitional information structure. Using *ex-ante* payoff, as is adopted by Crawford and Sobel (1982), necessarily involves preferences and the prior distribution. A "context-free" approach not adopted involves considering the fineness of the information structures.

[19] For conciseness, I assume in this illustration that the expert uses pure strategy.



(a) The CS Model

(b) The Amateur Model

Figure 1: Profile of Induced Actions

$\frac{\theta_1+1}{2} = \frac{\theta_1^{cs}+1}{2}$ $(\theta_1 EJK)$. But for the remaining types (EDH), the influence of m_2 is attenuated downward: those interval types take $\frac{\theta_1+t}{2} \leq \frac{\theta_1^{cs}+1}{2}$, $t \in (\theta_1, 1]$. Similarly, when θ_1 sends m_1, some interval types take $\frac{\theta_1}{2} = \frac{\theta_1^{cs}}{2}$ $(BCDE)$, and some - on which the message's influence is attenuated upward - take $\frac{t+\theta_1}{2} \geq \frac{\theta_1^{cs}}{2}$, $t \in [0, \theta_1]$ $(AE\theta_1)$. Message m_2 induces actions on average lower than $\frac{\theta_1^{cs}+1}{2}$; to maintain the indifference, this creates a force pulling θ_1 below θ_1^{cs}. On the other hand, m_1 induces actions on average higher than $\frac{\theta_1^{cs}}{2}$, creating a force pushing θ_1 above θ_1^{cs}. If the two forces balanced out (when $\theta^{cs} = \frac{1}{2}$), $\theta_1 = \theta_1^{cs}$ would indeed be the equilibrium boundary type in the amateur model. But for $b > 0$ so that $\theta_1^{cs} < \frac{1}{2}$, the pulling force is stronger than the pushing force, rendering the equilibrium boundary type in the amateur model strictly lower than θ_1^{cs}.[20] The expert has more room to effectively exaggerate for low states but less room for high states. Since he is biased upward, in equilibrium he exaggerates more by moving θ_1 down from θ_1^{cs}.

Let $N(b)$ $(N_{cs}(b))$ be the maximum number of steps an equilibrium in the amateur model (CS model) can have when the expert's bias is b. The above illustration suggests that the prevalence of less informative advice in the amateur model should not be limited to two-step equilibria:[21]

Theorem 1. *For all $b > 0$, $N(b) \leq N_{cs}(b)$. Furthermore, for a given b, in the respective partitional equilibria of the amateur model and the CS model in which $N = N_{cs}$, $2 \leq N \leq$*

[20] Admittedly, if the density of t is clustered around 0, the pushing force can well be stronger than the pulling force. While the uniform distribution of t provides a tractable but rather restrictive formulation, the results of less informative advice will continue to hold so long as the distribution of t is relatively even.

[21] Unlike the CS uniform-quadratic model, in which the indifference condition reduces to a solvable second-order linear difference equation, the corresponding difference equation in the amateur model is non-linear; the lack of explicit solution makes beyond reach a complete characterization of the relationship between b and $N(b)$ in the amateur model.

$N(b)$ and $2 \leq N_{cs} \leq N_{cs}(b)$, the boundary types in the two models are such that $\theta_i < \theta_i^{cs}$, $i = 1, \ldots, N-1$.

A partitional equilibrium with the maximum number of steps allowed by a given b is commonly considered the most informative in the class; Theorem 1 thus says that, for all $b > 0$, the most informative partitional equilibria of the amateur model are less informative than those of the CS model. Some doctors are reported to become less helpful when the patient is a "Googler."[22] There are certainly many ways in which a doctor may respond negatively to a "Googler-patient." And the factors driving the negative responses - many of which may be psychological - could vary. This paper contributes a rational, strategic account to the picture by demonstrating how the attenuated influence of an expert in face of an informed decision maker could render him less helpful.

4 Decision Maker's Welfare

4.1 Welfare Criteria and the Scope of Analysis

To evaluate the welfare change accrued to the amateur, we are essentially evaluating the value of her information in the game. Given the different interpretations of the amateur's "information" (Section 2.2), there are also different criteria with which we can measure the "value of information." To put the issues in perspective, I start by taking the expert out of the picture for a moment.

In a single-agent decision problem in which a Bayesian decision maker maximizes payoff by choosing her action, value of information typically refers to the value of the decision maker's information structure; it can be measured by the difference between the *ex-ante* payoff of a decision maker who chooses actions under the given information structure and that of a decision maker who does so under the prior.[23] In the present setting, this corresponds to $W_t - W$, where $W_t = \int_0^t -(\frac{t}{2} - \theta)^2 d\theta + \int_t^1 -(\frac{t+1}{2} - \theta)^2 d\theta$ is the *ex-ante* payoff of the amateur whose actions are chosen under an information structure characterized by t and $W = \int_0^1 -(\frac{1}{2} - \theta)^2 d\theta$, the base of comparison, is the *ex-ante* payoff of the novice whose action is chosen under the uniform prior.

[22]In Fox and Fallows (2003), a patient responds to the survey: ".....they [doctors] became irritated with me for having knowledge of the condition beyond what they chose to share with me.....they became defensive and short with me when I would question.....what I had found out on my own on the Internet." Despite the incentives to protect their professional images, negative responses are also noted from the doctors' side (Ahmad et al., 2006): "So, if your patient [is] having a $15 visit, you're not going to sit for 15 minutes going through all this [patient's findings], you're going to get them out of the office."

[23]See, for example, Laffont (1989, Chapter 4).

Stepping back into the strategic regime, I compare payoffs across the most informative partitional equilibria of the amateur model and the CS model. Evaluating the value of information structures in the game, I replace W_t with $W_t(b)$, the amateur's *ex-ante* payoff in the most informative partitional equilibrium when the expert's bias is b and when the amateur's information structure has threshold t; for the base of comparison, W is superseded by $W_{cs}(b)$, the corresponding *ex-ante* payoff of the novice in the CS model. Under this criterion the amateur is (weakly) better off compared to the novice if and only if the value of her information structure is nonnegative: $W_t(b) - W_{cs}(b) \geq 0$.[24]

In the amateur model in which t is a private information, the above interpretation allows us to address how different information structures may affect the amateur differently in the game. On the other hand, that t is modeled as a draw of a random variable opens up another interpretation for the amateur's "value of information": the *value of information opportunity.* The corresponding welfare criterion then entails evaluating $W_T(b) - W_{cs}(b)$, where $W_T(b) = \int_0^1 W_t(b)dt$ is the *ex-ante* payoff of the amateur before t is realized.[25]

The lack of explicit solution between b and $N(b)$ in the amateur model limits the scope of the welfare analysis. In the following, for how a particular information structure relates to the amateur's welfare and for evaluating the information opportunity, I shall focus on two-step (and babbling) equilibria, with level of bias $b \in (\frac{1}{12}, \frac{1}{4})$. In the spirit of Theorem 1 generalizing a qualitative aspect of Corollary 1, however, a more general (for all $b < \frac{1}{4}$) - though less specific - welfare result regarding the value of the amateur's information structures will ensue.

4.2 Welfare Analysis

Behind the comparisons between $W_t(b)$ and $W_{cs}(b)$ lie two opposing effects. On one hand, the amateur's information structure contributes positively to her payoff as in a single-agent decision problem; on the other, that the expert in the amateur model provides less informative advice counteracts such positive effect. Focusing on $b \in (\frac{1}{12}, \frac{1}{4})$, the following proposition character-

[24] Since it is performed after t is realized, one may be tempted to consider this evaluation of information structures as an evaluation in the *interim* stage. However, according to Holmström and Myerson (1983), in an *interim* stage *every* agent in the model has received his or her private information but does not know that of the others. Since the decision maker's payoff is always evaluated before θ is realized, there is one agent in the model - the expert - who has yet to receive his private information. Thus, even though the amateur has observed t when we evaluate the value of information structure, the game is still in the *ex-ante* stage.

[25] In reference to the doctor-patient example, it is natural to expect that most patients search for information only when they are sick, which corresponds to an alternative specification of the flow of events with θ realized before t. In this case, any evaluation involving *ex-ante* payoffs automatically uses the value of information opportunity. Yet it is not uncommon for completely healthy individuals to access medical websites (or health magazines) to obtain knowledge about certain diseases. Without any possibility of having the diseases at the moment, these individuals are simply acquiring an information structure that may be useful for the future. In this case, the value of information structures will be a more appropriate yardstick of welfare.

izes exactly which exogenous information structures of the amateur are not useful enough to compensate for the less informative information structure provided endogenously by the expert:

Proposition 2. *Consider two scenarios in the amateur model, when the expert babbles ($\theta_1 = 0$) and provides two-step advice ($\theta_1 > 0$) in the most informative partitional equilibria:*

1. *Suppose $b \in [\frac{1}{6}, \frac{1}{4})$ so that $\theta_1 = 0$. The amateur is better off compared to the novice in the most informative (two-step) equilibrium of the CS model if and only if she has access to information structure characterized by $t \in [\frac{1}{2} - 2b, \frac{1}{2} + 2b]$.*

2. *Suppose $b \in (\frac{1}{12}, \frac{1}{6})$ so that $\theta_1 > 0$. There exist $\theta_1 < \underline{t}' < \overline{t}' < 1$ such that, compared to the novice in the most informative (two-step) equilibrium of the CS model, the amateur is better off if and only if she has access to information structure characterized by $t \in [\underline{t}', \overline{t}']$.*

It is obvious that the decision maker in the amateur model is strictly worse off in the measure-zero event that $t = 0$ or 1 - when she turns out to be a novice. Her exogenous information structure provides no useful information at all while she suffers a less informative endogenous information structure from the expert. Proposition 2 indicates, however, that even when the amateur has $t \in (0, 1)$, she can still be strictly worse off if she does not draw the "right" threshold.

For $b \in [\frac{1}{6}, \frac{1}{4})$, the welfare comparison comes down to which of the two-step partitions - that generated exogenously by the threshold t in the amateur model or that generated endogenously by the boundary type θ_1^{cs} in the CS model - dominates in terms of payoffs. Given quadratic preferences and uniform state, a two-step partition of an interval renders the highest *ex-ante* payoff when it divides the interval at its mid-point, which, for unit interval, is $\frac{1}{2}$; the amateur is better off only when her t lies closer to $\frac{1}{2}$ than θ_1^{cs} does. Since $\theta_1^{cs} = \frac{1}{2} - 2b$, this defines the range $[\frac{1}{2} - 2b, \frac{1}{2} + 2b]$.

When $b \in (\frac{1}{12}, \frac{1}{6})$, the comparison involves a two-step partition (in the CS model) and a three-step partition (in the amateur model). Despite the additional step from the amateur's threshold, it comes at the expense of a less even two-step partition from the expert. When t is close to 0, θ_1, or 1, the amateur's own partition does not provide too much additional information. Given that 0 and θ_1 lie relatively close to each other, this rules out any $t \in [0, \theta_1]$ the possibility of dividing the expert's two-step partition into a three-step that is even enough to assimilate the negative strategic effect. Only when t lies close to the mid-point of the longer step $(\theta_1, 1]$ will the resulting three-step partition be even enough to render the amateur better off.[26]

In the single-agent world, the amateur's *ex-ante* payoff increases monotonically as t approaches $\frac{1}{2}$ from both sides. The above analysis suggests that such comparative statics no longer

[26] Refer to the proof of Proposition 2 in Appendix A for the exact values of $\underline{t}'$ and $\overline{t}'$.

holds when we evaluate payoffs in the game. To the extent that we use payoffs to measure informativeness, this implies that the most informative (binary partitional) information structure in a single-agent decision problem is no longer the most informative in the game. Moreover, the single-peaked property of the payoff with respect to t also does not hold. Figure 2 illustrates the case when $b = \frac{1}{11}$. In this two-step equilibrium with boundary type $\theta_1 = \frac{5}{22}$, the amateur's *ex-ante* payoff, $W_t(\frac{1}{11})$, increases in the comparative statics as t departs from 0, reaching a local maximum at $t = \frac{5}{44}$, the mid-point of the first step. As t further increases, the payoff decreases, reaching one of the global minimums when t reaches the boundary type at $\frac{5}{22}$. The payoff then increases again, reaching the global maximum at $t = \frac{27}{44}$, the mid-point of the second step.[27] Since the *ex-ante* payoff of the novice, $W_{cs}(\frac{1}{11})$, is independent of t, the value of the amateur's information structure, $W_t(\frac{1}{11}) - W_{cs}(\frac{1}{11})$, follows the same pattern as t increases from 0 to 1. And the value is non-negative - i.e., the amateur is better off - if and only if $t \in [0.304, 0.923]$.

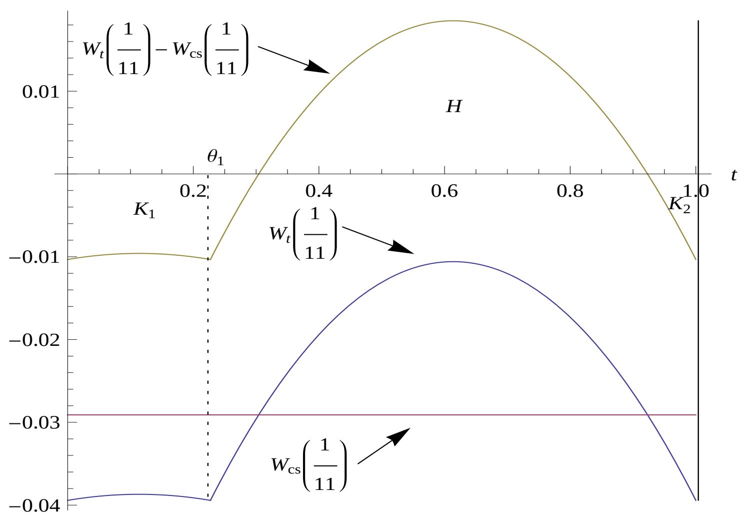


Figure 2: Decision Maker's *Ex-Ante* Payoff When $b = \frac{1}{11}$

Leveraging on Theorem 1, the following result generalizes a qualitative aspect of the above example to equilibria with more than two steps:

Theorem 2. *For $b \in (0, \frac{1}{4})$, there exists a set, $\mathcal{T}(b) \subset T$, with positive measure such that, in the respective most informative partitional equilibria of the amateur model and the CS model, the amateur with information structure characterized by $t \in \mathcal{T}(b)$ is strictly worse off compared to the novice.*

Unless the bias of the expert is so large ($b \geq \frac{1}{4}$) that both the amateur and the novice receive no useful information from the expert, the amateur, by being informed, is not necessarily better

[27] More generally for $b \in (\frac{1}{12}, \frac{1}{6}]$, this "optimal information structure," which gives the amateur the highest *ex-ante* payoff in the game, has threshold $t^* = \frac{3}{4}(1 - 2b)$. Note that $t^* = \frac{1}{2}$ (and the single-peaked property restored) when the expert in the amateur model babbles at $b = \frac{1}{6}$ (and indeed for $b > \frac{1}{6}$).

off. The multiple-peaked property of the amateur's *ex-ante* payoff, illustrated in Figure 2 for $N = 2$, extends to equilibria with more than two steps. Indeed, in an N-step equilibrium, the payoff will exhibit N "peaks" with respect to change in t. Without explicit solutions for the boundary types when $N > 2$, I have been unable to characterize exactly for which set of "peaks" the value of the amateur's information structure is nonnegative. Yet, when it comes to information structures with negative value, it can be certain that they exist when the threshold coincides with one of the boundary types; the amateur is strictly worse off at these "valleys" (e.g., $t = \frac{5}{22}$ in Figure 2) and their neighborhoods. The set $\mathcal{T}(b)$ is made up of those neighborhoods which in general is different for different values of b.

Focusing back to two-step and babbling equilibria, the following result indicates that, despite the possibility of drawing an information structure with negative value, the amateur is better off with the opportunity to access the "lottery" over information structures:

Proposition 3. *For $b \in (\frac{1}{12}, \frac{1}{4})$, the information opportunity always renders the amateur better off compared to the novice.*

Figure 2 provides a visualization of the forces at work. Despite the negative values of some information structures, Proposition 3 is a reflection of the fact that the measure of welfare gain (H) more than offsets the measure of welfare loss ($K_1 + K_2$). And this is true not just for $b = \frac{1}{11}$ as in the figure, but for all $b \in (\frac{1}{12}, \frac{1}{4})$. While intractability prevents a complete evaluation of the amateur's information opportunity when b takes on smaller values, I conclude the welfare analysis with an example of three-step equilibria, when $b = \frac{1}{15}$:

Example 1: Three-Step Equilibria. When $b = \frac{1}{15}$, the boundary types in the three-step equilibrium of the amateur model are $\{\theta_1, \theta_2\} = \{0.011, 0.307\}$, and those in the CS model are $\{\theta_1^{cs}, \theta_2^{cs}\} = \{0.067, 0.4\}$. The novice's *ex-ante* payoff is $W_{cs}(\frac{1}{15}) = -0.02111$. For the evaluation of information structures, $W_t(\frac{1}{15}) \geq -0.02111$ if and only if $t \in [0.390, 0.917] \subset [\theta_2, 1]$. And the amateur is better off with the information opportunity: $W_T(\frac{1}{15}) = -0.01996 > -0.02111$.

5 Related Literature

Pioneering work on cheap talk models with informed decision maker (receiver) shares a common finding that the decision maker's information can, unlike that in the present paper, improve communication. Seidmann (1990), Watson (1996), and Olszewski (2004) consider settings with discrete state space, and in the former two the expert's (sender's) ideal action is independent of the state. Seidmann's improvement-result leverages on altering the expert's effective preferences when messages induce distributions of actions under the decision maker's information. Watson's result exploits the correlation of the expert's and the decision maker's information. Olszewski shows that when the expert has a strong enough reputational concern for honesty, which the decision maker evaluates with her information, even full communication can be achieved.

A few recent papers paint a less rosy picture on the impacts of the decision maker's information on cheap-talk communication. Among them, the paper that is closest to the present paper is Chen (2009). To the best of my knowledge, we are the only two that introduce an informed decision maker into the CS model, in which the state space is a continuum and ideals actions are increasing functions of the state. In Chen's model, the decision maker has access to one out of two information structures, where the corresponding priors satisfy the monotone likelihood ratio. The major focus of her paper is on whether the decision maker can credibly reveal which information structure she has; in general a credible communication in that direction is found to be impossible. Chen (2005) and Ishida and Shimizu (2010) consider discrete models in which ideal actions depend on the state and the experts are allowed to be imperfectly informed. They also find that the decision maker's information can lead to worse communication. While in Chen (2005, 2009) it is also observed that the decision maker's opportunity to access information privately can reduce her welfare in the strategic setting, the decision maker's welfare is not explicitly analyzed in the above related papers.

Other papers that consider models in which the receiving end of the information transmission also possess information include Ottaviani and Sørensen (2006) and Feltovich et al. (2002). Ottaviani and Sørensen consider a "reputational" cheap-talk model in which an evaluator, who eventually observes the state, evaluates the ability of the expert in having accurate information. Feltovich et al. consider a costly signalling model in which the receiver observes private and noisy information about the sender's type. There is another strand of literature that consider an additional source of information for the decision maker: the opportunity to consult one more expert. Krishna and Morgan (2001a) show, in the setting of the CS model, that when the interests of the two experts oppose, it will be beneficial to have them both in place. On the contrary, in a model with binary state space and imperfectly informed experts, Austen-Smith (1993) show that full revelation of information can be achieved with a single expert but not when two experts are consulted simultaneously.

In a paper originally circulated in the early 80s, Green and Stokey (2007) study in a highly general setting how improvements in the expert's information quality may affect the players' welfare. They show that the decision maker can indeed be hurt by more informative information structures of the expert. In more specialized environments, Fisher and Stocken (2001) and Ivanov (2010a) show that a better quality of the expert's information does not necessarily translate into a better quality of information the decision maker receives. While the focus of the two papers is on the expert's information structures, they share the theme of the present paper that in strategic information transmission more information is not necessarily better.

Another strand of literature that extends the CS model studies how communication can be improved. Krishna and Morgan (2004) introduce an additional round of communication. Blume et al. (2007) introduce noise into the communication channel. Ottaviani and Squintani (2006) and Kartik et al. (2007) introduce credulous receivers (and in the latter cost of lying as well). Goltsman et al. (2009) and Ivanov (2010b) introduce a mediator. Broadly speaking, the additional components introduced into these models distort the actions of the receiver; the distortions bring into closer alignment the effective interests of the sender and the receiver, allowing communication to be improved. The underlying forces are similar in nature to the attenuated influence of the expert in the present paper. But the different direction of the distortions in the amateur model exacerbates the misalignment of interests and thus worsens communication.

Finally, in relation to the applications of doctor-patient relationship, Calcott (1999) and De Jaegher and Jergers (2001) are among the first to use cheap-talk models to study the interaction between doctors and patients, in particular the physician-induced demand. Kőszegi (2004, 2006) studies the emotional aspect of the doctor's advice. He shows that when the doctor takes into account the emotional impact of his advice on the patient's "anticipatory utility," the doctor will be prone to provide "good news."

6 Concluding Remarks

This paper explores the question of how information of a decision maker, who interacts with a biased expert, may affect the decision outcomes. The question is perhaps asked in an apt time, when recent developments in the Internet (e.g. information sharing as part of the Web 2.0) have created a group of amateurs who have access to information once available only to experts. Simply put, the analysis in this paper suggests that this development may not benefit every decision maker involved. When a decision maker also relies on a biased expert in making decision, becoming an amateur could indeed backfire because the expert may respond strategically by withholding more information. In the case that the decision maker's own information is not that useful on top of what the expert offers in advice, the negative strategic effect can dominate the benefit of being informed.

Appendix A - Proofs

Proof of Proposition 1. Consider $N = 2$, and the expert sends $m \in M_1$ for $\theta \in I_1$ and $m \in M_2$ for $\theta \in I_2$. The amateur's best responses - the induced actions - are

$$\rho(m,t_h) = \begin{cases} \frac{t+\theta_1}{2}, & \text{for } t \in I_1 \text{ and } m \in M_1, \\ \frac{\theta_1+1}{2}, & \text{for } t \in I_1 \text{ and } m \in M_2, \end{cases} \quad \text{and } \rho(m,t_l) = \begin{cases} \frac{\theta_1}{2}, & \text{for } t \in I_2 \text{ and } m \in M_1, \\ \frac{\theta_1+t}{2}, & \text{for } t \in I_2 \text{ and } m \in M_2. \end{cases}$$

In order for θ_1 to be indifferent between sending messages in M_1 and M_2, we need the indifference condition:

$$\begin{aligned} &\int_0^{\theta_1} \left(\left[\frac{t+\theta_1}{2} - (\theta_1+b)\right]^2 - \left[\frac{\theta_1+1}{2} - (\theta_1+b)\right]^2 \right) dt \\ &\quad + \int_{\theta_1}^{1} \left(\left[\frac{\theta_1}{2} - (\theta_1+b)\right]^2 - \left[\frac{\theta_1+t}{2} - (\theta_1+b)\right]^2 \right) dt = 0. \end{aligned} \tag{A.1}$$

Solving (A.1) for θ_1 gives $\theta_1 = \frac{1}{2}(1-6b)$; $N = 2$ requires $\theta_1 > 0$, satisfied if and only if $b < \frac{1}{6}$.

I verify next that it is a best response for all $\theta \in I_1$ to send messages in M_1 and all $\theta \in I_2$ to send messages in M_2. Note first that the induced actions under redundant advice are: $\rho(m,t_h) = \frac{t+1}{2}$ for $t \in I_2$ and $m \in M_2$, and $\rho(m,t_l) = \frac{t}{2}$ for $t \in I_1$ and $m \in M_1$. Given the specification that the amateur uses her pre-communication beliefs after false advice, the corresponding actions are: $\rho(m,t_h) = \frac{t+1}{2}$ for $t \in I_2$ and $m \in M_1$, and $\rho(m,t_l) = \frac{t}{2}$ for $t \in I_1$ and $m \in M_2$.

When $\theta \in [0,\theta_1]$ sends $m \in M_1$ and when he deviates by sending $m \in M_2$, his expected payoffs are, respectively,

$$\int_0^{\theta} -\left[\frac{t+\theta_1}{2} - (\theta+b)\right]^2 dt + \int_{\theta}^{\theta_1} -\left[\frac{t}{2} - (\theta+b)\right]^2 dt + \int_{\theta_1}^{1} -\left[\frac{\theta_1}{2} - (\theta+b)\right]^2 dt, \text{ and} \tag{A.2}$$

$$\int_0^{\theta} -\left[\frac{\theta_1+1}{2} - (\theta+b)\right]^2 dt + \int_{\theta}^{\theta_1} -\left[\frac{t}{2} - (\theta+b)\right]^2 dt + \int_{\theta_1}^{1} -\left[\frac{\theta_1+t}{2} - (\theta+b)\right]^2 dt. \tag{A.3}$$

Subtracting (A.2) from (A.3) we have, after imposing the equilibrium condition $\theta_1 = \frac{1}{2}(1-6b)$,

$$D_1 = \frac{\theta(27\theta - 10\theta^2 - 3) + b(9 + 96\theta - 30\theta^2) + 36b^2(4-3\theta) - 324b^3 - 4}{24}.$$

I show that $D_1 \le 0$ for all $(\theta,b) \in [0,\frac{1}{2}(1-6b)] \times (0,\frac{1}{6})$. Since $\frac{\partial^2 D_1}{\partial\theta^2} = \frac{9-10(b+\theta)}{4} > 0$ for $(\theta,b) \in [0,\frac{1}{2}(1-6b)] \times (0,\frac{1}{6})$, D_1 is convex in θ for all $b \in (0,\frac{1}{6})$. If the value of D_1 at the boundaries of $[0,\frac{1}{2}(1-6b)]$ is non-positive for all $b \in (0,\frac{1}{6})$, then $D_1 \le 0$ for all $(\theta,b) \in [0,\frac{1}{2}(1-6b)] \times (0,\frac{1}{6})$. By the indifference condition, $D_1 = 0$ when $\theta = \frac{1}{2}(1-6b)$. When $\theta = 0$, $D_1 = \frac{(1+6b)(33b-54b^2-4)}{24}$, and this can easily be verified to be negative for all $b \in (0, \frac{1}{6})$.

When $\theta \in (\theta_1, 1]$ sends $m \in M_1$ and $m \in M_2$, his expected payoffs are, respectively,

$$\int_0^{\theta_1} -\left[\frac{t+\theta_1}{2} - (\theta+b)\right]^2 dt + \int_{\theta_1}^{\theta} -\left[\frac{t+1}{2} - (\theta+b)\right]^2 dt + \int_{\theta}^{1} -\left[\frac{\theta_1}{2} - (\theta+b)\right]^2 dt, \text{ and} \quad \text{(A.4)}$$

$$\int_0^{\theta_1} -\left[\frac{\theta_1+1}{2} - (\theta+b)\right]^2 dt + \int_{\theta_1}^{\theta} -\left[\frac{t+1}{2} - (\theta+b)\right]^2 dt + \int_{\theta}^{1} -\left[\frac{\theta_1+t}{2} - (\theta+b)\right]^2 dt. \quad \text{(A.5)}$$

Subtracting (A.4) from (A.5) we have, after imposing the equilibrium condition $\theta_1 = \frac{1}{2}(1-6b)$,

$$D_2 = \frac{(1-\theta)[10\theta^2 + 7\theta + 30b(1+\theta) - 5] + (1-6b)[9\theta + 3b(5+6\theta) + 54b^2 - 5]}{24}.$$

Using a similar routine as above, it can be verified that $D_2 \geq 0$ for all $(\theta, b) \in [\frac{1}{2}(1-6b), 1] \times (0, \frac{1}{6})$.

□

Proof of Corollary 1. The results follow by comparing $\theta_1 = \frac{1}{2}(1-6b)$ with $\theta_1^{cs} = \frac{1}{2}(1-4b)$ for the two-step equilibria of the CS model.

□

Proof of Theorem 1. For clarity, I shall use $U^e(\cdot)$ to denote the expert's quadratic payoff. In the CS model, the indifference condition reduces to: for $i = 1, \ldots, N-1$

$$\begin{aligned} V_{cs}(\theta_{i-1}^{cs}, \theta_i^{cs}, \theta_{i+1}^{cs}, b) &= U^e\left(\frac{\theta_i^{cs} + \theta_{i+1}^{cs}}{2}, \theta_i^{cs}, b\right) - U^e\left(\frac{\theta_{i-1}^{cs} + \theta_i^{cs}}{2}, \theta_i^{cs}, b\right) \\ &= \int_0^1 \left[U^e\left(\frac{\theta_i^{cs} + \theta_{i+1}^{cs}}{2}, \theta_i^{cs}, b\right) - U^e\left(\frac{\theta_{i-1}^{cs} + \theta_i^{cs}}{2}, \theta_i^{cs}, b\right)\right] dt = 0, \end{aligned} \quad \text{(A.6)}$$

with $\theta_0^{cs} = 0$ and $\theta_N^{cs} = 1$.

The proof below parallels the proofs of Lemmas 4-6 in Crawford and Sobel (1982). I use the following two properties of the uniform-quadratic model. The first corresponds to the monotonicity condition in their Condition (M).

Property P1. *Suppose* $\{\theta_i^{cs}\}_{i=0}^N$ *and* $\{\theta_i^{cs\prime}\}_{i=0}^N$ *are two solutions to* (A.6)*, and* $\theta_0^{cs} = \theta_0^{cs\prime}$ *and* $\theta_1^{cs} > \theta_1^{cs\prime}$*. Then,* $\theta_i^{cs} > \theta_i^{cs\prime}$ *for all* $i \geq 2$*.*

Property P2. *Suppose* $\{\theta_i^{cs}\}_{i=0}^N$ *is a solution to* (A.6)*. Then,* $(\theta_{i+1}^{cs} - \theta_i^{cs}) > (\theta_i^{cs} - \theta_{i-1}^{cs})$*,* $i = 1, \ldots, N-1$*.*

In the amateur model, the indifference condition reduces to: for $i = 1, \ldots, N-1$

$$
\begin{aligned}
V(\theta_{i-1}, \theta_i, \theta_{i+1}, b) = &\int_0^{\theta_{i-1}} \left[U^e\left(\frac{\theta_i + \theta_{i+1}}{2}, \theta_i, b\right) - U^e\left(\frac{\theta_{i-1} + \theta_i}{2}, \theta_i, b\right)\right] dt \\
&+ \int_{\theta_{i-1}}^{\theta_i} \left[U^e\left(\frac{\theta_i + \theta_{i+1}}{2}, \theta_i, b\right) - U^e\left(\frac{t + \theta_i}{2}, \theta_i, b\right)\right] dt \\
&+ \int_{\theta_i}^{\theta_{i+1}} \left[U^e\left(\frac{\theta_i + t}{2}, \theta_i, b\right) - U^e\left(\frac{\theta_{i-1} + \theta_i}{2}, \theta_i, b\right)\right] dt \\
&+ \int_{\theta_{i+1}}^{1} \left[U^e\left(\frac{\theta_i + \theta_{i+1}}{2}, \theta_i, b\right) - U^e\left(\frac{\theta_{i-1} + \theta_i}{2}, \theta_i, b\right)\right] dt = 0,
\end{aligned}
\tag{A.7}
$$

with $\theta_0 = 0$ and $\theta_N = 1$.

Let $\theta^{cs}(K) = \{\theta_i^{cs}(K)\}_{i=0}^K$ with $\theta_{i-1}^{cs}(K) < \theta_i^{cs}(K)$, $i = 1, \ldots, K$, be a partial partition with $K+1$ elements that satisfy (A.6). Similarly, let $\theta(K) = \{\theta_i(K)\}_{i=0}^K$ with $\theta_{i-1}(K) < \theta_i(K)$, $i = 1, \ldots, K$, be a partial partition with $K+1$ elements that satisfy (A.7). I suppress "K" as the argument for the solution when it is clear in the context what K equals to.

I first prove the second part of the theorem by proving that, for $K \in \mathbb{N}$, if $\theta_0^{cs}(K) = \theta_0(K)$ and $\theta_K^{cs}(K) = \theta_K(K)$, then $\theta_i^{cs}(K) > \theta_i(K)$ for $i = 1, \ldots, K-1$. The proof proceeds by induction on K. This is true vacuously for $K = 1$. So, consider $K = 2$. Then, (A.6) becomes

$$
\begin{aligned}
V_{cs}(\theta_0^{cs}, \theta_1^{cs}, \theta_2^{cs}, b) &= U^e\left(\frac{\theta_1^{cs} + \theta_2^{cs}}{2}, \theta_1^{cs}, b\right) - U^e\left(\frac{\theta_0^{cs} + \theta_1^{cs}}{2}, \theta_1^{cs}, b\right) \\
&= \int_0^1 \left[U^e\left(\frac{\theta_1^{cs} + \theta_2^{cs}}{2}, \theta_1^{cs}, b\right) - U^e\left(\frac{\theta_0^{cs} + \theta_1^{cs}}{2}, \theta_1^{cs}, b\right)\right] dt = 0.
\end{aligned}
\tag{A.8}
$$

Substituting $\{\theta_0^{cs}, \theta_1^{cs}, \theta_2^{cs}\}$ into $V(\theta_{i-1}, \theta_i, \theta_{i+1}, b)$ in (A.7), we have, after using (A.8),

$$
\begin{aligned}
V(\theta_0^{cs}, \theta_1^{cs}, \theta_2^{cs}, b) = &\int_{\theta_0^{cs}}^{\theta_1^{cs}} \left[U^e\left(\frac{\theta_1^{cs} + \theta_2^{cs}}{2}, \theta_1^{cs}, b\right) - U^e\left(\frac{t + \theta_1^{cs}}{2}, \theta_1^{cs}, b\right)\right] dt \\
&+ \int_{\theta_1^{cs}}^{\theta_2^{cs}} \left[U^e\left(\frac{\theta_1^{cs} + t}{2}, \theta_1^{cs}, b\right) - U^e\left(\frac{\theta_0^{cs} + \theta_1^{cs}}{2}, \theta_1^{cs}, b\right)\right] dt.
\end{aligned}
\tag{A.9}
$$

Note that $\frac{t+\theta_1^{cs}}{2} > \frac{\theta_0^{cs}+\theta_1^{cs}}{2}$ for $t \in (\theta_0^{cs}, \theta_1^{cs})$. Thus, given $U_{11}^e(\cdot) < 0$, (A.8) implies that $U^e\left(\frac{\theta_1^{cs}+\theta_2^{cs}}{2}, \theta_1^{cs}, b\right) - U^e\left(\frac{t+\theta_1^{cs}}{2}, \theta_1^{cs}, b\right) < 0$ for $t \in (\theta_0^{cs}, \theta_1^{cs})$. On the other hand, that $\frac{\theta_1^{cs}+t}{2} < \frac{\theta_1^{cs}+\theta_1^{cs}}{2}$ for $t \in (\theta_1^{cs}, \theta_2^{cs})$ implies that $U^e\left(\frac{\theta_1^{cs}+t}{2}, \theta_1^{cs}, b\right) - U^e\left(\frac{\theta_0^{cs}+\theta_1^{cs}}{2}, \theta_1^{cs}, b\right) > 0$ for $t \in (\theta_1^{cs}, \theta_2^{cs})$. Property P2 implies that there is a net positive effect such that $V(\theta_0^{cs}, \theta_1^{cs}, \theta_2^{cs}, b) > 0$. It is straightforward to verify that then there exists $\theta > \theta_2^{cs}$ such that $V(\theta_0^{cs}, \theta_1^{cs}, \theta, b) = 0$. Since $V(\theta_0, \theta_1, \theta_2, b) = 0$, under the hypothesis that $\theta_0^{cs} = \theta_0$ and $\theta_2^{cs} = \theta_2$, we have $V(\theta_0^{cs}, \theta_1, \theta_2^{cs}, b) = 0$. Note that, given Property

P2 of the solution to (A.6), $\{\theta_0^{cs}, \theta_1^{cs}, \theta_2^{cs}\}$, Property P1 also holds for the two solutions to (A.7), $\{\theta_0^{cs}, \theta_1^{cs}, \theta\}$ and $\{\theta_0^{cs}, \theta_1, \theta_2^{cs}\}$. Thus, $\theta > \theta_2^{cs}$ implies that $\theta_1^{cs} > \theta_1$. Note that this is reminiscent of the result in Corollary 1 in which the explicit solutions of two-step equilibria are compared across the two models.

Suppose, as the induction hypothesis, the statement is true for $K-1$. Consider partial partition $\theta^{cs}(K) = \{\theta_i^{cs}(K)\}_{i=0}^{K}$ that satisfies (A.6) and $\theta(K) = \{\theta_i(K)\}_{i=0}^{K}$ that satisfies (A.7). Note first that $\{\theta_i^{cs}(K)\}_{i=0}^{K-1}$ is a partial partition with K elements that satisfies (A.6). Suppose that $\theta_0^{cs}(K) = \theta_0(K) = 0$ and $\theta_K^{cs}(K) = \theta_K(K)$. If $\{\theta_i'\}_{i=0}^{K-1}$ is a partial partition with K elements that satisfies (A.7) and $\theta_0' = \theta_0^{cs}(K) = 0$ and $\theta_{K-1}' = \theta_{K-1}^{cs}(K)$, then by the induction hypothesis $\theta_i^{cs}(K) > \theta_i'$, $i = 1, \ldots, K-2$. Since $V(\theta_{K-2}^{cs}(K), \theta_{K-1}^{cs}(K), \theta_K^{cs}(K), b) > 0$, we have that $V(\theta_{K-2}', \theta_{K-1}', \theta_K^{cs}(K), b) > 0$. Thus, there exists $\theta > \theta_K^{cs}(K)$ such that $V(\theta_{K-2}', \theta_{K-1}', \theta, b) = 0$. Given that $V(\theta_{K-2}(K), \theta_{K-1}(K), \theta_K(K), b) = V(\theta_{K-2}(K), \theta_{K-1}(K), \theta_K^{cs}(K), b) = 0$, that $\theta > \theta_K^{cs}(K)$ implies that $\theta_i' > \theta_i(K)$, $i = 1, \ldots, K-1$. Thus, that $\theta_i^{cs}(K) > \theta_i'$, $i = 1, \ldots, K-2$ and $\theta_{K-1}^{cs}(K) = \theta_{K-1}'$ imply that the statement is true for K.

Since the statement holds for any two partial partitions with $K+1$ elements so long as $\theta_0^{cs}(K) = \theta_0(K)$ and $\theta_K^{cs}(K) = \theta_K(K)$, it holds for $\theta_0^{cs}(K) = \theta_0(K) = 0$ and $\theta_K^{cs}(K) = \theta_K(K) = 1$. The second part of the theorem then holds when we set $K = N = N^{cs}$. For the first part of the theorem, note first that the second part, coupled with Property P1 of the solutions to (A.7) which follows from Properties P1 and P2 of the solutions to (A.6), implies that if $\theta_0^{cs}(K) = \theta_0(K)$ and $\theta_1^{cs}(K) = \theta_1(K)$, then $\theta_i^{cs}(K) < \theta_i(K)$, $i = 2, \ldots, K$. Suppose $\theta(N')$ is an equilibrium partition in the amateur model with $N'+1$ elements. Further suppose $\theta^{cs}(N')$ is a partition satisfying (A.6) with $\theta_0^{cs}(N') = \theta_0(N')$ and $\theta_1^{cs}(N') = \theta_1(N')$. Then, $\theta_i^{cs}(N') < \theta_i(N')$, $i = 2, \ldots, N'$. Thus, if $\theta^{cs}(N')$ is an equilibrium partition, it has at least $N'+1$ elements. Hence, $N_{cs}(b) \geq N(b)$.

□

Proof of Proposition 2. The novice's expected payoff is (Crawford and Sobel, 1982)

$$W_{cs}(b) = -\frac{1}{12N_{cs}(b)^2} - \frac{b^2(N_{cs}(b)^2 - 1)}{3}, \tag{A.10}$$

Since $N_{cs}(b) = 2$ for $b \in (\frac{1}{12}, \frac{1}{4})$, we have $W_{cs}(b) = -\frac{1}{48} - b^2$ for such values of b.

The amateur's expected payoff if her $t \in (\theta_1, 1]$ is

$$W_t(b) = \int_0^{\theta_1} -\left(\frac{\theta_1}{2} - \theta\right)^2 d\theta + \int_{\theta_1}^{t} -\left(\frac{\theta_1 + t}{2} - \theta\right)^2 d\theta + \int_t^1 -\left(\frac{t+1}{2} - \theta\right)^2 d\theta. \tag{A.11}$$

When $\theta_1 = 0$, the first term in (A.11) vanishes, and $W_t(b) = -\frac{1}{12}(3t^2 - 3t + 1)$. Thus, the amateur is better off if and only if $-\frac{1}{12}(3t^2 - 3t + 1) - (-\frac{1}{48} - b^2) \geq 0$ or $t \in [\frac{1}{2} - 2b, \frac{1}{2} + 2b]$.

Consider next that $\theta_1 > 0$. Consider first $t \in (\theta_1, 1]$. Using the equilibrium condition that $\theta_1 = \frac{1}{2}(1-6b)$, the expected payoff in (A.11) becomes $W_t(b) = -\frac{1}{48} - b^2 + A(t,b)$, where $A(t,b) = \frac{t(3-2t)+12bt(1-t)+4b^2(4-9t)-1}{16}$. And the amateur is better off if and only if $A(t,b) \geq 0$. Note that since for all $b \in (\frac{1}{12}, \frac{1}{6})$, $A(\frac{1}{2}(1-6b), b) = A(1,b) = -\frac{5b^2}{4} < 0$, $\frac{\partial^2 A(t,b)}{\partial t^2} < 0$, and $A_{\max}(b) = \max_{t' \in [\frac{1}{2}(1-6b),1]} A(t',b) = \frac{1+18b-52b^2-216b^3}{128} > 0$, solving $A(t,b) = 0$ gives that $A(t,b) \geq 0$ if and only if $t \in [\underline{t}', \overline{t}']$, where

$$\underline{t}' = \frac{3+12b-36b^2-\sqrt{1+24b+56b^2-96b^3+1296b^4}}{4(1+6b)} > \frac{1}{2}(1-6b), \text{ and}$$
$$\overline{t}' = \frac{3+12b-36b^2+\sqrt{1+24b+56b^2-96b^3+1296b^4}}{4(1+6b)} < 1.$$

Consider next $t \in [0, \theta_1]$. The amateur's expected payoff is

$$\begin{aligned} W_t(b) &= \int_0^t -\left(\frac{t}{2}-\theta\right)^2 d\theta + \int_t^{\theta_1} -\left(\frac{t+\theta_1}{2}-\theta\right)^2 d\theta + \int_{\theta_1}^1 -\left(\frac{\theta_1+1}{2}-\theta\right)^2 d\theta \\ &= -\frac{1}{48} - b^2 + B(t,b), \end{aligned} \tag{A.12}$$

where $B(t,b) = \frac{t(1-2t)-12bt(1-t)-4b^2(5-9t)}{16}$. The amateur is therefore better off if and only if $B(t,b) \geq 0$. Note that $B(0,b) = B(\frac{1}{2}(1-6b), b) = -\frac{5b^2}{4} < 0$ and $\frac{\partial^2 B(t,b)}{\partial t^2} < 0$ for $b \in (\frac{1}{12}, \frac{1}{6})$. Furthermore, for $b \in (\frac{1}{12}, \frac{1}{6})$, $B_{\max}(b) = \max_{t' \in [0, \frac{1}{2}(1-6b)]} B(t',b) = \frac{1-18b-52b^2-216b^3}{128} < 0$. Thus, every $t \in [0, \frac{1}{2}(1-6b)]$ is strictly worse off.

□

Proof of Theorem 2. For $b \in (\frac{1}{12}, \frac{1}{4})$, the result follows immediately from Proposition 2. I shall thus focus on $b \in (0, \frac{1}{12}]$. I shall follow the notational conventions of the proof of Theorem 1. I first establish several results pertaining to the *ex-ante* payoff of the novice, part of which parallels Theorem 3 of Crawford and Sobel (1982).

Let $\theta(x) = \{\theta_i(x)\}_{i=0}^K$ be a partial partition that satisfies (A.7) for $i = 2, \ldots, N$, with $\theta_0(x) = \theta_0$, $\theta_{K-1}(x) = x$, and $\theta_K(x) = \theta_K$. Let $\underline{\theta}$ be the type such that $\theta_1(\underline{\theta}) = \theta_0$ and $\overline{\theta}$ be the type such that $\theta(\overline{\theta})$ satisfies (A.7) for $i = 1, \ldots, N$. Note that under the partition $\theta(x)$ the novice's *ex-ante* payoff on $[\theta_0, \theta_K]$ is $EU^d_{cs}(x) = \sum_{i=1}^K \int_{\theta_{i-1}(x)}^{\theta_i(x)} U^d(\frac{\theta_{i-1}(x)+\theta_i(x)}{2}, \theta) d\theta$. Since $\frac{\theta_{i-1}(x)+\theta_i(x)}{2}$ is a result of optimization and $\theta_0(x)$ and $\theta_K(x)$ are fixed, applying the envelope theorem we have

$$\frac{dEU^d_{cs}(x)}{dx} = \sum_{i=1}^{K-1} \left[U^d\left(\frac{\theta_{i-1}(x)+\theta_i(x)}{2}, \theta_i(x)\right) - U^d\left(\frac{\theta_i(x)+\theta_{i+1}(x)}{2}, \theta_i(x)\right)\right] \frac{d\theta_i(x)}{dx}.$$

Given that Properties P1 and P2 of the solutions to (A.6) in the proof of Theorem 1 imply that

Property P1 also holds for the solutions to (A.7), we have $\frac{d\theta_i(x)}{dx} > 0$. Since $\frac{\theta_{i+1}(x)+\theta_i(x)}{2} > \frac{\theta_{i-1}(x)+\theta_i(x)}{2}$, $\frac{\theta_{i+1}(x)+\theta_i(x)}{2} \geq \frac{t+\theta_i(x)}{2}$ for $t \in (\theta_{i-1}(x), \theta_i(x)]$, and $\frac{\theta_i(x)+t}{2} \geq \frac{\theta i-1(x)+\theta_i(x)}{2}$ for $t \in (\theta_i(x), \theta_{i+1}(x)]$, the satisfaction of (A.7) and that $U^e(\cdot)_{13} > 0$ imply that $U^d(\frac{\theta_{i-1}(x)+\theta_i(x)}{2}, \theta_i(x)) - U^d(\frac{\theta_i(x)+\theta_{i+1}(x)}{2}, \theta_i(x)) > 0$ for $i = 2, \ldots, K-1$. Also, Property P1 of the solutions to (A.7) and that $U^e(\cdot)_{13} > 0$ imply that $U^d(\frac{\theta_0(x)+\theta_1(x)}{2}, \theta_1(x)) - U^d(\frac{\theta_1(x)+\theta_2(x)}{2}, \theta_1(x)) > 0$ for $x \in [\underline{\theta}, \overline{\theta}]$. Hence $EU^d_{cs}(x)$ is strictly increasing in $x \in [\underline{\theta}, \overline{\theta}]$.

I show that for $N_{cs} = N = K \geq 2$ under a given b, the novice is strictly better off under partition $\theta^{cs}(K)$ than under $\theta(K)$. The proof proceeds by induction on K. This is true vacuously for $K = 1$. For $K = 2$, the statement is true by Proposition 2 with $t = 0$ or $t = 1$. Suppose, as the induction hypothesis, the statement is true for $K - 1$. Consider two partitions $\theta^{cs}(K) = \{\theta^{cs}_i(K)\}_{i=0}^K$ and $\theta(K) = \{\theta_i(K)\}_{i=0}^K$, that satisfy, respectively, (A.6) and (A.7) and where $\theta^{cs}_0 = \theta_0 = 0$ and $\theta^{cs}_K = \theta_K = 1$. We can find a partition $\hat{\theta}^{cs}(K) = \{\hat{\theta}^{cs}_i(K)\}_{i=0}^K$ such that $\hat{\theta}^{cs}_1 = \theta_1$ and (A.6) is satisfied for $\{\hat{\theta}^{cs}_i(K)\}_{i=2}^K$. Note that it follows from the argument in the last paragraph that the novice is strictly better off under $\theta^{cs}(K)$ than under $\hat{\theta}^{cs}(K)$. For $\hat{\theta}^{cs}(K)$ and $\theta(K)$, it follows from the proof of Theorem 1 that $\hat{\theta}^{cs}_i > \theta_i$, $i = 2, \ldots, K-1$. That the statement is true for $K = 2$ and the induction hypothesis imply that the novice is strictly better off under $\hat{\theta}^{cs}(K)$ than under $\theta(K)$. The statement then follows. Note that since the statement is true for any $K \in \mathbb{N}$, it is true for $N_{cs}(b) = N(b)$. I next show that if $N_{cs}(b) > N(b)$, the novice is strictly better off under partition $\theta^{cs}(N_{cs}(b))$ than under $\theta(N(b))$. Construct a new partition $\tilde{\theta}(\tilde{N}(b))$ by extracting the elements $\{\theta^{cs}_1, \ldots, \theta^{cs}_{N_{cs}(b)-N(b)}\} \in \theta^{cs}(N_{cs}(b))$ and imposing them into $\theta(N(b))$. Note that then $\tilde{N}(b) = N_{cs}(b)$. Since $\hat{\theta}(\tilde{N}(b))$ is finer than $\theta(N(b))$ (i.e., $\theta(N(b)) \subset \tilde{\theta}(\tilde{N}(b))$), the novice is strictly better off under $\tilde{\theta}(\tilde{N}(b))$ than under $\theta(N(b))$. Now, for $\theta^{cs}(N_{cs}(b))$ and $\tilde{\theta}(\tilde{N}(b))$, Property P1 and the way of the above construction guarantee that $\theta^{cs}_i \geq \tilde{\theta}_i$, $i = 1, \ldots, N_{cs}(b) - 1$. By an induction argument similar to the above, the novice is strictly better off under $\theta^{cs}(N_{cs}(b))$ than under $\tilde{\theta}(\tilde{N}(b))$. The result then follows.

Finally, turning to the amateur, note that since $N(b) \geq 2$ for $b \in (0, \frac{1}{12}]$, we can find a $t' \in T$ such that $t' \in \{\theta_i\}_{i=1}^{N(b)-1}$. It then follows from the above that $W_{cs}(b) > W_{t'}(b)$. Since $W_t(b)$ is continuous in t, there exists a positive measure of t's such that $W_{cs}(b) > W_t(b)$.

□

Proof of Proposition 3. Consider first that $b \in [\frac{1}{6}, \frac{1}{4})$. Integrating $W_t(b) = -\frac{1}{12}(3t^2 - 3t + 1)$ with respect to t gives $W_T(b) = -\frac{1}{24}$. Thus, the amateur is better off if and only if $W_{cs}(b) \leq -\frac{1}{24}$ or $b \geq \frac{1}{4\sqrt{3}}$, which is satisfied for $b \in [\frac{1}{6}, \frac{1}{4})$. For $b \in (\frac{1}{12}, \frac{1}{6})$, integrating (A.11) and (A.12) with respect to t in the respective ranges and summing them up gives, after using that $\theta_1 = \frac{1}{2}(1-6b)$, $W_T(b) = -\frac{1+72b^2-432b^4}{64}$. It is straightforward to verify that $W_T(b) \geq W_{cs}(b)$ for all $b \in (\frac{1}{12}, \frac{1}{6})$.

□

Appendix B - Analysis of Off-Equilibrium Beliefs and Calculation for Example

B.1 Unused and Out-of-Equilibrium Messages

In the CS model, unused messages are synonyms for out-of-equilibrium messages, and vice versa. Since in cheap-talk models messages do not directly affect payoffs, for every equilibrium (in which there may be unused messages) there exists another equilibrium with the same outcome in which all messages are used. One can therefore restrict attention to equilibria with no unused message, and any specifications of off-equilibrium beliefs would have no effect on the equilibrium outcomes.[28] Crawford and Sobel (1982) enlist all the messages by assuming, as is adopted in this paper, that the expert randomizes uniformly over supports of distinct sets that exhaust the message space.

In the amateur model, even when there is no unused message, out-of-equilibrium messages can still arise when the decision maker receives a false advice. Before establishing this observation formally, the following lemma provides a link between the notion of information set and the notations used in this paper:

Lemma 1. *A type-t_s amateur is in an information set unreached in equilibrium if and only if she receives message $m \in M$ such that $\Theta_\sigma(m) \cap t_s = \emptyset$, where σ is an equilibrium strategy.*

***Proof*.** For all m sent on the equilibrium paths, by definition $\Theta_\sigma(m)$ contains the realized θ. Furthermore, if the amateur is of type t_s, the realized $\theta \in t_s$. Thus, for any m that t_s receives in information sets reached in equilibrium, $\Theta_\sigma(m)$ and t_s contain at least one common element. Conversely, if $\Theta_\sigma(m)$ and t_s contain some common elements, in equilibrium t_s receives m with positive probability. The lemma follows by the contrapositive of the above.

□

The following proposition establishes that the amateur's information set can be empty - that there are out-of-equilibrium messages - even when all messages are used:

Proposition 4. *Suppose there is an informative equilibrium in the amateur model with no unused message. There exists a deviation of the expert from this equilibrium that leaves some interval types of amateur in information sets unreached in equilibrium.*

[28]See, for example, the discussion in Farrell (1993).

Proof. When there is an informative equilibrium with no unused message, for all $m \in M$, $\Theta_\sigma(m)$ is a strict non-empty subset of Θ. Suppose the expert deviates from the equilibrium by sending m' for $\theta'' \notin \Theta_\sigma(m')$; $\Theta_\sigma(m')$ being a strict subset of Θ ensures that there exists such a θ''. We can find a t_s such that $\theta'' \in t_s$ and $t_s \subset \Theta \setminus \Theta_\sigma(m')$: if $\theta'' > \sup \Theta_\sigma(m')$, choose a t_h with $t \in (\sup \Theta_\sigma(m'), \theta'')$, and if $\theta'' < \inf \Theta_\sigma(m')$ choose a t_l with $t \in (\theta'', \inf \Theta_\sigma(m'))$; $\Theta_\sigma(m')$ being non-empty ensures that $\sup \Theta_\sigma(m')$ and $\inf \Theta_\sigma(m')$ are well-defined in Θ. When t_s receives m' in the deviation, because $\Theta_\sigma(m') \cap t_s = \emptyset$, she will, by Lemma 1, be in an information set unreached in equilibrium even though m' is a used message.

□

The following proposition states that, same as the CS model, it is without loss of generality to consider that all messages are used in the amateur model:

Proposition 5. *In the amateur model, for every equilibrium (σ, ρ, μ) there exists another equilibrium (σ', ρ', μ') with no unused message of which the equilibrium outcome is equivalent to that of (σ, ρ, μ).*

Proof. Consider an equilibrium (σ, ρ, μ) in which there may be unused messages. Denote the set of such unused messages by M_0. Consider a message $m \in M \setminus M_0$ that induces the set of actions $\mathcal{A}(m)$ in this equilibrium. Construct another equilibrium (σ', ρ', μ') as follows: prescribe σ' to the expert in which he is asked to randomize over $M_0 \cup \{m\}$ for all $\theta \in \Theta_\sigma(m)$ such that every $m' \in M_0 \cup \{m\}$ is used with positive probability. Upon receiving any $m' \in M_0 \cup \{m\}$, every interval type that the expert could face updates beliefs μ' with the same conclusion as when m is received. Given that ρ' best responds to μ', $\mathcal{A}(m') \equiv \mathcal{A}(m)$. It is thus a best response to ρ' that the expert adopts the prescribed σ'. We then have an equilibrium (σ', ρ', μ') with the same outcome as (σ, ρ, μ), but every $m \in M$ is used with positive probability.

□

B.2 Off-Equilibrium Beliefs and Existence of Equilibria

The analysis in this section is performed under more general payoff functions. Without taking an explicit form, the payoffs of the expert and the decision maker, $U^e(a, \theta, b)$ and $U^d(a, \theta)$, are assumed to satisfy the following conditions: 1) $U^e(\cdot)$ and $U^d(\cdot)$ are twice continuously differentiable, 2) $U^i_{11}(\cdot) < 0$, 3) $U^i_1(a, \cdot) = 0$ for some $a \in \mathbb{R}$, 4) $U^i_{12}(\cdot) > 0$, $i = e, d$, 5) $U^d(a, \theta) = U^e(a, \theta, 0)$ for all (a, θ), and 6) $U^e_{13}(\cdot) > 0$ everywhere. Given $b > 0$, Condition 6) implies that the ideals actions of the expert and the decision maker are such that $a^e(\theta, b) > a^d(\theta)$ for all θ. Also define

$$a(r, s) = \begin{cases} \underset{a'}{\text{argmax}} \int_r^s U^d(a', \theta) d\theta, & \text{if } r < s, \\ a^d(r), & \text{if } r = s. \end{cases}$$

The emergence of off-equilibrium beliefs in the amateur model, which cannot be eliminated by having all messages used, poses a challenge in the characterization of equilibria. A full characterization requires consideration of all possible off-equilibrium beliefs, which constitute a very large set. In this section, I shall illustrate the issues and provide a specification of beliefs that, coupled with a mild condition on the expert's payoff, guarantees the existence of partitional equilibria.

I first show that, regardless of what beliefs the amateur holds after receiving a false advice, the expert never fully reveals his information. Intuitively, if the expert reveals θ, the amateur's own information will become useless, implying that the amateur will respond in exactly the same way as does the novice. Since a biased expert does not fully reveal his information to the novice, he also will not do so to the amateur.

Proposition 6. *There exists no separating equilibrium in the amateur model.*

Proof. If the expert in the amateur model fully reveals his information, for all $\theta \in \Theta$, θ induces $a^d(\theta)$ on all interval types. Accordingly, a type-θ expert's payoff is $\int_0^1 U^e(a^d(\theta), \theta, b)dt$. Suppose there exists a fully separating equilibrium. For any $\eta > 0$, we can find a $\bar{\theta} < \eta$ that induces $a^d(\bar{\theta})$ in the equilibrium. Suppose $\bar{\theta}$ deviates by sending m' that in the equilibrium is reserved for $\theta' = \bar{\theta} + \epsilon$, where $\epsilon > 0$ is such that $a^d(\bar{\theta}) < a^d(\bar{\theta} + \epsilon) < a^e(\bar{\theta}, b)$. The continuity of the payoff functions and that $U^d(a, \theta) \equiv U^e(a, \theta, 0)$ guarantee that such an ϵ exists. Upon receiving m', all interval types with $t \notin (\bar{\theta}, \bar{\theta} + \epsilon)$ will take action $a^d(\bar{\theta} + \epsilon)$, which, given the choice of ϵ and that $U_{11}^e(\cdot) < 0$, is strictly preferred over $a^d(\bar{\theta})$ by $\bar{\theta}$. To the remaining types with $t \in (\bar{\theta}, \bar{\theta} + \epsilon)$, all being low-interval types, m' is a false advice, and they take action under off-equilibrium beliefs $\psi(\theta|t_l)$. For any such beliefs, given that $U_{12}^d(\cdot) > 0$, the set of induced actions taken by these interval types is bounded by $a^d(0)$ and $a^d(\bar{\theta} + \epsilon)$. Since $U_{11}^e(\cdot) < 0$, we have $U^e(a^d(0), \bar{\theta}, b) \leq U^e(a', \bar{\theta}, b)$ for all $a' \in [a^d(0), a^d(\bar{\theta} + \epsilon)]$. Thus, the payoff for $\bar{\theta}$ to send m' is, regardless of the specification of $\psi(\theta|t_l)$, bounded below by

$$\int_0^{\bar{\theta}} U^e(a^d(\bar{\theta} + \epsilon), \bar{\theta}, b)dt + \int_{\bar{\theta}}^{\bar{\theta}+\epsilon} U^e(a^d(0), \bar{\theta}, b)dt + \int_{\bar{\theta}+\epsilon}^1 U^e(a^d(\bar{\theta} + \epsilon), \bar{\theta}, b)dt. \tag{B.1}$$

Subtracting $\bar{\theta}$'s equilibrium payoff from (B.1) gives:

$$\int_0^{\bar{\theta}} [U^e(a^d(\bar{\theta}+\epsilon), \bar{\theta}, b) - U^e(a^d(\bar{\theta}), \bar{\theta}, b)]dt + \int_{\bar{\theta}}^{\bar{\theta}+\epsilon} [U^e(a^d(0), \bar{\theta}, b) - U^e(a^d(\bar{\theta}), \bar{\theta}, b)]dt \\ + \int_{\bar{\theta}+\epsilon}^{1} [U^e(a^d(\bar{\theta}+\epsilon), \bar{\theta}, b) - U^e(a^d(\bar{\theta}), \bar{\theta}, b)]dt. \tag{B.2}$$

We can choose η sufficiently small such that, for any ϵ that satisfies the above criterion of choosing the deviating message, the first and the third positive terms in (B.2) dominate the second negative terms for some $\bar{\theta} < \eta$. And the strict incentive for some θ to deviate poses a contradiction to the existence of fully separating equilibrium.

□

In the CS model, when the indifference condition is satisfied under a *partitional strategy* of the expert (p.9) where $N \geq 2$, it follows that the following incentive-compatibility conditions are satisfied: 1) every boundary type θ_i will (weakly) prefer sending messages in M_i over any other messages; and 2) θ in the interior of every I_i - the *interior types* - will (strictly) prefer the same. The expert's strategy thus constitutes an informative partitional equilibrium. In the amateur model, even if the indifference condition holds under such strategy, in which[29]

$$V^e(M_i, \theta_i, b) = V^e(M_{i+1}, \theta_i, b), \quad i = 1, \ldots, N-1, \, \theta_0 = 0 \text{ and } \theta_N = 1, \tag{B.3}$$

because the off-equilibrium beliefs may generate a "benefit of lying" under false advice the incentive-compatibility conditions may not be satisfied.

Before illustrating how "benefit of lying" can arise in the amateur model, I first pause to characterize the induced actions of the amateur under a partitional strategy. I distinguish between two types of induced actions, effectively induced and ineffectively induced. An action a is effectively induced by m if the amateur updates her beliefs $\mu(\theta|m, t_s)$ using Bayes's rule and there exists $\theta \in \Theta$ such that, in her maximization problem of which a is the solution, $\mu(\theta|m, t_s) \neq \phi(\theta|t_s)$.

Lemma 2. *A high-interval type t_h takes effectively induced actions if and only if she receives*

1. *substituting advice: her threshold $t \in I_i$, $i = 1, \ldots, N-1$, and the expert reveals that $\theta \in I_j$, $i < j \leq N$; or*

2. *complementary advice: her threshold $t \in I_i$, $i = 1, \ldots, N-1$, and the expert reveals that $\theta \in I_i$,*

[29]With a slight abuse of notations, I use $V^e(M_i, \theta_i, b)$ to stand for $V^e(m, \theta_i, b)$ for all $m \in M_i$.

and she takes ineffectively induced actions if and only if she receives

3. *redundant advice: her threshold $t \in I_N$, and the expert reveals that $\theta \in I_N$; or*

4. *false advice: her threshold $t \in I_i$, $i = 2, \ldots, N$, and expert reveals that $\theta \in I_k$, $1 \leq k < i$.*

A low-interval type t_l takes effectively induced actions if and only if she receives

5. *substituting advice: her threshold $t \in I_i$, $i = 2, \ldots, N$, and the expert reveals that $\theta \in I_k$, $1 \leq k < i$; or*

6. *complementary advice: her threshold $t \in I_i$, $i = 2, \ldots, N$, and the expert reveals that $\theta \in I_i$,*

and she takes ineffectively induced actions if and only if she receives

7. *redundant advice: her threshold $t \in I_1$, and the expert reveals that $\theta \in I_1$;*

8. *false advice: her threshold $t \in I_i$, $i = 1, \ldots, N-1$, and the expert reveals that $\theta \in I_j$, $i < j \leq N$.*

Proof. I prove the cases for high-interval types; the cases for low-interval types are similar. Consider first Conditions 1 and 2. Suppose t_h with $t \in I_i$, $i = 1, \ldots, N-1$, receives m indicating that $\theta \in I_j$. For $j > i$, t_h's updated beliefs under Bayes's rule are $\mu(\theta|m, t_h) = 1/(\theta_j - \theta_{j-1})$ for $\theta \in (\theta_{j-1}, \theta_j]$ and zero elsewhere. For $j = i$, t_h's updated beliefs under Bayes's rule are $\mu(\theta|m, t_h) = 1/(\theta_i - t)$ for $\theta \in [t, \theta_i]$ and zero elsewhere. In both cases, there exists $\theta \in [0, 1]$ such that $\mu(\theta|m, t_h) \neq \phi(\theta|t_h)$, and the resulting actions are effectively induced. This proves the sufficiencies.

The necessities is proved by contrapositive. Suppose t_h with $t \in I_i$, $i = 1, \ldots, N-1$, receives m indicating that $\theta \in I_j$, $j < i$. Note that then $\Theta_\sigma(m) \cap t_h = \emptyset$; Bayes's rule cannot be applied in updating t_h's beliefs, and the resulting action cannot be effectively induced. Finally, suppose t_h with $t \in I_N$ receives m indicating that $\theta \in I_N$. Her updated beliefs are $\mu(\theta|m, t_h) = 1/(1-t)$ for $[t, 1]$ and zero elsewhere. This is equivalent to $\phi(\theta|t_h)$ for all $\theta \in [0, 1]$, and the resulting action cannot be effectively induced.

Since there are only two types of actions, effectively induced and ineffectively induced, and they are mutually exclusive, the sufficiencies (necessities) in Conditions 3 and 4 for ineffectively induced actions follow from the above necessities (sufficiencies) for effectively induced actions.

□

Using Lemma 2, the profile of actions effectively induced on t_h, $t \in I_i$, $i = 1, \ldots, N-1$, is

$$\rho(m, t_h) = \begin{cases} a(t, \theta_i), & \text{for } m \in M_i \text{ (complementary advice)}, \\ a(\theta_{j-1}, \theta_j), & \text{for } m \in M_j,\ i < j \leq N \text{ (substituting advice)}; \end{cases}$$

and that induced on t_l, $t \in I_i$, $i = 2, \ldots, N$ is

$$\rho(m, t_l) = \begin{cases} a(\theta_{i-1}, t), & \text{for } m \in M_i \text{ (complementary advice)}, \\ a(\theta_{k-1}, \theta_k), & \text{for } m \in M_k,\ 1 \leq k < i \text{ (substituting advice)}. \end{cases}$$

The profile of actions ineffectively induced by redundant advice is $\rho(m, t_h) = a(t, 1)$ and $\rho(m, t_l) = a(0, t)$. The profile of actions induced by false advice depends on the specifications of off-equilibrium beliefs.

To illustrate the "benefit of lying," suppose the indifference condition holds with $N = 3$ in a proposed equilibrium. Consider boundary type θ_1 (Figure 3).[30] If θ_1 is the true state, all high-interval types will have $t \leq \theta_1$ and all low-interval types will have $t > \theta_1$. Suppose θ_1 sends $m \in M_1$, indicating to the amateur that $\theta \in [0, \theta_1]$. Then, according to the above profile of actions, all high-interval types will take action $a(t, \theta_1)$, $t \in [0, \theta_1]$, and all low-interval types will take action $a(0, \theta_1)$ (the second line in Figure 3). Given that θ_1 satisfies the indifference condition, he will be indifferent between inducing these actions and those induced by $m \in M_2$, which are $a(\theta_1, \theta_2)$ and $a(\theta_1, t)$, $t \in (\theta_1, \theta_2]$ (not shown in the figure).

Now, suppose θ_1 lies by sending $m \in M_3$, indicating that $\theta \in (\theta_2, 1]$. All high-interval types will take $a(\theta_2, 1)$ and all low-interval types with $t \in (\theta_2, 1]$ will take $a(\theta_2, t)$ (the first line in Figure 3). These interval types are not able to detect the lie and are effectively induced to take these actions. To θ_1, these actions are less favorable than those induced by M_1 (or M_2) because they are positioned farther away from his ideal action. To the low-interval types with $t \in (\theta_1, \theta_2]$, however, the advice can be detected as false because it contradicts with what they know. Off-equilibrium beliefs $\psi(\theta|t_l)$ come into play in determining what ineffectively induced actions these low-interval types will take. Without any restriction, one can come up with beliefs so that the actions taken will be closer to θ_1's ideal than is $a(0, \theta_1)$, the effectively induced action these low-interval types take in the proposed equilibrium. This creates a benefit of lying that is absent in the CS model. It is conceivable, especially in equilibria with more steps, that such benefit of lying may outweigh the cost of inducing less favorable actions, breaking the equilibrium even though the indifference condition is satisfied. What equilibria will emerge instead in situations

[30] There could be a benefit of lying for boundary types whenever $N \geq 3$. However, for the interior types, such benefit also arises for $N = 2$. Thus, the indifference condition is not always sufficient even for two-step equilibria. Indeed, in the CS model, incentive compatibility for the interior types is a consequence of that for the boundary types. As will be discussed below, this is also not true in the amateur model.

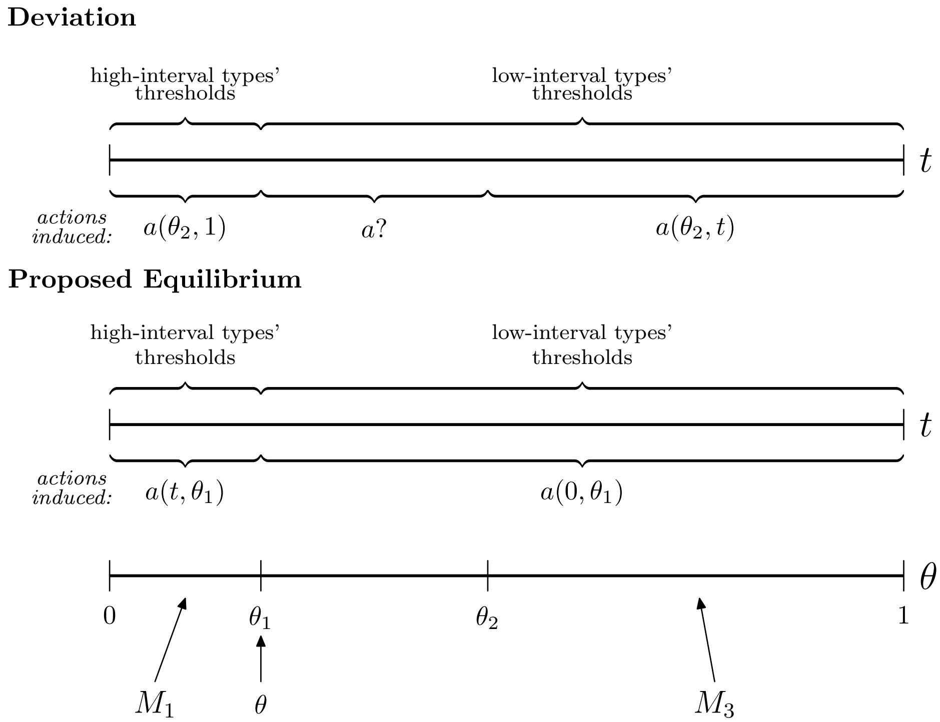


Figure 3: Incentives for Deviations

of this sort require *ad hoc* and detailed specifications of beliefs.

The following proposition states, however, that there is a set of off-equilibrium beliefs that, together with a mild restriction on the expert's payoff, guarantees the sufficiency of the indifference condition for the existence of partitional equilibria. Denote ψ to be the set of off-equilibrium beliefs of all interval types: $\psi = \bigcup_{\{t,s\}\in T\times\{l,h\}} \psi(\theta|t_s)$.

Proposition 7. *There exists a set of off-equilibrium beliefs ψ^* such that, provided $U^e_{12}(\cdot)$ is sufficiently large, the boundary types $\{\theta_i\}_{i=1}^{N-1}$ that satisfy* (B.3) *always constitute an equilibrium.*

Proof. The outline of the proof is as follows. I shall first construct ψ^* and state the cases of the expert's payoff $V^e(m,\theta,b)$ under ψ^*. I shall then show that, if $\psi = \psi^*$ and $U^e_{12}(\cdot)$ is sufficiently large, then (B.3) is sufficient for the following to always hold: for all $\theta \in [\theta_{i-1}, \theta_i]$,

$$V^e(M_i,\theta,b) = \max_j V^e(M_j,\theta,b),\ i,j = 1,\dots,N. \tag{B.4}$$

The set of off-equilibrium beliefs ψ^* is constructed as follows. Suppose there exists a monotone solution, $\{\theta_1,\dots,\theta_{N-1}\} \subset (0,1)$, to (B.3). If a high-interval type t_h with $t \in (\theta_i, \theta_{i+1}]$, $i = 0,\dots,N-1$, receives a false advice, her beliefs are that θ is distributed on $[t, \theta_{i+1}]$ with density $1/(\theta_{i+1} - t)$ and zero elsewhere; if a low-interval type t_l with $t \in (\theta_i, \theta_{i+1}]$ receives a false advice, her beliefs are that θ is distributed on (θ_i, t) with density $1/(t - \theta_i)$ and zero elsewhere. Then, when $\theta \in [\theta_{i-1}, \theta_i]$ sends $m \in M_i$ under the partitional strategy and deviates from it by sending

$m \in M_g$, $g \neq i$, the profile of his expected payoff will be

$$V^e(m,\theta,b) = \begin{cases} \int_0^{\theta_{k-1}} U^e(a(\theta_{k-1},\theta_k),\theta,b)dt \\ \quad + \sum_{r=k-1}^{i-1} \int_{\theta_r}^{\theta_{r+1}} U^e(a(t,\theta_{r+1}),\theta,b)dt & \text{if } m \in M_k,\ 1 \leq k < i, \\ \quad + \int_{\theta_i}^{\theta} U^e(a(t,\theta_{i+1}),\theta,b)dt \\ \quad + \int_{\theta}^{1} U^e(a(\theta_{k-1},\theta_k),\theta,b)dt, \\ \int_0^{\theta_{i-1}} U^e(a(\theta_{i-1},\theta_i),\theta,b)dt \\ \quad + \int_{\theta_{i-1}}^{\theta} U^e(a(t,\theta_i),\theta,b)dt \\ \quad + \int_{\theta}^{\theta_i} U^e(a(\theta_{i-1},t),\theta,b)dt & \text{if } m \in M_i, \\ \quad + \int_{\theta_i}^{1} U^e(a(\theta_{i-1},\theta_i),\theta,b)dt, \\ \int_0^{\theta} U^e(a(\theta_{j-1},\theta_j),\theta,b)dt \\ \quad + \int_{\theta}^{\theta_i} U^e(a(\theta_{i-1},t),\theta,b)dt \\ \quad + \sum_{r=i}^{j-1} \int_{\theta_r}^{\theta_{r+1}} U^e(a(\theta_r,t),\theta,b)dt & \text{if } m \in M_j,\ i < j \leq N. \\ \quad + \int_{\theta_j}^{1} U^e(a(\theta_{j-1},\theta_j),\theta,b)dt, \end{cases} \tag{B.5}$$

Using the second and the third cases in (B.5), the expected payoff for θ_i to send $m \in M_i$ and $m \in M_{i+1}$ are, respectively,

$$\int_0^{\theta_{i-1}} U^e(a(\theta_{i-1},\theta_i),\theta_i,b)dt + \int_{\theta_{i-1}}^{\theta_i} U^e(a(t,\theta_i),\theta_i,b)dt + \int_{\theta_i}^{1} U^e(a(\theta_{i-1},\theta_i),\theta_i,b)dt \tag{B.6}$$

$$\int_0^{\theta_i} U^e(a(\theta_i,\theta_{i+1}),\theta_i,b)dt + \int_{\theta_i}^{\theta_{i+1}} U^e(a(\theta_i,t),\theta_i,b)dt + \int_{\theta_{i+1}}^{1} U^e(a(\theta_i,\theta_{i+1}),\theta_i,b)dt. \tag{B.7}$$

Thus, the indifference condition (B.3) becomes the following second-order difference equation:

$$\begin{aligned} V(\theta_{i-1},\theta_i,\theta_{i+1},b) = \int_0^{\theta_{i-1}} & [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i-1},\theta_i),\theta_i,b)]dt \\ &+ \int_{\theta_{i-1}}^{\theta_i} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(t,\theta_i),\theta_i,b)]dt \\ &+ \int_{\theta_i}^{\theta_{i+1}} [U^e(a(\theta_i,t),\theta_i,b) - U^e(a(\theta_{i-1},\theta_i),\theta_i,b)]dt \\ &+ \int_{\theta_{i+1}}^{1} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i-1},\theta_i),\theta_i,b)]dt = 0, \end{aligned} \tag{B.8}$$

$i = 1,\ldots,N-1$, $\theta_0 = 0$, $\theta_N = 1$. Suppose there is a strictly increasing partition, $\theta_0,\ldots,\theta_i$, that satisfies (B.8). That $U^e_{11}(\cdot) < 0$, $a(\cdot,\cdot)$ is strictly increasing in its arguments, and the continuity of $V(\theta_{i-1},\theta_i,\theta',b)$ in θ' guarantee that there exists a unique $\theta_{i+1} > \theta_i$ that satisfies (B.8).

Turning to incentive compatibility, I begin by showing that (B.4) holds for θ_i, $i = 1,\ldots,N-1$, that satisfy (B.3). If $N = 2$, there exists no other set of messages that θ_i can send, and (B.4) is satisfied vacuously. So, consider $N \geq 3$. Suppose θ_i sends message $m \in M_{i+n}$, $2 \leq n \leq N-i$. Then, from the third case in (B.5) his expected payoff is

$$\begin{aligned} \int_0^{\theta_i} U^e(a(\theta_{i+n-1},\theta_{i+n}),\theta_i,b)dt + \sum_{r=i}^{i+n-2}\int_{\theta_r}^{\theta_{r+1}} U^e(a(\theta_r,t),\theta_i,b)dt \\ + \int_{\theta_{i+n-1}}^{\theta_{i+n}} U^e(a(\theta_{i+n-1},t),\theta_i,b)dt + \int_{\theta_{i+n}}^{1} U^e(a(\theta_{i+n-1},\theta_{i+n}),\theta_i,b)dt. \end{aligned} \tag{B.9}$$

Subtracting (B.9) from (B.7), we have

$$\begin{aligned} D_3 = \int_0^{\theta_i} & [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i+n-1},\theta_{i+n}),\theta_i,b)]dt \\ &+ \int_{\theta_i}^{\theta_{i+1}} [U^e(a(\theta_i,t),\theta_i,b) - U^e(a(\theta_i,t),\theta_i,b)]dt \\ &+ \sum_{r=i+1}^{i+n-2}\int_{\theta_r}^{\theta_{r+1}} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_r,t),\theta_i,b)]dt \\ &+ \int_{\theta_{i+n-1}}^{\theta_{i+n}} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i+n-1},t),\theta_i,b)]dt \\ &+ \int_{\theta_{i+n}}^{1} [U^e(a(\theta_i,\theta_{i+1}),\theta_i,b) - U^e(a(\theta_{i+n-1},\theta_{i+n}),\theta_i,b)]dt. \end{aligned}$$

Note that (B.8) implies that the expert's ideal action $a^e(\theta_i,b) \in (a(\theta_{i-1},\theta_i), a(\theta_i,\theta_{i+1}))$. Since $a(\theta_{i+n-1},\theta_{i+n}) > a(\theta_i,\theta_{i+1})$, $a(\theta_{i+n-1},t) > a(\theta_i,\theta_{i+1})$ for $t \in (\theta_{i+n-1},\theta_{i+n})$, and $a(\theta_j,t) >$

$a(\theta_i, \theta_{i+1})$ for $t \in (\theta_j, \theta_{j+1})$, $j = i+1, \ldots, i+n-2$, given $U_{11}^e(\cdot) < 0$ and the maximum of $U^e(a, \theta_i, b)$ is achieved for $a \in (a(\theta_{i-1}, \theta_i), a(\theta_i, \theta_{i+1}))$, the first, third, fourth and fifth terms are positive. Also, the second term vanishes. Thus, $D_3 > 0$.

Next, suppose θ_i sends $m \in M_{i-\eta}$, $1 \leq \eta \leq i-1$. From the first case in (B.5), his expected payoff is

$$\begin{aligned}\int_0^{\theta_{i-\eta-1}} U^e(a(\theta_{i-\eta-1}, \theta_{i-\eta}), \theta_i, b)dt + \int_{\theta_{i-\eta-1}}^{\theta_{i-\eta}} U^e(a(t, \theta_{i-\eta}), \theta_i, b)dt \\ + \sum_{r=i-\eta}^{i-1} \int_{\theta_r}^{\theta_{r+1}} U^e(a(t, \theta_{r+1}), \theta_i, b)dt + \int_{\theta_i}^{1} U^e(a(\theta_{i-\eta-1}, \theta_{i-\eta}), \theta_i, b)dt.\end{aligned} \tag{B.10}$$

Subtracting (B.10) from (B.6), we have

$$\begin{aligned}D_4 = &\int_0^{\theta_{i-\eta-1}} [U^e(a(\theta_{i-1}, \theta_i), \theta_i, b) - U^e(a(\theta_{i-\eta-1}, \theta_{i-\eta}), \theta_i, b)]dt \\ &+ \int_{\theta_{i-\eta-1}}^{\theta_{i-\eta}} [U^e(a(\theta_{i-1}, \theta_i), \theta_i, b) - U^e(a(t, \theta_{i-\eta}), \theta_i, b)]dt \\ &+ \sum_{r=i-\eta}^{i-2} \int_{\theta_r}^{\theta_{r+1}} [U^e(a(\theta_{i-1}, \theta_i), \theta_i, b) - U^e(a(t, \theta_{r+1}), \theta_i, b)]dt \\ &+ \int_{\theta_{i-1}}^{\theta_i} [U^e(a(t, \theta_i), \theta_i, b) - U^e(a(t, \theta_i), \theta_i, b)]dt \\ &+ \int_{\theta_i}^{1} [U^e(a(\theta_{i-1}, \theta_i), \theta_i, b) - U^e(a(\theta_{i-\eta-1}, \theta_{i-\eta}), \theta_i, b)]dt.\end{aligned}$$

Similar to the above, since $a(\theta_{i-1}, \theta_i) > a(\theta_{i-\eta-1}, \theta_{i-\eta})$, $a(\theta_{i-1}, \theta_i) > a(t, \theta_{i-\eta})$ for $t \in (\theta_{i-\eta-1}, \theta_{i-\eta})$, and $a(\theta_{i-1}, \theta_i) > a(t, \theta_{j+1})$, for $t \in (\theta_j, \theta_{j+1})$, $j = i-\eta, \ldots, i-2$, the first, second, third and fifth terms are positive, while the fourth term vanishes. Thus, $D_4 > 0$. That $D_3 > 0$ and $D_4 > 0$ imply that (B.4) holds for θ_i, $i = 1, \ldots, N-1$.

I show next that given (B.8) and for sufficiently large $U_{12}^e(\cdot)$, all $\theta \in (\theta_{i-1}, \theta_i)$ prefer sending $m \in M_i$ over $m \in M_{i+1}$, and all $\theta \in (\theta_i, \theta_{i+1})$ prefer sending $m \in M_{i+1}$ over $m \in M_i$, $i = 1, \ldots, N-1$, so that (B.4) holds for all interior θ. Consider an arbitrary $\theta \in (\theta_{i-1}, \theta_i)$. From the third case in (B.5), his expected payoff from sending $m \in M_{i+1}$ is

$$\begin{aligned}\int_0^{\theta} U^e(a(\theta_i, \theta_{i+1}), \theta_i, b)dt + \int_{\theta}^{\theta_i} U^e(a(\theta_{i-1}, t), \theta_i, b)dt \\ + \int_{\theta_i}^{\theta_{i+1}} U^e(a(\theta_i, t), \theta_i, b)dt + \int_{\theta_{i+1}}^{1} U^e(a(\theta_i, \theta_{i+1}), \theta_i, b)dt.\end{aligned} \tag{B.11}$$

Subtracting his expected payoff from sending $m \in M_i$ in (B.5) from (B.11), we have

$$\begin{aligned} D_5 = & \int_0^{\theta_{i-1}} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt \\ & + \int_{\theta_{i-1}}^{\theta} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(t, \theta_i), \theta, b)]dt \\ & + \int_{\theta}^{\theta_i} [U^e(a(\theta_{i-1}, t), \theta, b) - U^e(a(\theta_{i-1}, t), \theta, b)]dt \\ & + \int_{\theta_i}^{\theta_{i+1}} [U^e(a(\theta_i, t), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt \\ & + \int_{\theta_{i+1}}^{1} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt. \end{aligned} \tag{B.12}$$

Differentiating D_5 with respect to θ gives

$$\begin{aligned} \frac{\partial D_5}{\partial \theta} = & \int_0^{\theta_{i-1}} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\ & + \int_{\theta_{i-1}}^{\theta} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(t, \theta_i), \theta, b)]}{\partial \theta} dt \\ & + \int_{\theta_i}^{\theta_{i+1}} \frac{\partial [U^e(a(\theta_i, t), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\ & + \int_{\theta_{i+1}}^{1} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\ & + [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta, \theta_i), \theta, b)]\theta. \end{aligned}$$

Since $a(\theta_i, \theta_{i+1}) > a(\theta_{i-1}, \theta_i)$, $a(\theta_i, \theta_{i+1}) > a(t, \theta_i)$ for $t \in (\theta_{i-1}, \theta)$, and $a(\theta_i, t) > a(\theta_{i-1}, \theta_i)$ for $t \in (\theta_i, \theta_{i+1})$, $U^e_{12}(\cdot) > 0$ implies that the first four terms are positive. The last term, derived under Leibniz rule that θ is in the range of integration of the second term in (B.12), is, however, negative. Thus, when θ decreases from θ_i, there are negative effects on D_5 from the first four term and a positive effect from the last term. However, for a sufficiently large $U^e_{12}(\cdot)$ at θ, the negative effects outweigh the positive effect. A sufficiently large $U^e_{12}(\cdot)$ then guarantees, given (B.8), $D_5 \leq 0$ for θ.

Consider next an arbitrary $\theta \in (\theta_i, \theta_{i+1})$. From the first case in (B.5), his expected payoff from sending $m \in M_i$ is

$$\begin{aligned} & \int_0^{\theta_{i-1}} U^e(a(\theta_{i-1}, \theta_i), \theta, b)dt + \int_{\theta_{i-1}}^{\theta_i} U^e(a(t, \theta_i), \theta, b)dt \\ & + \int_{\theta_i}^{\theta} U^e(a(t, \theta_{i+1}), \theta, b)dt + \int_{\theta}^{1} U^e(a(\theta_{i-1}, \theta_i), \theta, b)dt. \end{aligned} \tag{B.13}$$

Subtracting (B.13) from the expected payoff from sending $m \in M_{i+1}$ in (B.5), we have

$$\begin{aligned}
D_6 = &\int_0^{\theta_{i-1}} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt \\
&+ \int_{\theta_{i-1}}^{\theta_i} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(t, \theta_i), \theta, b)]dt \\
&+ \int_{\theta_i}^{\theta} [U^e(a(t, \theta_{i+1}), \theta, b) - U^e(a(t, \theta_{i+1}), \theta, b)]dt \\
&+ \int_{\theta}^{\theta_{i+1}} [U^e(a(\theta_i, t), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt \\
&+ \int_{\theta_{i+1}}^{1} [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]dt.
\end{aligned} \tag{B.14}$$

Differentiating D_6 with respect to θ gives

$$\begin{aligned}
\frac{\partial D_6}{\partial \theta} = &\int_0^{\theta_{i-1}} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\
&+ \int_{\theta_{i-1}}^{\theta_i} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(t, \theta_i), \theta, b)]}{\partial \theta} dt \\
&+ \int_{\theta}^{\theta_{i+1}} \frac{\partial [U^e(a(\theta_i, t), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\
&+ \int_{\theta_{i+1}}^{1} \frac{\partial [U^e(a(\theta_i, \theta_{i+1}), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]}{\partial \theta} dt \\
&- [U^e(a(\theta_i, \theta), \theta, b) - U^e(a(\theta_{i-1}, \theta_i), \theta, b)]\theta.
\end{aligned}$$

Since $a(\theta_i, \theta_{i+1}) > a(\theta_{i-1}, \theta_i)$, $a(\theta_i, \theta_{i+1}) > a(t, \theta_i)$ for $t \in (\theta_{i-1}, \theta)$, and $a(\theta_i, t) > a(\theta_{i-1}, \theta_i)$ for $t \in (\theta_i, \theta_{i+1})$, $U_{12}^e(\cdot) > 0$ implies that the first four terms are positive. While the last term is negative, similar to the above, for a sufficiently large $U_{12}^e(\cdot)$ at θ, the positive effects on D_6 from the first four terms outweigh the negative effect from the last term; (B.8) then implies $D_6 \geq 0$ at θ. When $U_{12}^e(\cdot)$ is sufficiently large for all interior types, (B.4) holds for all of them.

□

The off-equilibrium beliefs specified in the proof - that for $t \in (\theta_i, \theta_{i+1}]$, t_h and t_l believe that θ is uniformly distributed on, respectively, $[t, \theta_{i+1}]$ and (θ_i, t) - are sufficient for the incentive compatibility condition to hold for the boundary types. However, since the interior types induce a set of actions not induced by the boundary types, a sufficiently large $U_{12}^e(\cdot)$ comes into the picture to guarantee that incentive compatibility also holds for them. Figure 4 illustrates the rationale with an example of two-step equilibrium.

Given that the indifference condition holds, the boundary type θ_1's expected payoff from the

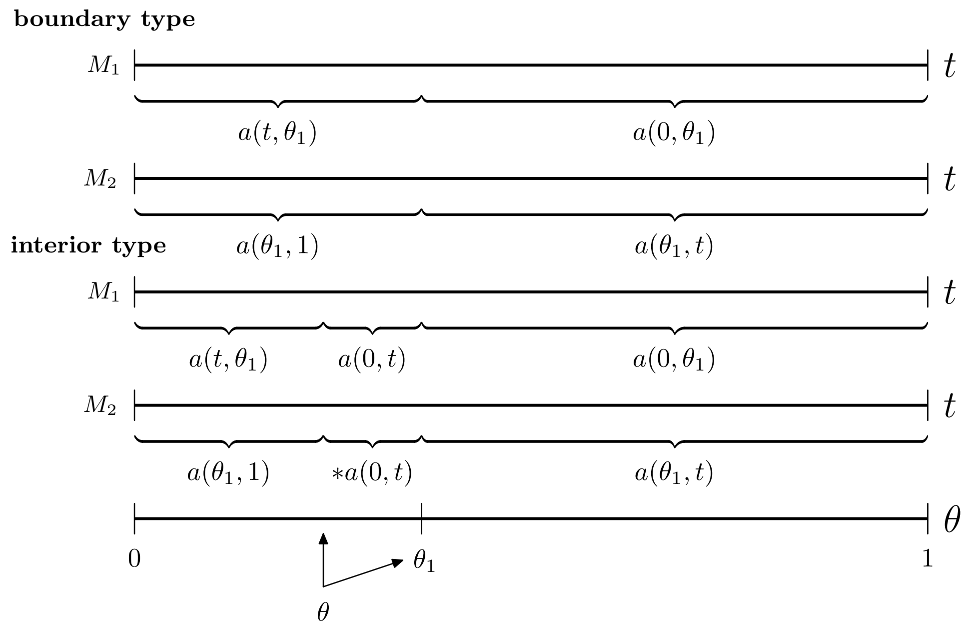


Figure 4: Actions Induced by Boundary and Interior Types

profile of actions $a(0,\theta_1)$ and $a(t,\theta_1)$, $t \in [0,\theta_1]$, is the same from that from $a(\theta_1,1)$ and $a(\theta_1,t)$, $t \in (\theta_1,1]$ (the two upper lines). Consider the actions induced when the interior type θ sends messages in M_1 and M_2. If we compare the profile of actions in the lower pair of lines with those in the upper pair, we can see that they are the same except for $t \in (\theta,\theta_1]$. While θ_1 gives no false advice when he sends messages in either M_1 and M_2, there is one when θ sends $m \in M_2$. The specification of ψ^*, which allows incentive compatibility to hold for θ_1, (ineffectively) induce the action $a(0,t)$ (with asterisk) for the interior type if he sends messages in M_2, which is the same as the action effectively induced by $m \in M_1$.

If we could fix the profile of actions, that $U^e_{12}(\cdot) > 0$ would have guaranteed that $\theta < \theta_1$ strictly prefers to send messages in M_1 over M_2. However, when the expert's type changes, the profile of actions also changes, and, insofar as the actions taken by $t \in (\theta,\theta_1]$ are concerned, θ is indifferent between M_1 and M_2. Thus, we have to ensure that, overall, θ prefers M_1 enough for $t \notin (\theta,\theta_1]$ so that even with the indifference for $t \in (\theta,\theta_1]$ the incentive compatibility still holds. For this, a sufficiently large $U^e_{12}(\cdot)$ is required. A large $U^e_{12}(\cdot)$ means that the ideal action of a higher θ is sufficiently higher than that of a lower θ. This additional restriction is nothing but a strengthening of the already existing sorting condition.

B.3 Calculation for Example 1

The indifference condition reduces to

$$\begin{aligned}\int_0^{\theta_1}&\left(\left[\frac{t+\theta_1}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_1+\theta_2}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2\right)dt\\&+\int_{\theta_1}^{\theta_2}\left(\left[\frac{\theta_1}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_1+t}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2\right)dt\\&+\int_{\theta_2}^{1}\left(\left[\frac{\theta_1}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_1+\theta_2}{2}-\left(\theta_1+\frac{1}{15}\right)\right]^2\right)dt=0;\end{aligned}\tag{B.15}$$

$$\begin{aligned}\int_0^{\theta_1}&\left(\left[\frac{\theta_1+\theta_2}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_2+1}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2\right)dt\\&+\int_{\theta_1}^{\theta_2}\left(\left[\frac{t+\theta_2}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_2+1}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2\right)dt\\&+\int_{\theta_2}^{1}\left(\left[\frac{\theta_1+\theta_2}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2-\left[\frac{\theta_2+t}{2}-\left(\theta_2+\frac{1}{15}\right)\right]^2\right)dt=0.\end{aligned}\tag{B.16}$$

The solution that satisfies $0<\theta_1<\theta_2<1$ is $\theta_1=0.011122$ and $\theta_2=0.307109$. Subtracting, for $\theta\in[0,\theta_1)$, the expected payoff from sending $m\in M_1$ from that from sending $m\in M_2$ gives $D_7=-0.00287+0.255508\theta+0.27656\theta^2-0.41667\theta^3$, and it is straightforward to verify that $\partial D_7/\partial\theta>0$ for $\theta\in[0,\theta_1)$ so that, given, (B.15), $D_7\leq 0$ for θ in this range. Subtracting, for $\theta\in(\theta_1,\theta_2)$ the expected payoff from sending $m\in M_2$ from that from sending $m\in M_3$ gives $D_8=-0.11957+0.099964\theta+1.228334\theta^2-1.41667\theta^3$, and it can be shown that $\partial D_8/\partial\theta>0$ for $\theta\in(\theta_1,\theta_2)$ so that, given, (B.16), $D_8\leq 0$ for θ in this range.

Next, subtracting, for $\theta\in(\theta_2,1]$, the expected payoff from sending $m\in M_2$ from that from sending $m\in M_3$ gives $D_9=-0.22272+0.747777\theta+0.054566\theta^2-0.41667\theta^3$. While the second derivative $\partial^2D_9/\partial\theta^2<0$ for $\theta\in(\theta_2,1]$, $D_9\geq 0$ for $\theta=\theta_2$ and $\theta=1$. Thus, $D_9\geq 0$ for all $\theta\in(\theta_2,1]$. Finally, subtracting, for $\theta\in(\theta_1,\theta_2)$, the expected payoff from sending $m\in M_1$ from that from sending $m\in M_2$ gives $D_{10}=-0.00292+0.263331\theta-0.030555\theta^2-0.41667\theta^3$, and it can be shown that $\partial D_{10}/\partial\theta>0$ for $\theta\in(\theta_1,\theta_2)$ so that, given, (B.15), $D_{10}\geq 0$ for θ in this range. Incentive compatibility is thus satisfied for all interior θ.

Note that $N_{cs}(\frac{1}{15})=3$. Accordingly, from (A.10) the novice's *ex-ante* payoff is $W_{cs}(\frac{1}{15})=-0.02111$. For the evaluation of information structures, for $t\in[0,\theta_1]$ the amateur is better off if and only if

$$\begin{aligned}W_t\left(\frac{1}{15}\right)-W_{cs}\left(\frac{1}{15}\right)&=-\frac{t^3}{12}+0.08333(t-0.01112)(t^2-0.02224t+0.00012)\\&\quad-0.00877\geq 0.\end{aligned}\tag{B.17}$$

However, since $\frac{\partial^2[W_t(\frac{1}{15})-W^{cs}(\frac{1}{15})]}{\partial t^2} < 0$ for $t \in [0, \theta_1]$ and the maximum of $W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})$ for $t \in [0, \theta_1]$, $[W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})]_{|t=0.00561} = -0.00877 < 0$, (B.17) is not satisfied. For $t \in (\theta_1, \theta_2]$, the amateur is better off if and only if

$$W_t\left(\frac{1}{15}\right) - W_{cs}\left(\frac{1}{15}\right) = t^3 - 0.07400t^2 + 0.02355t - 0.00902 \geq 0. \tag{B.18}$$

Similar to the above, $\frac{\partial^2[W_t(\frac{1}{15})-W^{cs}(\frac{1}{15})]}{\partial t^2} < 0$ for $t \in (\theta_1, \theta_2]$, and the maximum of $W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})$ for $t \in (\theta_1, \theta_2]$, $[W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})]_{|t=0.159116} = -0.00715 < 0$. Thus, (B.18) is also not satisfied. For $t \in (\theta_2, 1]$, the amateur is better off if and only if

$$\begin{aligned} W_t\left(\frac{1}{15}\right) - W_{cs}\left(\frac{1}{15}\right) &= -\frac{(1-t)^3}{12} - 0.08333(t - 0.307111) \\ &\quad \times (t^2 - 0.61422t + 0.09432) + 0.01895 \geq 0. \end{aligned} \tag{B.19}$$

Note that $\frac{\partial^2[W_t(\frac{1}{15})-W_{cs}(\frac{1}{15})]}{\partial t^2} < 0$ for $t \in (\theta_2, 1]$, but the maximum of $W_t(\frac{1}{15}) - W_{cs}(\frac{1}{15})$ for $t \in (\theta_2, 1]$, $[W_t(\frac{1}{15}) - W^{cs}(\frac{1}{15})]_{|t=0.65356} = 0.01202 > 0$. Solving the equation $W_t(\frac{1}{15}) - W_{cs}(\frac{1}{15}) = 0$ in (B.19) gives that the amateur is better off if and only if $t \in [0.39014, 0.91697]$.

Finally, the amateur's *ex-ante* payoff before t is realized, derived by integrating the expressions in (B.17)-(B.19) with respect to t in the respective ranges and summing them up, is $W_T(\frac{1}{15}) = -0.01996 > -0.02111$. Thus, the amateur is better off with the information opportunity.

Appendix B is Not Intended for Publication

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