# Table of Integrals

$$\int (a+bx)^n\,dx = \frac{(a+bx)^{n+1}}{b(n+1)} \qquad \text{for n} \neq -1$$

$$\int \frac{dx}{(a+bx)} = \frac{1}{b}\ln|a+bx|$$

$$\int \frac{xdx}{(a+bx)} = \frac{1}{b^2}\left[a+bx-a\ln|a+bx|\right]$$

$$\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

$$\int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2}\left[\ln(a+bx)+\frac{a}{(a+bx)}\right]$$

$$\int \frac{dx}{(a+bx)^{1/2}} = \frac{2}{b}(a+bx)^{1/2}$$

$$\int \frac{xdx}{(a+bx)^{1/2}} = \frac{2}{b^2}\left[\frac{(a+bx)^{3/2}}{3}-a(a+bx)^{1/2}\right]$$

$$\int \frac{dx}{(a+bx)^{3/2}} = \frac{-2}{b(a+bx)^{1/2}}$$

$$\int \frac{xdx}{(a+bx)^{3/2}} = \frac{2}{b^2}\left[(a+bx)^{1/2}+\frac{a}{(a+bx)^{1/2}}\right]$$

$$\int \frac{dx}{(a^2+x^2)} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{xdx}{(a^2+x^2)} = \frac{1}{2}\ln(a^2+x^2)$$

$$\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)}+\frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{xdx}{(a^2+x^2)^2} = -\frac{1}{2(a^2+x^2)}$$

$$\int \frac{dx}{(a^2-x^2)} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|$$

$$\int \frac{xdx}{(a^2-x^2)} = -\frac{1}{2}\ln|a^2-x^2|$$

$$\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)}+\frac{1}{4a^3}\ln\left|\frac{a+x}{a-x}\right|$$

$$\int \frac{xdx}{(a^2-x^2)^2} = \frac{1}{2(a^2-x^2)}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x+\sqrt{x^2+a^2}) = \sinh^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{xdx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

$$\int \frac{x^2dx}{\sqrt{x^2+a^2}} = \frac{x\sqrt{x^2+a^2}}{2} - \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})$$

$$\int \frac{x^3dx}{\sqrt{x^2+a^2}} = \frac{(x^2+a^2)^{3/2}}{3} - a^2\sqrt{x^2+a^2}$$

$$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{dx}{x^2\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{a^2x}$$

$$\int \frac{dx}{x^3\sqrt{x^2+a^2}} = -\frac{\sqrt{x^2+a^2}}{2a^2x^2} + \frac{1}{2a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \sqrt{x^2+a^2}\,dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})$$

$$\int x\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{3/2}}{3}$$

$$\int x^2\sqrt{x^2+a^2}\,dx = \frac{x(x^2+a^2)^{3/2}}{4} - \frac{a^2x\sqrt{x^2+a^2}}{8} - \frac{a^4}{8}\ln(x+\sqrt{x^2+a^2})$$

$$\int x^3\sqrt{x^2+a^2}\,dx = \frac{(x^2+a^2)^{5/2}}{5} - \frac{a^2(x^2+a^2)^{3/2}}{3}$$

$$\int \frac{\sqrt{x^2+a^2}}{x}\,dx = \sqrt{x^2+a^2} - a\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{\sqrt{x^2+a^2}}{x^2}\,dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln(x+\sqrt{x^2+a^2})$$

$$\int \frac{\sqrt{x^2+a^2}}{x^3}\,dx = -\frac{\sqrt{x^2+a^2}}{2x^2} - \frac{1}{2a}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$$

$$\int \frac{xdx}{(x^2+a^2)^{3/2}} = \frac{-1}{\sqrt{x^2+a^2}}$$

$$\int \frac{x^2 dx}{(x^2+a^2)^{3/2}} = \frac{-x}{\sqrt{x^2+a^2}} + \ln(x+\sqrt{x^2+a^2})$$

$$\int \frac{x^3 dx}{(x^2+a^2)^{3/2}} = \sqrt{x^2+a^2} + \frac{a^2}{\sqrt{x^2+a^2}}$$

$$\int \frac{dx}{x(x^2+a^2)^{3/2}} = \frac{1}{a^2\sqrt{x^2+a^2}} - \frac{1}{a^3}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \frac{dx}{x^2(x^2+a^2)^{3/2}} = -\frac{\sqrt{x^2+a^2}}{a^4 x} - \frac{x}{a^4\sqrt{x^2+a^2}}$$

$$\int \frac{dx}{x^3(x^2+a^2)^{3/2}} = \frac{-1}{2a^2x^2\sqrt{x^2+a^2}} - \frac{3}{2a^4\sqrt{x^2+a^2}} + \frac{3}{2a^5}\ln\left(\frac{a+\sqrt{x^2+a^2}}{x}\right)$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax)$$

$$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax)$$

$$\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int \sin(ax)\cos(ax)dx = \frac{1}{2a}\sin^2(ax)$$

$$\int \sin(ax)\cos^2(ax)dx = -\frac{\cos^3(ax)}{3a}$$

$$\int \sin^2(ax)\cos(ax)dx = \frac{\sin^3(ax)}{3a}$$

$$\int \sin^2(ax)\cos^2(ax)dx = -\frac{1}{32a}\sin(4ax) + \frac{x}{8}$$

$$\int x\sin^2(ax)dx = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$$

$$\int x\cos^2(ax)dx = \frac{x^2}{4} + \frac{x\sin(2ax)}{4a} + \frac{\cos(2ax)}{8a^2}$$

$$\int x^2\sin^2(x)dx = \frac{x^3}{3} - \frac{x\cos(2x)}{4} - \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin(2x)$$

$$\int x^2\cos^2(x)dx = \frac{x^3}{3} + \frac{x\cos(2x)}{4} + \left(\frac{x^2}{4} - \frac{1}{8}\right)\sin(2x)$$

$$\int_0^a \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi x}{a}\right)dx = \frac{a}{2}\delta_{mn}$$

$$\int_0^a \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi x}{a}\right)dx = \frac{a}{2}\delta_{mn}$$ (note: if $m = n = 0$, then integral = $a$)

$$\int_{-a}^a \cos\left(\frac{m\pi x}{2a}\right)\cos\left(\frac{n\pi x}{2a}\right)dx = a\delta_{mn}$$ (note: if $m = n = 0$, then integral = $2a$)

$$\int_0^\pi P_\ell(\cos\theta)P_m(\cos\theta)\sin\theta d\theta = \int_{-1}^1 P_\ell(x)P_m(x)dx = \frac{2}{2\ell+1}\delta_{\ell m}$$

$$\int e^{ax}dx = \frac{1}{a}e^x$$

$$\int x\,e^{ax}dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right]$$

$$\int x^2\,e^{ax}dx = e^{ax}\left[\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right]$$

$$\int x^3\,e^{ax}dx = e^{ax}\left[\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right]$$

$$\int x^n\,e^{ax}dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}\,e^{ax}dx$$

$$\int_0^\infty x^n e^{-x/\alpha}dx = n!\alpha^{n+1}$$

$$\int_0^\infty e^{-a^2x^2}dx = \frac{\sqrt{\pi}}{2a}$$

$$\int_0^\infty x\,e^{-a^2x^2}dx = \frac{1}{2a^2}$$

$$\int_0^\infty x^2 e^{-a^2x^2}dx = \frac{\sqrt{\pi}}{4a^3}$$

$$\int_0^\infty x^3 e^{-a^2x^2}dx = \frac{1}{2a^4}$$

$$\int_0^\infty x^4 e^{-a^2x^2}dx = \frac{3\sqrt{\pi}}{8a^5}$$

$$\int_0^\infty x^{2n+1} e^{-a^2x^2}dx = \frac{n!}{2a^{2n+2}}$$

$$\int_0^\infty x^{2n} e^{-a^2x^2}dx = \frac{[1\times3\times5\times\ldots\times(2n-1)]\sqrt{\pi}}{2^{n+1}a^{2n+1}}$$