Polarization

Scott Freese

Physics 262

19 March 2008

Partner: John Yamrick

Abstract

The fundamental goals for this experiment were to learn the physics behind the polarization of light, and the manipulation of this polarized light. By calculating Brewster's angle for a glass plate, and using Snell's law, we were able to determine the index of refraction for the plate. We calculated an angle of 55º, which corresponds to an index of refraction of $n = 1.43$ for the glass plate. Many of the other goals for this experiment were mathematically based. Given the equations for linearly polarized light along with those for $\lambda/4$ and $\lambda/2$ plates, we proved that $\lambda/4$ plates will convert linearly polarized light into circularly polarized, and that a $\lambda/2$ will change the direction of polarization of the linearly polarized light. From these proofs, we developed and performed tests for determining the type of retardation plate; whether they were $\lambda/4$, $\lambda/2$, or something else. For the plates determined to be something else, a procedure was determined and proven for determining the quantitative retardation factor for the plate. The plate we tested was shown to have a retardation factor of $(0.13)\lambda$, or $\approx \lambda/7.69$.

Introduction

The direction of polarization of a polarized wave of light, an electromagnetic wave, is defined by the direction in which its corresponding electric field vector ($\boldsymbol{E}$) points. Since all electromagnetic waves are transverse by nature, it is known that this $\boldsymbol{E}$ vector will always point in the direction perpendicular to the direction of motion. When the direction of the electric field vector remains constant in time, the wave is said to be linearly polarized. In considering a wave that is propagating in the $\boldsymbol{z}$ direction, the $\boldsymbol{E}$ vector is expressed by the equation:

$$\boldsymbol{E}(z,t) = \mathrm{Re}(\boldsymbol{E}_{0x}\boldsymbol{x} + \boldsymbol{E}_{0y}\boldsymbol{y})e^{i(kz-\omega t)} \quad (1)$$

where ω is the angular frequency of the linearly polarized plane wave. While the amplitude of the $\boldsymbol{E}$ vector may oscillate over time, the direction remains the same as long as $\boldsymbol{E}_{0x}$ and $\boldsymbol{E}_{0y}$ are both real. A model of the $\boldsymbol{E}$ vector field for a linearly polarized wave is shown in Figure 1 below. In the diagram, the angle Θ is found by the association of the x and y components of the $\boldsymbol{E}$ vector: $\tan\Theta = \boldsymbol{E}_{0y}/\boldsymbol{E}_{0x}$.

For the light to be linearly polarized, both the x and y components of $\boldsymbol{E}$ must be in phase with one another. When they are out of phase, the $\boldsymbol{E}$ vector no longer remains in the same direction. Instead, the $\boldsymbol{E}$ field rotates. A vector of this form is expressed by the following equation:

$$\boldsymbol{E}(z,t) = \mathrm{Re}(\boldsymbol{E}_{0x}\boldsymbol{x} + i\boldsymbol{E}_{0y}\boldsymbol{y})e^{i(kz-\omega t)} \quad (2)$$

For the case when $\boldsymbol{E}_{0x} = \boldsymbol{E}_{0y}$, the wave is referred to as circularly polarized light. The two components are equal, but 90º out of phase. The result of this is an $\boldsymbol{E}$ field that has an amplitude that remains constant with time, but has a direction that rotates clockwise in a circle with the angular frequency, ω.

However, when $|\boldsymbol{E}_{0x}| \neq |\boldsymbol{E}_{0y}|$, the amplitude of the $\boldsymbol{E}$ field is no longer constant with time. Under this circumstance, the amplitude of the vector traces out an ellipse when viewed head on. The major and minor axes of this ellipse are the given by $\boldsymbol{E}_{0x}$ and $\boldsymbol{E}_{0y}$, as can be seen in Figure 2.

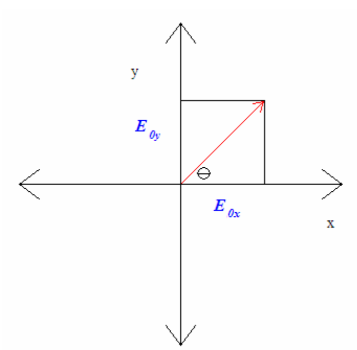

Figure 1 – Linearly polarized light

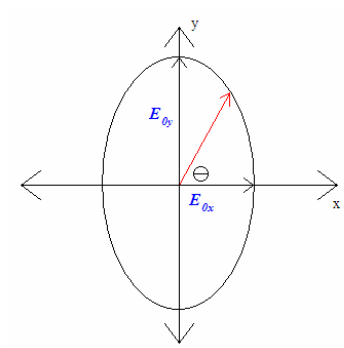

Figure 2 – Elliptically polarized light

It is possible to create these various forms of polarization by using polarizers and wave plates, but one of the simplest means is by just using a piece of glass. When the incoming light ray is incident upon the surface of the glass at a specific angle, called Brewster's angle, Θ_B, the reflected light is linearly polarized. Whether the incoming ray is unpolarized, or partially polarized, the component polarized in the plane of incidence passes through the glass. However, the light that is polarized in the direction perpendicular to this plane is partially transmitted and partially reflected. Therefore, the only light that is reflected by the surface is light polarized in the plane perpendicular to the plane of incidence. Regardless of the material the light is incident upon, Brewster's angle is separated by exactly 90º from the path of the ray through the material. Therefore $\Theta_B + \Theta_2 = 90^\circ$ (see Figure 3), which by using Snell's law, reduces to:

$$\tan \Theta_B = n_2 / n_1 \tag{3}$$

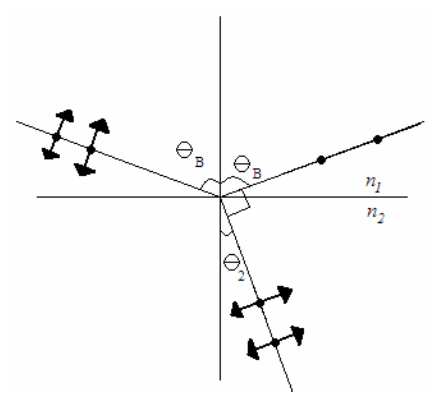

Figure 3 – Light rays, angles, and polarization associated with Brewster's angle

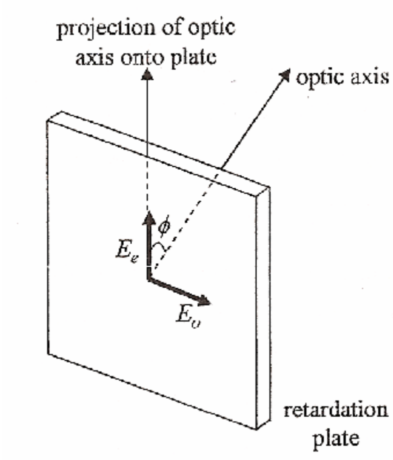

Figure 4 – A birefringent retardation plate

Another method for altering the state of polarization is with birefringent retardation plates. In these plates, and all birefringent materials, there is a direction associated with them call the optic axis. A light ray that is polarized perpendicular to this optic axis travels through the plate with a velocity $v_o = c/n_o$, which is characterized by the "ordinary" index of refraction of the plate, n_o. A ray with a polarization vector perpendicular to this ordinary ray has a component of its ***E*** vector along the optic axis. This is referred to as an "extraordinary ray," and travels through the wave with a velocity of $v_e = c/n_e$. It is important to note that even though n_e is a function of the φ, this angle is a constant for a given retardation plate (See Figure 4). Also, since the ordinary and extraordinary rays travel with different velocities, they are set out of phase with one another. Given a plate of thickness, d, the equation for the effects of the plate is given as:

$$(n_o - n_e)d = (m + 1/x)\lambda_{vac} \tag{4}$$

When $x=4$, the plate is called a λ/4 plate. These plates can be used to convert linearly polarized light into circularly polarized light. When $x=2$, the plate is called a λ/2 plate, and can be used to rotate the direction of polarization for linearly polarized light.

Apparatus

The primary equipment used during the proceedings of this experiment is listed as follows:

- 633nm polarized HeNe laser
- Multiple birefringent retardation plates ($\lambda/4$, $\lambda/2$, and unknown types)
- Polarizer and analyzer
- Glass plate with unknown index of refraction

I. Measurement of Brewster's angle and index of refraction for a glass plate

Method

First, Equation 3 was derived from Snell's Law to ensure thorough understanding of the reasoning behind Brewster's angle (See page 7 of the lab book). Upon completion of this, a test was developed for measuring Brewster's angle. The layout of the equipment used for this test is shown in the lab book. The plan for the test was to reflect a beam of laser light off of the glass plate, and by passing the reflected light through an analyzer, determine whether this reflected light was fully polarized in just one direction. By finding the angle at which the reflected light was polarized in this manner, Brewster's angle was found. Once this position was discovered, the distance was measured from the incidence point on the glass plate to the point on a wall at which the beam struck. By also measuring from this wall point to the line on which the ray of light was initially traveling, an imaginary triangle can be created, allowing for the use of trigonometry to solve for Brewster's angle.

Upon determining Brewster's angle, Equation 3 was used to calculate the index of refraction for the glass plate.

Results

Using a meter stick for measurement, the following distances were gathered, with a confidence of ± 2 cm:

Distance (from plate to wall) = 1.52m

Distance (from wall to ray line) = 1.43m

The resulting angle of the imaginary triangle and the corresponding Brewster's angle are:

$\varphi = 19.815^{o}$ (extra digits held for calculation)

$\Theta_B \approx 55.0^{o}$

Therefore, by using Equation 3, and n_{air} = 1.0, the index of refraction for the glass plate was determined:

$$n_{glass} = 1.43$$

II. Mathematically prove the effects of both λ/4 and λ/2 plates and predict the effect of other various retardation plates

Method

To prove that a λ/4 plate will turn linearly polarized light into circularly polarized, and that a λ/2 plate will change the direction of polarization for linearly polarized light, the respective equations (Eq. 4, replacing *x* by 4 and 2) for each are used in conjunction with the equation for linearly polarized light (Eq. 1). Carrying through the mathematics should reveal that for λ/4, using Equation 4 with x=4, Equation 1 becomes Equation 2. And for λ/2, using Equation 4 with x=2, one of the terms in Equation 1 becomes negative, thereby changing the direction of polarization by 90º.

Upon determining the proofs for these plates, a method can be determined for predicting the effects of other various retardation plates.

Results

The proofs for the λ/4 and λ/2 plates are given on pages 8-9 in the lab book. Both proofs show exactly what was described above in the Method for this section of the experiment.

Also on page 9 of the lab book are the predictions for other various retardation plates.

On page 10, these results were used to create test methods for determining whether a plate was λ/4, λ/2, or something else entirely.

III. Devise a measurement procedure to quantitatively determine the retardation of a plate at 633 nm.

Method

While the following test method was given, the physical reasoning behind was needed to be proven. This proof is given on pages 11-13 of the lab book.

First, the analyzer was set in line with the beam of laser light, and then rotated until extinction of the laser occurred. After this, a $\lambda/4$ plate was placed between the laser and the analyzer, and this too was rotated until extinction occurred. With this in place, another wave plate was placed between the source and the $\lambda/4$ plate; however the retardation factor of this new plate was unknown. As occurred with the others, this new plate was rotated until extinction of the laser occurred. At this point, either the fast or slow axis of both retardation plates was aligned with the polarization direction of the laser.

Next, the unknown plate was rotated 45º degrees from its initial position to give the ordinary and extraordinary rays equal amplitudes. This procedure actually creates elliptically polarized light. When this light then strikes and passes through the $\lambda/4$ plate, the polarization of the light is reverted back to linearly polarized light, but with a shift from the initial polarization direction from the laser source. This is the effect which is proven on pages 11-13 of the lab book.

By using the analyzer, the angle of the new polarization direction can be determined, and the displacement can be measured. This angle (φ) is directly related to the retardation (δ) of the unknown retardation plate. The two are related by the equation:

$$\varphi = \pi\delta / \lambda \tag{5}$$

This relationship was also proven in the methods shown in the lab book.

Results

The displacement angle, φ, for this experiment was measured to be 23.0º with an uncertainty of 0.5º.

From this angle, the retardation, δ, was calculated to be 80.88 nm. Since the light passing through the retardation plate was 633 nm, this relates to a retardation factor of $\approx 1/7.69$. This is the same expression as $1/x$ in Eq. 4. Therefore, it can be said that $\delta = \lambda/x$.

Error Analysis

Part I

(Calculations for these errors are shown on Page 13 of the lab book.)

The distance measurements in this part of the experiment were calculated with an uncertainty of 2 cm for both measurements. Because these measurements were used to directly calculate Brewster's angle for the glass plates, these errors carry through to the determination of the angle.

The error in the calculated angle is $\varphi = 19.815^{o} \pm 4.31^{o}$

And due to the relationship of $\varphi/2$, the error in Brewster's angle is: $\theta_B = 55.0^{o} \pm 2.16^{o}$

Following through, the index of refraction of the glass has an overall error of: n_{glass} 1.43 ± 0.114

Part III

(Calculations are shown on Page 14 of the lab book.)

The angle of displacement for this experiment was measured to be 23.0^{o} with a confidence of ± 0.5^{o}.

From this, the retardation had an error of: $\delta = 80.88 \pm 1.76$ nm

And therefore, the error in the 1/x expression is: $1/x = \delta/\lambda = 0.13 \pm .00278$

Conclusion

While at times, this experiment proved to be very frustrating, it can certainly be considered a success. The challenges encountered with the mathematical proofs were overcome to reach a much greater understanding of the processes involved with the polarization of light, in all its forms.

The results obtained in the first part of this experiment for the glass plate are very accurate with standard values for the index of refraction for standard glass. Most glass ranges from $n = 1.52$ – 1.62, but the typical standard for generic glass is 1.52. While we obtained an index of refraction of $n = 1.43$, the standard value, along with part of the corresponding range for glass, is well within our error set.