Optical Tweezers

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Abstract

The overall goal for this experiment was to learn the basic physics behind the technique of optical tweezers. Beyond this, two principle objectives were proposed. The first was to detect the motion of a trapped, oscillating particle, and to calibrate the effective spring constant of the optical trap. The second of these objectives was dependent upon the first. This was to calculate the viscoelastic moduli of a bio-polymer.

Introduction

Despite the fact that the process of optical tweezing is very young, the principle behind it has been known for a few centuries now. Kepler was able to theorize that solar irradiance was the cause behind the tail of comets. By 1873, James Clerk Maxwell was able to prove by theory that light is capable of exerting a force on matter, something that is now called radiation pressure. In the next century, Otto R. Frisch was able to deflect a beam of sodium atoms with nothing but light from a sodium lamp. In the 1980's, Stephen Chu, William Phillips, and Claude Cohen-Tannoudji would win the Nobel Prize in Physics for the laser cooling and trapping of atoms. While none of these experiments or theories is an optical tweezers system, they implement the same ideas. In 1986, Bell Laboratories' scientist Arthur Ashkin showed that he could accelerate latex spheres suspended in water, using only laser beams. A year later, he was able to trap biological cells using just one laser.

A tightly focused single beam like the one Ashkin used creates a very high electric field gradient around the focal point. Any dielectric particle, like a transparent silica bead or bacterial cell, will undergo a force in the direction of the focus of the beam, essentially locking it in place. By adjusting the focus, the particle can be moved.

The principle that allows this to work is the fact that light carries momentum with it. When light passes through a material with a different index of refraction than the medium through which it is traveling, like water or air, momentum is transferred from the light beam to the particle. This is the radiation pressure theorized by Maxwell. To successfully use the laser for tweezing, the particle must be controlled in all three spatial dimensions. Ashkin realized that this could be performed with a steep intensity gradient in these three directions. A laser beam with this property creates a Gaussian beam profile, since it cannot be fully focused. This Gaussian profile creates an intensity gradient that forces the particle toward the center of the beam, locking it in the x-y direction. When the laser is used in conjunction with a microscope with an objective lens with a high numerical aperture, a steep intensity gradient is formed in the z-direction as well, forcing the particle toward the focal point of the lens.

To determine the equation of motion for the particle in the optical trap, two forces must be taken into account: the viscous drag force, and the force exerted by the optical trap on the particle. Using Hookes' law, and a spring constant of k_{ot}, the force of the optical trap can be approximated very well. To determine the spring constant, the particle is set into an oscillating motion with frequency, ω.

$$F_{drag} = 6\pi\eta_o a v(\omega, t)$$

$$F_{trap} = k_{ot}(Ae^{i\omega t} - x(\omega,t))$$

Where η_o is the zero shear viscosity of the liquid, $v(\omega, t)$ is the velocity of the particle, and a is the radius of the particle. By using Newton's second law of motion, these equations can lead to an equation for the displacement of the particle, as well as the phase shift.

$$D(\omega) = \frac{A}{\sqrt{\tau^2\omega^2 + (1-\omega^2/\omega_0^{\ 2})^2}}$$

$$\delta(\omega) = \tan^{-1}\left(\frac{\tau\omega}{1-\omega^2/\omega_0^{\ 2}}\right)$$

Where $\tau = 6\,\pi\eta_o a/\,k_{ot}$ and $\omega_0 = (k_{ot}\,/\,m)^{1/2}$.

To measure the displacement and frequency of oscillation, two lasers are used. One is used to act as the optical tweezers. This is filtered out in the end, before reaching a position sensing detector. This filter only transmits the frequency associated with the second laser, which acts primarily as a position indicator. When this is matched with a reference frequency in a Lock-in Amplifier, and analyzing the phase difference, the oscillation of the particle can be measured very accurately. The phase and displacement, along with the frequency associated with each, allows for the calculation of the spring constant k_{ot}..

In using a gel solution, the spring constant determined for a bead can lead to the calculation of the gel's viscoelastic properties, the storage modulus and the loss modulus. To do this, the displacement of the bead and the phase shift must be again measured in the gel. The following equations can then be used to measure the gel's properties:

Storage Modulus: $$\boldsymbol{G' = \frac{k_{ot}}{6\pi a}\left(\frac{A}{D}\cos\delta - 1\right)}$$

Loss Modulus: $$\boldsymbol{G'' = \frac{A}{D}\frac{1}{6\pi a}k_{ot}\sin\delta}$$

Apparatus

The primary equipment used throughout this experiment are listed as follows:

- 980 nm laser
- 1064 nm laser
- optical mirrors and lenses
- neutral density filters
- Piezo Electric controlled mirror
- Olympus IX81 Microscope
- 1.5 μm silica beads in water solution
- 980 nm filter
- Position Sensing Detector
- Oscilloscope
- SRS SR830 Lock-In Amplifier
- Computer using LabVIEW software

I. Detection of the Motion of a particle, and calibration of spring constant of an optical trap

Method

Upon setting up the equipment in the manner shown by the diagram in the lab book, experiments were set to be run. To perform determine the motion of the silica beads, a sample was created placing an amount of diluted solution containing the beads onto a microscope slide, which was then placed on the microscope. The next required step was to "catch" a bead with the laser. Upon this, the particle is moved away from the surface of the glass containing it, to reduce any frictional forces. This is done by merely adjusting the focus of the laser. At this point, the LabVIEW program is run to collect data for both the phase and displacement of the oscillation of the particle.

To determine the spring constant, k_{ot}, is calculated by plotting the normalized displacement and the phase as a function of frequency, ω. This can be performed since the only unknown in the equations for either is the spring constant. The average of the phase and displacement spring constants is used.

Results

Two runs were performed in oscillating the silica beads. The first resulted in the calculations of spring constants that were quite different between the phase and the displacement. It was only noticed after the calculations were performed that the experiment was performed with two beads caught in the optical tweezers. Therefore these data are by no means reliable for the calculation of the spring constant for just one bead.

The second run proved much more favorable. The microscope display was monitored throughout the test, to ensure the presence of only one bead. The calculations at the end of the experiment determined a very precise spring constant. The results are listed as follows:

Phase: $k_{ot,} = 0.01405 \pm 0.00032$
Displacement: $k_{ot,} = 0.01446 \pm 0.00014$

Average: $k_{ot,} = 0.01426$

Frequency (rad/s)	**Phase**	**Normalized Displacement**
6283.185307	83.6331	0.156967226
4988.849134	82.02696333	0.20857249
3964.689929	77.85423333	0.25969917
3147.875839	74.245	0.315378554
2500.707752	69.79463333	0.394054692
1985.486557	64.13836667	0.463254738
1577.079512	59.0004	0.546143078
1256.637061	52.8103	0.637862919
992.7432785	48.60173333	0.679623452
791.6813487	39.49237	0.779366456
628.3185307	27.07832	0.858368788
496.3716393	22.487	0.900285554
395.8406744	19.04695	0.931190525
314.1592654	15.90112	0.94399461
251.3274123	12.73187333	0.977209937
201.0619298	10.41669667	0.986894193
157.0796327	8.22663	1.000837386
125.6637061	6.364633333	0.989963251
100.5309649	4.802096667	1.00186826
79.16813487	3.695333333	1.005104713
62.83185307	3.85679	0.999939735
49.63716393	3.30992	1.007332023
39.58406744	3.03908	0.993560548
31.41592654	2.489596667	0.995957741
25.13274123	1.8698	0.999983078
20.10619298	1.734385	1.005453264
15.70796327	1.549485433	0.996060742
12.56637061	0.80209	0.989498564
10.05309649	1.174473333	0.993805267
8.168140899	1.5547	0.992905944

Table 1 – Recorded values from the second run. The normalized displacement is the displacement for each frequency divided by the maximum displacement for all frequencies, which was calculated to be A = 6.025.

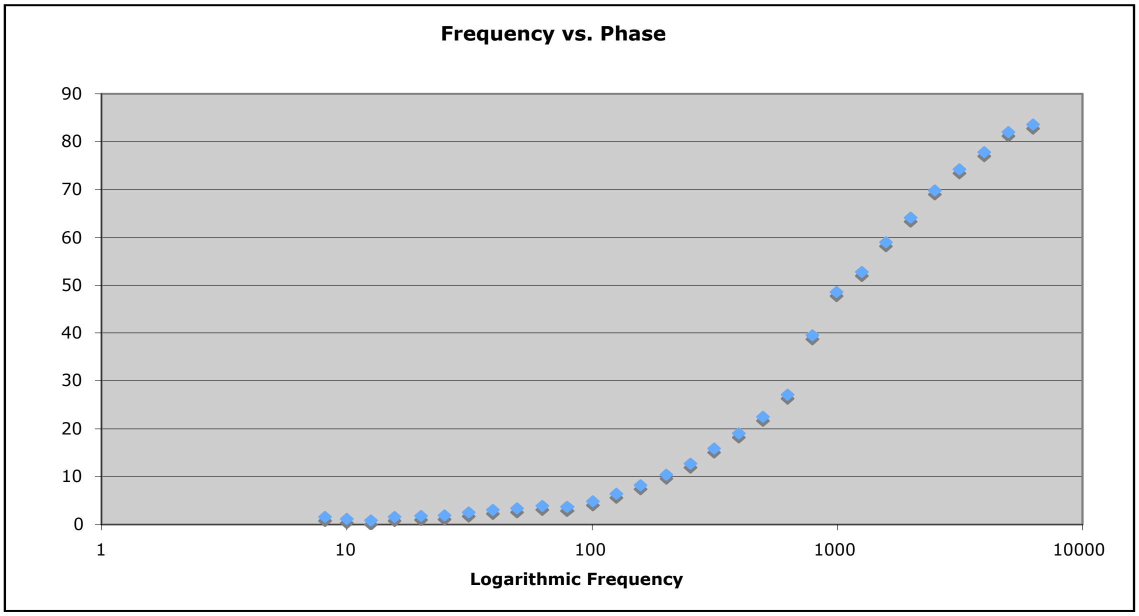

Chart 1 – The log-linear plot of frequency and phase for the second run. The fit for the spring constant was calculated from the Origin program, which calculated a $k_{ot} = 0.01405$.

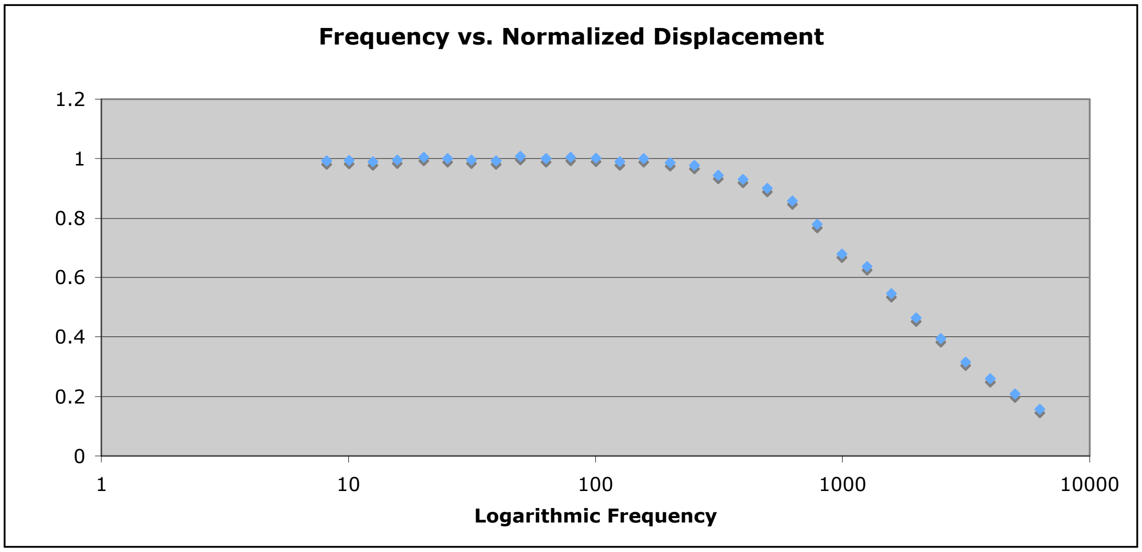

Chart 2 – The log-linear plot of frequency and the normalized displacement for the second run. The spring constant fit, calculated by the Origin program, was determined to be $k_{ot} = 0.01446$.

II. Calculation of the viscoelastic moduli of a bio-polymer

Method

The procedure for this section of the experiment is much the same as in Part I. The main difference is that instead of a water solution, a 4% gelatin solution as a bio-polymer. The solution is placed on a microscope plate, which is then placed in the microscope. The laser again captures a bead, and the LabVIEW program is run. The data for displacement and phase are used to calculate G' and G", thereby giving the viscoelastic properties of the gel solution.

Results

Using the spring constant measured in Part I of this experiment, the viscoelastic properties of storage and loss modulus were calculated for the gel solution. The moduli were expected to change with frequency, and both increased quite dramatically with the increase of frequency. The data are shown below in both table and chart format.

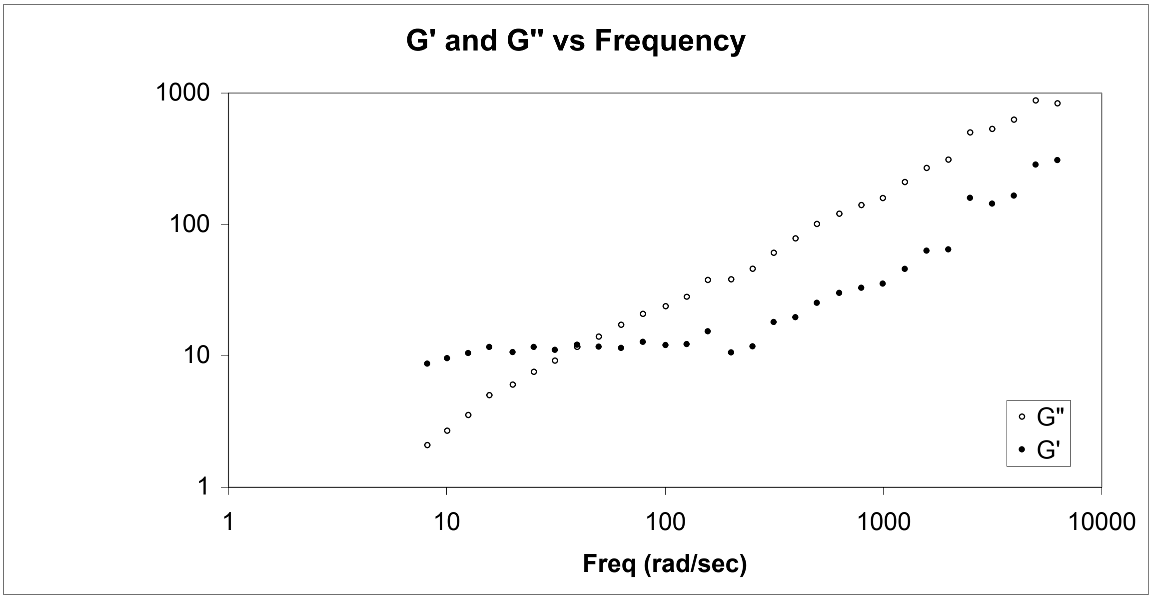

Chart 3 – The variability of the two moduli are distinct in this chart, which plots each against frequency. The loss modulus, represented by the open circles, ascends much more rapidly at low frequencies, but has nearly the same change as the storage modulus, represented by the solid circles, for higher frequencies.

Frequecy (rad/s)	**G' (dyne/cm)**	**G'' (dyne/cm)**
6283.185307	277.7595512	755.4208494
4988.849134	257.0326307	796.9320189
3964.689929	149.1151324	567.2621123
3147.875839	129.0704706	482.9021571
2500.707752	143.4894765	455.010639
1985.486557	57.44134148	282.1835193
1577.079512	56.30170022	244.4360222
1256.637061	40.58859196	191.1077947
992.7432785	31.14863461	144.3290533
791.6813487	28.88037659	127.563162
628.3185307	26.33323309	109.2800812
496.3716393	21.86229829	91.437311
395.8406744	16.82319712	71.24461307
314.1592654	15.37162831	55.35719448
251.3274123	9.744847126	41.88226406
201.0619298	8.663190065	34.68800284
157.0796327	12.9551357	34.27226526
125.6637061	10.14498318	25.46126628
100.5309649	9.986300591	21.61646913
79.16813487	10.61640555	18.87624123
62.83185307	9.455078901	15.59389273
49.63716393	9.696120203	12.68685879
39.58406744	10.05443872	10.56772913
31.41592654	9.112655993	8.315058051
25.13274123	9.620970024	6.848196445
20.10619298	8.744252518	5.473017693
15.70796327	9.65024214	4.532132234
12.56637061	8.57533895	3.20939152
10.05309649	7.744158854	2.44404695
8.168140899	6.944822396	1.893494539

Table 2 – The calculated values for storage and loss modulus listed with their respective frequencies.

Error Analysis

Error values in the spring constants for phase and displacement were quite small:

Phase: $k_{ot,} = 0.01405 \pm 0.00032 = 2.3\%$ error
Displacement: $k_{ot,} = 0.01446 \pm 0.00014 = 0.97\%$ error

Average: $k_{ot,} = 0.01426 \pm 0.00023 = 1.6\%$ error

Since this $k_{ot,}$ value is also used in the calculation of the storage and loss modulus, these values also have a 1.6% error associated with them.

Conclusions

This experiment proved highly successful. The errors associated with the determined values are quite small, giving great confidence in the experiment's results. Also, the similarity between the spring constant for the phase and the displacement also provides confidence in the gathered data. In fact, the error ranges for the two spring constants overlap in the range $0.01432 < k_{ot} < 0.01437$.

This experiment created a great experience for learning the technique of optical tweezing. Beyond this introduction, the experiment also reinforced previous concepts of the use of lasers, radiation pressure, and LabVIEW.