Polarization

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In this lab we analyze the effects of altering the polarization of a linearly polarized source (a polarized He-Ne laser). Using the He-Ne, Brewster's angle is measured for a thick glass slab. Then, using data gathered from known $\lambda/2$ and $\lambda/4$ wave plates, "unknown" wave plates are characterized as $\lambda/2$, $\lambda/4$ or λ/x. For the cases in which the plates have been identified as λ/x, the retardation δ (and thereby x) was measured by using a system of other known wave plates.

I. INTRODUCTION

The polarization of an electromagnetic wave is defined as the unit vector pointing in the direction of the electric field of the wave. If this vector is constant in time, the wave is termed to be "linearly polarized" and can be written in the form $\vec{E}(z,t) = Re\left((E_{0x}\hat{x} + E_{0y}\hat{y})e^{i(kz-\omega t)}\right)$, where the wave is defined to be traveling in the z direction. In this case the x and y components of the electric field are in phase, and thus the magnitude of the electric field is constant. However, we may introduce a phase difference between the two components. For example, if the wave can be written as $\vec{E}(z,t) = Re\left((E_{0x}\hat{x} + iE_{0y}\hat{y})e^{i(kz-\omega t)}\right)$, there is an overall phase difference of 90 degrees between the x and y components of the field. If $E_{0y} = E_{0x}$, the magnitude of the electric field is constant in time, but is rotating in the x-y plane at a frequency ω. Thus, light of this type is termed (right) circularly polarized. However, if the x and y components are not equal, the electric field traces out an ellipse in the x-y plane, with the major and minor axes determined by the magnitude of the x and y components; light of this type is termed elliptically polarized.

Light incident at an angle θ to the normal of a glass plate will have a portion of the beam transmitted through the glass and a portion reflected, the portions depending on the initial polarization with respect to the geometry of the system. Specifically, if the beam is incident at Brewster's angle, denoted θ_B, the component of the light polarized in the plane of incidence will be completely transmitted. The component polarized perpendicular to the plane of incidence will be partly transmitted and partly reflected: thus, all reflected light will be polarized perpendicular to the plane of incidence. Thus, for all polarization angles, the reflected beam has minimum intensity at Brewster's angle.

A wave plate is a plate of a certain thickness, denoted d, which contains two indices of refraction. Along one ("horizontal" or "vertical") axis of the plate, termed the extraordinary axis, has an index of refraction n_e, which is smaller than that of the other axis, the ordinary axis, which has an index of n_o. Thus, the component of the light along the extraordinary axis will travel faster than the component along the extraordinary axis. A plate is termed $\lambda/4$ if $(n_o - n_e)d = (m + 0.25)\lambda_{vac}$, (where m is any integer) which is shown in the lab notebook to introduce a phase difference of $\pi/2$ between the two components along the axes of the plate, which can produce elliptically or circularly polarized light. A wave plate is called $\lambda/2$ if $(n_o - n_e)d = (m + 0.5)\lambda_{vac}$, which is shown in the lab notebook to introduce a phase difference between the components along the axes of the plate of 180 degrees, which maintains linear polarization, but can alter its direction. It should be noted that plates are made for a specific wavelength, λ_{vac}, 633 nm in this case, however, the use of a wave plate for a different wavelength than that of the laser could be characterized by a similar equation, $(n_o - n_e)d = (m + \frac{1}{x})\lambda_{vac}$ in which x is not an integer in general.

II. PROCEDURE

A. Brewster's Angle Measurement

Pass the linearly polarized He-Ne beam though a linear polarizer such that the axis of the polarizer is very nearly aligned with the axis of polarization of the laser, to "clean up" the laser's inherent polarization. Do this by rotating the polarizer until extinction, which occurs when the polarization is perpendicular to the axis through which the polarizer passes the components therein. Maximize intensity by then rotating the polarizer another 90 degrees, which brings the two axis into alignment. Next, with the room lights out and the beam incident on a glass plate capable of rotation about an axis perpendicular to the optical axis, rotate the laser in its holder such that the reflected beam is of a low enough intensity that a minimum of intensity would be visible. Next, rotate the mirror in it's holder, while carefully watching the reflected beam on the wall. At the point of minimum reflected beam intensity, the normal of the glass surface makes an angle of θ_B with the optical axis. Measure this angle by fastening the down the glass plate at this angle, and using a meter stick up against the glass plate, mark the normal line of the mirror on the optical table using the screw holes. Do the same for the optical axis, which should merely be horizontal line on the table. Drop a perpendicular line from the normal line to the optical axis, and by measuring the sides of the resulting right triangle, calculate θ_B.

B. Characterizing Unknown Wave Plates

Maintain the laser and initial polarizer, but remove the glass plate. At one end of the optical path, place another linear polarizer, hereinafter termed the analyzer. Rotate the analyzer until extinction of the beam occurs. Next, in between the initial polarizer and the analyzer, place the unknown wave plate. Rotate the unknown plate until extinction occurs again, which aligns either the ordinary or extraordinary axis of the plate with the initial polarization direction. Next, rotate the unknown wave plate an additional 45 degrees, which equates the components along the axes of the unknown. If the plate is a $\lambda/4$ plate for 633 nm light, light exiting the unknown wave plate is circularly polarized, and thus rotation of the analyzer should yield no change in intensity of the exiting beam through a full rotation. This is because for circularly polarized light the magnitude of the field is constant in the plane of the screen (perpendicular to the optical axis), and is rotating at a frequency far too great too see a change in with only the eye. If the plate is a $\lambda/2$ plate for 633 nm light, rotation of the analyzer should produce two zeroes of intensity separated by 180 degrees. This is because light emerging from the $\lambda/2$ is linearly polarized in a direction 45 degrees off from the initial polarization, and thus when the axis of the analyzer is perpendicular to the polarization exiting the $\lambda/2$ (which happens once every 180 degrees), extinction of the exiting beam occurs. If the plate is λ/x for 633 nm light, then light emerging from the plate will be elliptically polarized (shown in the lab notebook). Thus, rotation of the analyzer should produce two maxima and two minima, corresponding to when the analyzer axis is parallel to the major and minor axes (respectively) of the elliptical pattern.

Following this procedure, characterize a known $\lambda/4$ plate by analyzing as above. Then repeat this entire procedure for three more unknown plates, determining whether they are $\lambda/2$, $\lambda/4$, or λ/x for He-Ne laser light.

C. Measuring Retardation δ for a λ/x Plate

Of all the unknown wave plates, select one of those determined to be a λ/x for 633 nm light, and record its number. Set up the laser, initial polarizer and analyzer as before, in which the analyzer is rotated until extinction. Place a known $\lambda/4$ plate between the initial polarizer and the analyzer, and rotate until extinction, which aligns either the ordinary or extraordinary axes of the $\lambda/4$ with the initial polarization direction. Next, insert the unknown plate between the $\lambda/4$ and the initial polarizer, and rotate to extinction, and then (similarly as above) another 45 degrees, to equate the components of the electric field across the ordinary and extraordinary axes of the unknown. Thus, light emerging from the unknown will be elliptically polarized, and upon exiting the $\lambda/4$ (which introduces a phase difference of 90 degrees between its two components), will be linearly polarized (shown in the lab book) at an angle $\varphi = \pi\delta/\lambda$ to the original source polarization direction. Thus, at this stage, if the analyzer is rotated until the next extinction, it will have traveled though an angle φ, and thus by measuring this angle, one can determine the retardation δ, which is related to x by $x = 2\lambda/\delta$, which is shown in the lab notebook.

III. EXPERIMENTAL RESULTS

A. Measuring Brewster's Angle

Only one measurement run of Brewster's angle was performed. The legs of the triangle used for measurement, x and y, where $\tan^{-1}(y/x) = \theta_B$ were measured to be $x = 5.0$ in, and $y = 8.0$ in, which yields $\theta_B = 58.0$ degrees. However, using the holes on the lab bench as markers in this procedure could introduce an error of as much as a quarter inch in each length measurement, which after propagation of error through the formula for θ_B yields $\theta_B = 58.00$ degrees ± 0.04 degrees. However, this is misleading in that this does not take into account the ambiguity of the choice of the "minimum" of intensity of the reflected beam, and then the subsequent plotting of the normal of the surface on to the table, which I assess may have introduced an error in the angle itself of up to 3 degrees. Thus, I assess the measurement of Brewster's angle to lie within the range $\theta_B = 58$ degrees ± 3 degrees.

B. Characterizing Known and Unknown Wave Plates

A total of four wave plates, one known and three unknown, were characterized as either $\lambda/2$, $\lambda/4$, and λ/x for 633 nm light. The first plate analyzed was a known $\lambda/4$ for 633 nm light, and thus upon rotation of the analyzer in the configuration described in the procedure, no change in intensity was noted through a full rotation. This is expected, as no change in intensity is characteristic of circularly polarized light, which is expected from a $\lambda/4$ plate in such a configuration. The second plate was an unknown, and upon rotation of the analyzer in the same configuration as above, two ”zeroes” of intensity were noted per full revolution. This is characteristic of linearly polarized light, which would be consistent with the second unknown as a $\lambda/2$ plate. After measurement, the label of the plate was noted, and it was found that this plate was indeed a $\lambda/2$ plate for 633 nm light. It should be noted that the two ”zeroes” were not points of pure extinction, but were so much dimmer in intensity than all other minima that they were assumed to be zero; this is accounted for by the imperfection of the wave plates in purely polarizing all incident light. The third plate (labeled A_0 *323.8*) was an unknown, and in the configuration above, rotation of the analyzer produced two maxima and two minima (not zeroes) of intensity per full rotation. As the plate was oriented 45 degrees to that of the incident polarization, this elliptically polarized light is indicative of a λ/x plate. The fourth plate (labeled A_0 *323.6*) yielded similar intensity patterns as the third, indicative of a λ/x', where x' is not necessarily the same as x.

C. Measuring the Retardation of an Unknown Wave Plate

The fourth unknown plate from the previous section, labeled A_0 *323.6* was used in this section. Four measurements of the the Retardation $\delta = \varphi\lambda/\pi$ were taken, by rotating the analyzer through an angle φ until extinction occurred as described in the procedure. Thus, we may perform a statistical analysis of the error. The mean value of the retardation was calculated to be 79 nm, with a standard deviation of the mean of 1 nm, which after propagating error, yields a value of $x^{-1} = 0.0617$ m^{-1} ± 0.0008 m^{-1}.

IV. CONCLUSIONS

In this lab I was successfully able to derive that linearly polarized light incident on a $\lambda/2$ yields linearly polarized light in a different direction, and that linearly polarized light incident on a $\lambda/4$ plate yields elliptically polarized light in general, and circularly polarized light if the components of the electric field are the same along the fast and the slow axes of a plate. Furthermore, I was able to show that linearly polarized light incident on a λ/x (where x is not necessarily an integer) exits as elliptically polarized in general, and that if this elliptical light is then incident upon a $\lambda/4$ plate, it is converted back to linearly polarized light rotated an angle φ from it’s original polarization direction, and that this angle can be related to x. I was able to measure Brewster’s angle for a slab of glass by observing the minima of intensity of the reflected He-Ne beam. Next, I was able to verify the intensity pattern with an analyzing polarizer for circularly polarized light using a known $\lambda/4$ plate, and from this was able to characterize three unknown wave plates as either $\lambda/4$, $\lambda/2$, or λ/x. Finally, selecting one of the λ/x plates, I was then able to calculate the retardation, δ of the plate, and from that the value of x.