Physics 262

Lab #3: Polarization

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Abstract

This experiment studied the concept of polarized light. It was shown that reflecting a beam of light off of a plane of glass at a particular angle (the Brewster angle) produces linearly polarized light. Additionally, the effects of polarization in the face of birefringent retardation plates were examined. The Brewster angle for an unknown sample piece of glass was determined to be $\Theta_B = 54.7^o \pm 2.3^o$, which indicated an index of refraction of $n_g = 1.42 \pm 0.12$ for the material, and the retardation of an unknown waveplate was determined to be a $\lambda/(7.9 \pm 0.4)$.

Introduction

Polarization of light refers to the fact that a wave travelling through space can take on various orientations. While many properties of polarized and unpolarized light are similar, they behave quite differently when they come into contact with certain types of regularly order crystalline structures. Also, depending on the source, newly created light might come into existence with some particular type of polarization. Understanding polarization and how it can be used to manipulate light path or intensity thus opens the door for a plethora of optical applications.

Theoretical Background

Travelling light can be represented as an electromagnetic wave as a pair of oscillating electric and magnetic fields whose directions are in the plane transverse to the direction of propagation. If light could be broken up into individual waves, then each wave would have a particular direction in which the electric and magnetic field vectors point at any given point in time. The direction of the electric field would be considered the direction of polarization for that particular wave. Since light is composed of many such waves superimposed, the aggregate effect of these individual polarizations is what is physically significant. In linearly polarized light they are all oriented in the same direction. In circularly and elliptically polarized light, the net direction of the electric field rotates with time. In normal, unpolarized light, waves of every polarization coexist with equal distribution.

One way to represent polarization mathematically is in terms of complex exponentials. Below is the equation of the electric field in linearly polarized light:

$$\vec{E}(z,t) = Re\left(E_{0x}\hat{x} + E_{0y}\hat{y}\right)e^{i(kz-\omega t)}$$

Note that the two field components are in phase. E_{0x} and E_{oy} are constants whose relative amplitude determine the net direction of the electric field (and thus polarization).

Elliptically polarized light occurs when the two field components become out-of-phase with each other. This contributes to a rotating electric field direction. It can always be represented by the following equation:

$$\vec{E}(z,t) = Re\left(E_{0x}\hat{x} + iE_{0y}\hat{y}\right)e^{i(kz-\omega t)}$$

In the special case where $E_{ox} = E_{oy}$, the light is said to be circularly polarized.

The use of these mathematical structures is quite helpful in the analysis of birefringent materials. Such materials have different indices of refraction along different axes. One particular axis, called the optical axis, has the minimum index of refraction for the material. When a wave plate is manufactured, the intended incident location is typically rotated an angle ϕ off of this axis. This angle allows manufacturers to have some control over the ordinary (minimum) and extraordinary (maximum) indices of refraction that appear within the plane of the wave plate. The different indices of refraction create different velocities for the components of waves travelling through the medium, and this introduces a phase shift in one wave relative to the other. By controlling the width of the plate, different phase shifts can be obtained. Waveplates are defined by the equation:

$$(n_o - n_e)d = \left(m + \frac{1}{x}\right)\lambda_{vac}$$

Where m is an integer, d is the width of the plate, and x defines the 'type' of waveplate (λ/x).

The Brewster angle refers to a particular incident angle at which light will hit a smooth surface such that only light with a polarity perpendicular to the plane of incidence is reflected, and all other light is transmitted through the surface. The angle is given with respect to the normal of the surface. It is related to the index of refraction of the surface and the surrounding medium by:

$$\tan\theta_B = \frac{n_{surface}}{n_{medium}}$$

Experimental Procedure

Apparatus

Polarized 633nm He-Ne laser, adjustable polarizers, unknown glass sample, quarter-wave plates, unknown wave plate, measuring stick

Procedure

The first task of this experiment was to determine the Brewster Angle for the unknown glass sample. One way to accomplish this was to arrange the laser so that it made an acute angle with the normal of the plane of glass. As mentioned earlier, when the Brewster angle is obtained, only light perpendicular to the plane of incidence will be reflected by the glass. If the incident beam is polarized such that there is a minimal amount of light in this polarization, then the beam reflected off of the sample will disappear when the Brewster angle is reached. Through a process of refining the angle of polarization and the incident angle of the laser beam, the reflected beam was finally minimized and a value for the Brewster angle recorded. The angle itself was determined by using simple trigonometry on the triangle made from the point where the reflected beam hit the wall, the sample itself, and the shortest projection from the wall point to the continued straight path of the transmitted beam. The index of refraction was then calculated using the formula given in the Theoretical Background.

The second step of the experiment was to prove mathematically that a λ/4 birefringent plate could produce circularly polarized light, and that a λ/2 plate would rotate the polarization of linearly polarized light by 90°. In each case the incident light is assumed to be linearly polarized in the form:

$$\vec{E}(z,t) = Re\left(E_{0x}\hat{x} + E_{0y}\hat{y}\right)e^{i(kz-\omega t)}$$

For the λ/4 plate this must become:

$$\vec{E}(z,t) = Re\left(E_{0x}\hat{x} + iE_{0y}\hat{y}\right)e^{i(kz-\omega t)}$$

And for the λ/2 plate this must become:

$$\vec{E}(z,t) = Re\left(E_{0x}\hat{x} - E_{0y}\hat{y}\right)e^{i(kz-\omega t)}$$

In both cases, start with the birefringence equation:

$$(n_o - n_e)d = \left(m + \frac{1}{x}\right)\lambda_{vac}$$

$$\left(\frac{n_o}{\lambda_{vac}} - \frac{n_e}{\lambda_{vac}}\right) d = \left(m + \frac{1}{x}\right)$$

$$(k_o - k_e)\, d = \left(m + \frac{1}{x}\right)_{2\pi}$$

$$k_o = (k_o - k_e) + k_e = \left(m + \frac{1}{x}\right)\frac{2\pi}{d} + k_e$$

Choosing z=0 for start of plate and z=d for end of plate:

$$\vec{E}(z,t) = Re\left(E_{0x}e^{i(k_o d-\omega t)}\hat{x} + E_{0y}e^{i(k_e d-\omega t)}\hat{y}\right)$$

$$\vec{E}(z,t) = Re\left(E_{0x}e^{i\left\{\left(\left(m+\frac{1}{x}\right)\frac{2\pi}{d}+k_e\right)d-\omega t\right\}}\hat{x} + E_{0y}e^{i(k_e d-\omega t)}\hat{y}\right)$$

$$\vec{E}(z,t) = Re\left(E_{0x}e^{i\left\{\left(m+\frac{1}{x}\right)2\pi+k_e d-\omega t\right\}}\hat{x} + E_{0y}e^{i(k_e d-\omega t)}\hat{y}\right)$$

$$e^{i\left\{\left(m+\frac{1}{x}\right)2\pi\right\}} = \begin{cases} i & if\ x = 4 \\ -1 & if\ x = 2 \end{cases}$$

Thus, for a λ/4 plate this becomes:

$$\vec{E}(z,t) = Re\left(iE_{0x}\hat{x} + E_{0y}\hat{y}\right)e^{i(k_e d-\omega t)}$$

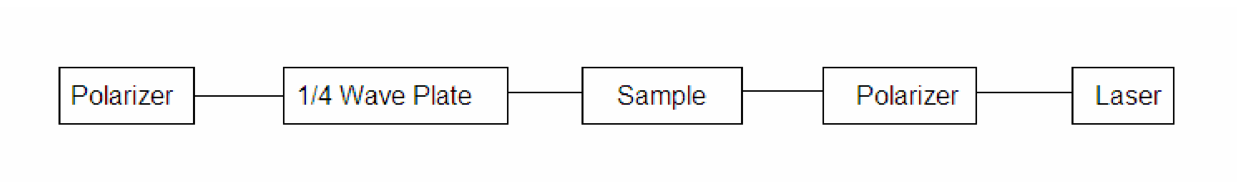
And for the λ/2 plate it becomes:

$$\vec{E}(z,t) = Re\left(-E_{0x}\hat{x}E_{0y}\hat{y}\right)e^{i(k_e d-\omega t)}$$

Part 3 of the lab was to design simple tests to distinguish between λ/2 and λ/4 retardation plates. The method developed was to put two crossed polarizers in the path of the laser beam (completely blocking the light leaving the second polarizer). The unknown retardation plate should be placed between them and rotated until all light is once again blocked. At this point, the plate is aligned with either its fast or slow axis and is having no effect on the relative phase between components of the incident light beam (as there is only one component). If the sample is rotated 45^{o} from this orientation, then it will either be producing circular or 90^{o} shift linear light at its other side. These two types of light can be distinguished by rotating the second polarizer. If the light is linear, then the intensity of the beam coming out of the second polarizer (the analyzer) will go from zero to full intensity as the polarizer is shifted. If the light is circular, then no change of intensity will be noticed. In the event that an odd sized waveplate was used, then elliptical light would produce a series of minima and maxima of intensity.

Part 4 of the experiment was to use the procedures developed to test a series of unknown samples and determine whether they were λ/2 , λ/4, or unknown. It was discovered that, with the exception of two λ/4 plates, all the provided samples were of odd lengths (likely intended to be more conventional sized waveplates for a different laser frequency). The proof showing that these odd-wavelength waveplates would produce elliptically polarized light is provided on pages 19 and 21 of the lab notebook.

Part 5 of the experiment was to take one of the unknown waveplates and determine what type of waveplate it was. The following apparatus was used:



The basic premise of the idea is that the linearly polarized light passing through the sample will produce elliptically polarized light coming out the other side, and that the λ/4 plate will take away the phase difference between the two components of the field and return them to a linear polarization that can be picked up by the analyzer.

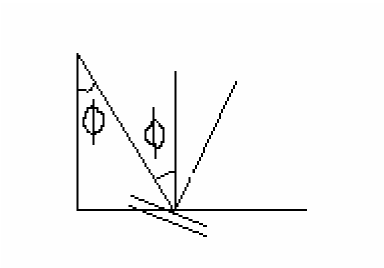
First, the two polarizers are crossed as in the previous experimental setup. Then the sample is inserted and turned until extinction occurs (lining it up with one of the two optical axes). The same is done as the λ/4 plate is put into place. Next, the sample is rotated to 45^{o}. This causes equal electric field components to hit both its fast and slow axis. This produces the elliptically polarized light of the form:

$$\bar{E}(z,t) = Re(E_{0x}\hat{x} + iE_{0y}\hat{y})e^{i(kz-\omega t)}$$

Where E_{0x} does not equal E_{0y}. When this light passes through the quarter wave plate, the phase difference between the x and y components falls away, but E_{0x} still does not equal E_{0y}. This is what causes the new direction of linear polarization that presents itself to the analyzer. The angle that the analyzer must be rotated to recover extinction can be used to calculate the relative values of E_{0x} and E_{0y}. These in turn can be used to determine the waveplate type x.

Experimental Results and Discussion

In the part of the experiment calculating the Brewster angle of the glass sample, the distance from the sample to the reflected beam on the wall was measured to be 1.52 m ± 0.02m (a single meter stick was used and moved forward to measure the extra distance) and the distance from the reflected beam to the straight path of the original beam was 1.43m ± 0.02m.



The angle ϕ is determined to be $\cos^{-1}(1.43m\pm0.02m / 1.52m\pm0.02m)$. This is somewhere between 14.8^o(minimum) to 23.7^o (maximum), OR $\phi = 19.3^o \pm 4.5^o$

Therefore the Brewster angle is between:

$\Theta_B = (14.8^o+90^o)/2 = 52.4^o$ and $\Theta_B = (23.7^o+90^o)/2 = 56.9^o$

$\Theta_B = 54.7^o \pm 2.3^o$

The index of refraction (counting that of air to be equal to 1) is therefore:

$n_g = (1)\tan(52.4^o) = 1.30$ or $n_g = (1)\tan(56.9^o) = 1.53$

$n_g = 1.42\pm0.12$

For the birefringence experiment (Part 5), the unknown wave plate (marked #6) took $23^o\pm1^o$ of rotation for the final analyzer to produce extinction. Using $\phi=\pi/x$, we determine x to be:

$X = \pi/[(24^o)*(\pi/180^o)] = 7.5$ OR $X = \pi/[(22^o)*(\pi/180^o)] = 8.2$

$X = 7.9 \pm 0.4$

Summary

Using the fact that light incident on a surface at the material's Brewster angle produces linearly polarized light, the Brewster angle of a glass sample was determined to be $\Theta_B = 54.7^o \pm 2.3^o$ and the index of refraction to be $n_g = 1.42 \pm 0.12$. A method was developed to test quarter, half, and unknown waveplates and to differentiate between them. A more involved test procedure was used to calculate the type of waveplate (beyond simply labeling it as 'unknown') and that using this method a sample waveplate was determined to be a $\lambda/(7.9 \pm 0.4)$ plate.