Interferometry

Scott Freese

Physics 262

21 April 2008

Abstract

The primary purpose of these experiments was to gain a basic physical understanding of the process of interferometry and its use in the measurement of the wavelength of light, the coherence length of lasers, and also the index of refraction of materials. The wavelength of an unknown laser was calculated by first calibrating an electrostrictive actuator with a HeNe laser of known wavelength (632.8 nm). This calibration allowed for the accurate determination of the unknown wavelengths of a green laser, which was calculated to be 537 nm. The coherence length of multiple lasers was tested by increasing the length of one of the arms of the Michelson interferometer until diffraction was no longer detected. For both the 537 nm green laser, and the 632.8 nm HeNe laser, the experimental setup was not large enough to determine the precise coherence length, but it was determined that it must be greater than 3.98 m. Another red laser of undetermined wavelength had a coherence length measured to be 1.4 m. The process of interferometry was finally used to determine the index of refraction for the glass of a microscope slide, and also for air. The refractive index for the glass was calculated to be $n_{glass} = 1.04$, while that for air was calculated as $n_{air} = 1.00025$.

Introduction

In this experiment, a Michelson interferometer is used. This instrument, composed of optical components is extremely versatile and allows for highly precise measurements. The interferometer is actually capable of accurately measuring down to the scale of the wavelength of the light being measured. So for visible light waves, the precision is on the scale of nanometers. The instrument is normally used for experiments that involve very small changes in the optical path length, and is so extremely useful because of its ability to produce circular and straight-line fringes for both monochromatic and white light. These fringes can be used to accurately compare the wavelengths of different sources, measure the index of refraction for transparent materials, and even measure small length changes.

Perhaps the best known among the many types of interferometers, the Michelson interferometer operates using the principle of division of amplitude instead of the division of wave front. The basis of this principle is the splitting of the light source. In the case of a beam of laser light, a beam splitter reflects approximately half of the beam's intensity in one direction, while transmitting the rest in another. These two individual beams, traveling different optical lengths, are eventually recombined in a common region. In this area, interference between the two beams occurs, and fringes are formed. The nature of the fringes is directly related to the optical path lengths that each of the beams travel, and in that sense, is related to anything that causes a difference in the optical path lengths.

This setup takes advantage of electrostrictive actuators to cause this change in optical path length. Electrostrictive materials behave in a fairly similar way to the more commonly referred to piezoelectric materials. Also known as pressure electricity, the materials that exhibit this property produce a voltage when they are compressed that is proportional to the applied pressure. In reverse, when an electric field is applied across the material, the object changes shape as if a pressure were being exerted upon it. Although multiple natural materials exhibit such properties, most modern devices use polycrystalline ceramics such as lead zirconate titanate (PZT). Similar materials are used in both types of devices, it is important to note that piezoelectric types operate in the ferroelectric region below the Curie temperature, and electrostrictive are those operating in the paraelectric region above the Curie temperature. Electrostrictive materials show less hysteresis that their piezoelectric counterparts and can therefore measure distance more accurately. One of the only fallbacks is their difficulty of use at very low voltages. Another important difference between the materials is that in the use of actuators, electrostrictive materials expand according to the square of the field, whereas piezoelectric materials expand linearly (both to first order).

The purpose of the actuator is to reliably and controllably change the optical length of one of the arms of the Michelson interferometer. In doing so, the interference patterns change, which affects the fringes. For the monochromatic light associated with lasers, the following interference conditions apply. When the two optical path lengths are the same, or when they differ by a whole number of wavelengths, λ, constructive interference occurs, and a bright fringe is formed. This condition is expressed by:

$$L_1 - L_2 = p\lambda \tag{1}$$

where L_1 and L_2 are the optical lengths, and p is the order number, or whole number of wavelengths (0, ±1, ±2, ±3, etc.). Conversely, when the two optical path lengths differ by a whole odd number of half wavelengths, destructive interference occurs, and dark fringes are observed. This condition is expressed by:

$$L_1 - L_2 = (p/2)\lambda \tag{2}$$

where, in this case, $p = \pm 1, \pm 3, \pm 5, etc.$

Based on this principle, it is quite simple to determine the precise wavelength of an object if the number of fringes, p, is recorded, as well as the difference in length of the two optical paths. Even if the optical path length change is not initially known, it can be determined by using a laser of known wavelength, and counting the number of fringes crossed during the change in length. The length change is related to this number and the wavelength by Eq. 1. The only difference is that the length change is double that in Eq. 1, because of the fact that the light is traveling the interferometer arms twice, both forwards and backwards.

$$2(L_1 - L_2) = p\lambda \tag{3}$$

With this optical path length change known and kept constant, the wavelength for any laser or other monochromatic source can be determined by the relationship expressed in Eq. 3.

The coherence length of a laser is the distance from the source that an electromagnetic wave can propagate and still maintain a degree of coherence, which is the correlation of the electric fields of the rays. This feature of a laser is useful for determining the spectral bandwidth of the laser. It is possible to calculate the bandwidth from the following equation:

$$L = c / n\Delta f' \tag{4}$$

where L is the coherence length, c is the speed of light, n is the index of refraction of the medium (1 for air), and $\Delta f'$ is the bandwidth of the source.

Apparatus

The primary equipment used in the proceeding experiments is listed as follows:

- Three lasers: 632.8 nm HeNe, unknown wavelength green laser, unknown wavelength red laser
- Multiple flat plane mirrors
- Beam splitting cube
- Electrostrictive actuator controlled by function generator
- Photodetector in conjunction with an oscilloscope
- Glass plate of unknown refractive index
- Air cell for producing a vacuum

I. Calibration of the Electrostrictive actuator for HeNe laser for the measurement of wavelengths of unknown lasers

Method

First, the Michelson interferometer is prepared with each of the arms set at equal length, as is shown in the diagram on Page 19 in the laboratory book. Once this is prepared, the electrostrictive actuator, with an attached mirror, was set up to work with a voltage control which was linked to a function generator. This setup allowed for the actuator to repeat over a set voltage range, which correlated to a set distance range for the actuator to move. Using the HeNe laser, with the known wavelength of 632.8 nm, the number of interference fringes counted over the distance change is directly related to this distance by Eq. 3. This distance was calculated, and then a fringe count was determined for the unknown green laser over the same distance change. This information then allowed for the calculation of the wavelength of this green laser.

Results

The voltage range over which the data was taken was from 30-60V for both the HeNe laser, and the green laser.

Three runs were taken for counting the number of fringes moved over this range. The mean of the three is:

$$M = 102.33$$

Using Eq. 3, this number determined the actual distance changed over the voltage range.

$$\Delta D = 32378\ nm$$

Three runs were also taken for the unknown green laser to determine the number of fringes changed over this distance and voltage range. Again, the mean was taken of these three.

$$M = 120.67$$

Once again, using Eq. 3, with the distance change calculated above, along with the mean of the fringes, the wavelength of the green laser was calculated.

$$\lambda_{green} = 537\ nm$$

II. Measurement of Coherence Length for the Lasers and Spectral Bandwidth of Red Diode laser

Method

For the determination of the coherence length for each of the lasers, the Michelson interferometer is set up with each of the arms at equal length, just like the set up from Part I. One of the arms is then moved away from the beam splitter to increase its length. This is increased until diffraction is no longer observed once the two beams are recombined. Upon determining the coherence length, the spectral bandwidth can be calculated by using Eq. 4.

Results

Using the above method, the coherence lengths were measured for the 632.8nm HeNe laser, the green laser with determined wavelength of 537nm, and the unknown wavelength red diode laser. For both the HeNe laser and the green laser, the same result was obtained: The set up did not allow for a large enough distance to determine the coherence length.

For both the 632.8nm HeNe laser and the 537nm green laser, the coherence length was measured to be:

$$L > 3.98m$$

However, for the red diode laser, the coherence length was measured to be:

$$L = 1.4m$$

This number is used in Eq. 4 to determine the spectral bandwidth for the red diode laser. The calculated bandwidth is:

$$\Delta f' = 2.1 \times 10^{8}\ Hz$$

which correlates to a time for coherence of:

$$T = 4.7\ ns$$

III. Measurement of Refractive Index for a Glass Plate and Atmospheric Air

Method

The methods for measuring the refractive index for both the glass plate and the air cell are quite similar. First, the Michelson interferometer is prepared like in Part I, with both arms set at equal length. The only difference is that in one of the arms the object being tested is placed. So, for testing the glass plate, it is placed in the path of the laser, and likewise for the air cell. For the glass plate, the data needed are the wavelength of the laser being used (632.8nm HeNe laser), the thickness of the plate, the number of fringes moved, and the angle range over which the change occurs. Very similar measurements are needed for the air cell. However, instead of an angle, a pressure difference must be measured, while counting the number of fringes changed.

The indices of refraction for both are calculated by the equations determined on Pages 21-23 of the laboratory book.

Results

For measuring the index of refraction for the glass plate, two tests were performed to determine the angle change and the number of fringes that are changed. The same results were obtained for both.

$$M = 50$$

$$\Delta\theta = 18^{o}$$

The thickness of the glass plate was measured to be:

$$T = 1.15mm$$

Using this data in the equations derived in the laboratory book results in an index of refraction for the glass plate of

$$N_{glass} = 1.042$$

For the measurement of the index of refraction for air, using the air cell, four separate tests were performed. For all of the tests, the air pressure within the cell was reduced by 15 in.Hg. The mean of the number of fringes counted for the four tests is

$$M = 30.5$$

Using an atmospheric pressure of

$$P_{air} = 29.9 \text{ in.Hg}$$

an refractive index for air was calculated to be

$$n_{air} = 1.00025$$

Error Analysis

(Error calculations are shown on Page 24 of the lab notebook)

Part I

The multiple trials for the fringe counts allowed for standard deviation measurements which helped improve accuracy for the measurements.

The error in the counts for the HeNe laser is: $N = 102.3 \pm 0.9$.

This number is used to calculate the distance change of the actuator. Therefore, the error associated with this value is: $\Delta D = 32378nm \pm 285nm$.

The error in the counts for the green laser of unknown wavelength is: $N = 120.7 \pm 0.3$.

To calculate the wavelength of the green laser, all of these values are used, and therefore there errors are involved as well. The error in the calculation of the wavelength is:

$$\lambda = 537nm \pm 6.06nm$$

Part III

For the measurement of the refractive index of air, the multiple trials also helped reduce error. The pressure change was kept at a constant for each trial, while counting fringe changes.

The error in the counts is: $N = 30.5 \pm 0.5$

This accounts for an error in the calculation of the index of refraction of air of

$$n = 1.00025 \pm 0.000004$$

Conclusion

Overall, this experiment proved to be very successful. The measurement of wavelength of the green laser had very low error, and the value fits perfectly in the expected range of green light. However, the calculated index of refraction for glass did not fit so well with expected values. Standard indices of refraction for glass tend to be about 1.5, while the calculated value here was 1.042. Although there is a chance of experimental error, it is more likely that the discrepancy between the calculated value and the expected is in the calculations themselves. The derivation of the equations proved challenging, and mistakes may have been made in the process. Again on the positive side, the calculation of the index of refraction for air was very precise. The calculated error is extremely low, and the value for the refractive index is spot on with expected results.