Geometric Optics

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Abstract

The primary goal for this experiment was to learn the basic physics of the concept of geometric optics. The specific concepts to be focused on were the focal length of a lens, along with the nodal and principal points, and also the magnification capabilities of the same lens. By the use of the auto-collimation method, the focal length of a telescope lens was determined. The focal length calculated by experimentation was determined to be 37.9 cm, with principal points separated by 1.6 cm. To determine the focal length of the eyepiece, a different method was used, this time taking advantage of the difference in magnification at different lengths from the image. The focal length of the eyepiece was determined to be 4.0 cm. Based on these two focal lengths, a telescope using both lenses has a theoretical magnification of 9.5 for objects at infinite distance. However, experimentation determined a mean magnification of just 7.27 for such cases. It intermediate length, the telescope was determined to have a magnification of 9.16.

Introduction

In a geometric system of optics, if the indices of refraction, the positions, and radii of curvature of all objects in the system are known, it is possible to determine the size and position of the image of any object. However, even if this information is not known, the cardinal points and focal length can be measured, which in turn will lead to the ability to determine the image size and position.

The focal lengths of a lens in a system are one of the most important features for determining image size and position. These are defined as the distances measured from the principal points to the respective focal points. Assuming a constant radius of curvature on both sides of the lens, when the media on either side are different, the focal lengths are related by their indices of refraction in the equation:

$$n/f = n'/f' \qquad (1)$$

When the media are the same, as in the case of a system immersed in air, the indices of refraction are the same, and therefore $f = f'$, and so the two focal lengths are of the same magnitude. In both situations, primed values refer to quantities of the image space, whereas those without are of the object space.

Because of this information, to determine the size and position of nearly any image, all that needs to be known of a system is the location of the focal and principal points of the lens, as long as the size and position of the object are also known. To find the principal points, all that must be done is determine the positions of the nodal points, which can be done with relative ease by experimentation.

There are two systems of equations which are commonly used to relate object and image in an optical system. The Gaussian system measures object and image distances from their corresponding principal points, whereas in the Newtonian system these values are measured from the focal points. The equations for each are listed below, with *Figure 1* showing the quantities used in each.

Gaussian Equations:

$$f'/S' + f/S = 1 \qquad (2)$$

$$M = y'/y = (-n/n')(S'/S) \qquad (4)$$

Newtonian Equations:

$$xx' = ff' \qquad (3)$$

$$M = y'/y = -x'/f' = -f/x \qquad (5)$$

As with Eq. 1, these equations can be simplified by remembering that for a system in which the media on each side of a lens are the same, $f = f'$. Also, in the above equation, S and S' are the object and image distance measured from the principal points, P and P', while x and x' are the same, but measured from the focal points, F and F'. The values f and f' are the focal lengths, y and y' are the heights of the object and the image, M is the lateral magnification, and d is the distance both between the two principal points and the nodal points, N and N'.

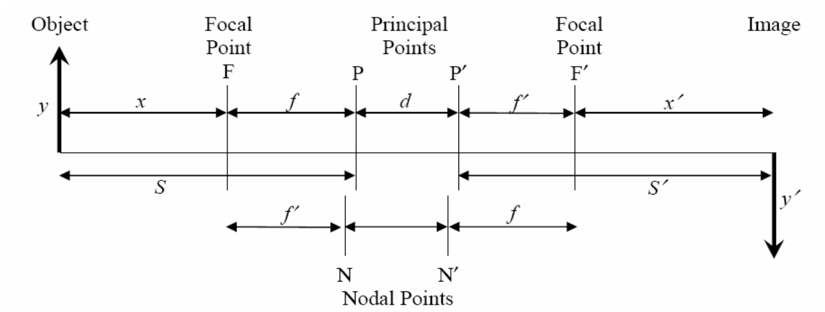

Figure 1 – A general optic system, showing the physical meaning of the symbols in the Gaussian and Newtonian Equations. The values for all quantities in this image are positive, except for y'.

The focal points of the system can be easily measured by locating the image of an object placed infinitely far away. If this is not actually possible, the object can be placed at infinity by artificial means with the use of another lens, or through a process known as auto-collimation. In this method, a planar mirror is used to send the light of the object back through the lens. The mirror is adjusted until the image lies in the same plane as the object. This plane, in which both the object and the image lie, is known as the focal plane. When configured in this manner, the lens produces a set of parallel rays for every point in the object. This is in fact where the term auto-collimation comes from, since collimation refers to the formation of a parallel beam of light.

To determine the position of the nodal points, and therefore the principal points as well, a nodal slide must be used. In such a set up, the system is shifted about the axis of rotation, *R*, of the lens, until a small rotation creates no change in the image, *I*, of a distant object. At this point, the nodal point in image space, or *N'*, is coincident with *R*.

Upon location of the focal points, the position of the Newtonian object and image distances, x and x', can be measured, which in turn can lead to the calculation of the focal length by means of the reduced version of Eq. 3: $xx' = ff' = ff$, since the system is immersed in the same medium, air, on all sides. An extremely useful method of measuring x and x' is by the three step process shown in Figure 2, and described below.

The first step uses auto-collimation again to locate the position of the system when the focal point is at the object-image screen on the left hand side. The second step determines the position of the system is determined for when it forms an image on the screen to the right. The third step, again using auto-collimation, determines the position of the system when the secondary focal point is now at the object-image screen on the right hand side, which is just the opposite of the first step. Keeping the two screens at the same positions throughout the test, x is the distance the system is moved between the first and second step, and x' is the distance it is moved between the second and third.

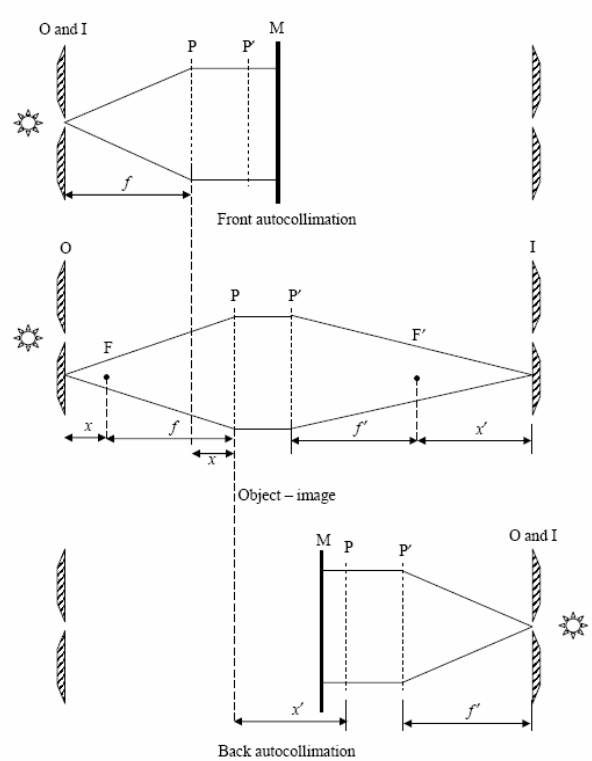

Figure 2 – Three step process for determining x and x'.

For the determination of a system such as an eyepiece, for which the focal length is very short, the above process does not work accurately. Instead, the focal length is determined by using an object of known dimensions, and measuring the size of the image at two distinct locations. The relationship between the difference in magnifications at the two points, and the distance between them is directly related to the focal length of the system. This can be expressed by

$$f = -d / (M_2 - M_1) \tag{6}$$

A full optical system with multiple lenses is often used for the purpose of magnification. The human eye determines the apparent size of an object based upon the size of the image on the retina. The size of this image is determined by the angle on the eye subtended by the object, or in the case of optical experimentation, by the image formed by an instrument. This angle defines the magnifying power of such an optical instrument. In precise terms, the magnifying power is defined as the angle subtended at the eye by the image formed from the instrument divided by the angle subtended at the eye by just the object alone. This is expressed mathematically in Eq. 8.

$$MP = \theta'/\theta = (y'/y)(D/D') \tag{7}$$

It is important to note that the magnifying power and the magnification, or the ratio of the size of the image to the size of the object, are not equal unless the viewing distance is the same as without the instrument. Certain assumed conditions are often used to develop a set of standard formulas for determining magnifying power of optic systems. For a simple magnifier, the formula is

$$MP = (25cm)/f \tag{8}$$

where the distance of 25 cm is assumed to be the standard distance for the viewing of an object by just the eyes. For a telescope, the formula is

$$MP = f_o/f_e \tag{9}$$

but only when the telescope is located an infinite distance from the object.

Apparatus

The primary equipment used in this experiment are listed as follows:

- Telescope objective lens
- Telescope eyepiece
- Planar mirror
- Objective lamps
- Nodal slide
- Fine graded glass plate
- Horizontally adjustable scope

I. Determination of focal length for telescope objective and eyepiece, and principal points for each

Method

The first thing performed in this experiment was the measurement of the values of *x* and *x'* for the objective lens of the telescope. This was carried about by using the three-step process as described in the introduction to this report. Auto-collimation was used to find the point at which the object and image were in the same plane on the left side of the lens. Then the objective was moved until an image was produced on the screen to the right of the lens from the object on the left. The distance between these two positions gave the value *x*. Auto-collimation was again used to determine the point at which the object and image on the right are in the same plane. The difference between this position and the second position gave the value of *x'*.

These two values were then used to calculate the focal length of the objective lens.

To determine the nodal points for the objective lens, the nodal slide was used in the system using the technique of auto-collimation. The nodal slide allowed for slight adjustment in of the axis of rotation along the line of propagation for the light. The position of the lens was gradually adjusted, while slightly rotating it until little to no change was observed in the image. At this point, the nodal point for the image space of the lens was measured. The process was repeated after turning the lens around, to determine the nodal point on the other side. By calculating the difference in the distances, the separation was determined, which is also the separation of the principal points.

For the measurement of the focal length of the telescope eyepiece, two new pieces of equipment were introduced. A very finely scaled glass plate (100 points/5 mm) was used in conjunction with the eyepiece and a scope which allowed for very accurate measurement of lengths across a horizontal translation. This scope is used to measure the horizontal length of the image in relation to the length of the object. This is then used to calculate the magnification of the object. By performing this measurement at two separate locations, and calculating the distance between the two positions, the focal length can be determined.

Results

The three positions of the objective lens throughout the three-step process for determining the focal length of the objective lens are

$$P_1 = 10.5\ cm$$
$$P_2 = 36.4\ cm$$
$$P_3 = 91.9\ cm$$

all with estimated precision of ± *0.5 cm.*

The values for x and x' are calculated by the following means

$$x = P_2 - P_1 = 36.4\ cm - 10.5\ cm = 25.9\ cm$$

$$x' = P_3 - P_2 = 91.9\ cm - 36.4\ cm = 55.5\ cm$$

And since these two values are related to the focal length by Eq. 3, which reduces to $xx' = f^2$ with a system immersed in air, the focal length can be determined

$$(25.9cm)(55.5cm) = 1437.45\ cm^2 = f^2$$

$$f_0 = 37.9\ cm$$

The process described above was used to determine the locations of the nodal points, and thereby determine the distance between the principal points. The following positions were determined:

$N' = 0.011m$ *into image space*
$N = 0.005m$ *into object space*

Therefore, the distance between the nodal points, and therefore the principal points for the objective lens is

$$\Delta P_{21} = 0.005m - (-0.011m) = 1.6\ cm$$

In determining the focal length of the telescope eyepiece, magnifications were measured at two different distances from the object. Four individual measurements were made at each of these positions, to allow for the greatest accuracy in calculations. The mean magnifications for each are listed here. By using Eq. 6, the focal length was calculated.

$$M_1 = -1.300$$
$$M_2 = -2.128$$
$$d = 4.80\ cm$$

$$f_e = (4.8cm)*(-2.128cm + 1.300cm) = 4.0\ cm$$

II. Determination of Magnification and Magnifying Power for Optical Systems

Method

First, a telescope was prepared by combining the objective lens and eyepiece along a straight path. Next, the telescope was placed in position to view a meter stick from a distance great enough to be considered at infinity. In this position, the image formed by the telescope was compared to the telescope itself to determine the magnitude of magnification. This was performed by relating the angle subtended by the meter stick on the unaided eye to the angle subtended by its image created by the telescope.

This process was also repeated with the telescope at an intermediate distance from the meter stick.

Results

The theoretical magnification of the telescope with an object located an infinite distance away is

$$M = 9.5$$

With the object located at an infinite distance, the magnification of the telescope was determined to be

$$M = 7.27$$

Since the object is at an infinite distance, both *D* and *D'* in Eq. 7 can be considered equal, in which case the magnifying power of the telescope is the same as the magnification.

At an intermediate distance of 3.5 m between the object and the focal point of the objective lens, the magnification of the telescope was calculated as

$$M = 8.78$$

Error Analysis

Part I

(Calculations for these errors are shown on the loose leaf paper in the geometric optics section of the lab book.)

The position of the objective lens at the three locations used for determining the focal length of the lens were all recorded with an uncertainty of 0.5 cm. This uncertainty carried through to the focal length due to the direct relationship between these positions and the focal length calculations.

The error in $x = 25.9\ cm \pm 1.0\ cm$, and the error in $x' = 55.5 \pm 1.0\ cm$.

These values are used to determine the focal length and the error associated with it, which is:

$$f_o = 37.9\ cm \pm 1.1\ cm$$

Based on the estimated accuracy in the positioning of the nodal points, the error in the distance between principal points is: $\Delta P_{21} = 1.6cm \pm 0.1\ cm$.

For the focal length of the eyepiece, two different magnifications were tested at two different positions. The errors in these values and the distance between the two positions are:

$$M_1 = -1.300 \pm 0.001$$
$$M_2 = -2.128 \pm 0.008$$
$$d = 4.8\ cm \pm 1.0\ cm$$

Using these three values, the focal length of the eyepiece is calculated with an accuracy of:

$$f_e = 4.0\ cm \pm 0.9\ cm$$

Part II

(Calculations for these errors are also shown on the loose leaf paper in the geometric optics section of the lab book.)

The theoretical magnification of a telescope with an object at infinity, which uses the objective lens and eyepiece from the previous section of the experiment, can be calculated by using the focal lengths calculated. The error in this value is related to the error in these focal lengths, and is

$$M_{\infty t} = 9.5 \pm 2.4$$

The experimental value for this magnification was determined by performing four tests, resulting in a magnification of

$$M_{\infty e} = 7.27 \pm 0.09$$

Only one experimental test was performed for the telescope at an intermediate distance, with an error in the measurements of: $y' = 39.5 \pm 1\ in.$; $y = 4.5\ in. \pm 0.5\ in.$ Resulting in an error of the magnification of

$$M_{Ie} = 8.78 \pm 1.2$$

Conclusion

Although the error in some of the calculations of this experiment was as high as ~25%, it can certainly be considered successful. The major source of error was in the calculation of the focal length of the telescope eyepiece. For such a small focal length, which is expected for an eyepiece, slight errors can propagate to be quite substantial. Despite this, the error range of the theoretical value for the magnification of the telescope overlaps with the experimental value. The error could certainly have been reduced further in the calculation of focal lengths by repeating the full process a number of times. However, time did not allow for this to be done.