

Table of Integrals

$$\int (a+bx)^n dx = \frac{(a+bx)^{n+1}}{b(n+1)} \quad \text{for } n \neq -1$$

$$\int \frac{dx}{(a+bx)} = \frac{1}{b} \ln|a+bx|$$

$$\int \frac{xdx}{(a+bx)} = \frac{1}{b^2} [a+bx - a \ln|a+bx|]$$

$$\int \frac{dx}{(a+bx)^2} = -\frac{1}{b(a+bx)}$$

$$\int \frac{xdx}{(a+bx)^2} = \frac{1}{b^2} \left[\ln(a+bx) + \frac{a}{(a+bx)} \right]$$

$$\int \frac{dx}{(a+bx)^{1/2}} = \frac{2}{b} (a+bx)^{1/2}$$

$$\int \frac{xdx}{(a+bx)^{1/2}} = \frac{2}{b^2} \left[\frac{(a+bx)^{3/2}}{3} - a(a+bx)^{1/2} \right]$$

$$\int \frac{dx}{(a+bx)^{3/2}} = \frac{-2}{b(a+bx)^{1/2}}$$

$$\int \frac{xdx}{(a+bx)^{3/2}} = \frac{2}{b^2} \left[(a+bx)^{1/2} + \frac{a}{(a+bx)^{1/2}} \right]$$

$$\int \frac{dx}{(a^2+x^2)} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{xdx}{(a^2+x^2)} = \frac{1}{2} \ln(a^2+x^2)$$

$$\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3} \tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{xdx}{(a^2+x^2)^2} = -\frac{1}{2(a^2+x^2)}$$

$$\int \frac{dx}{(a^2-x^2)} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$$

$$\int \frac{xdx}{(a^2-x^2)} = -\frac{1}{2} \ln|a^2-x^2|$$

$$\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{4a^3} \ln \left| \frac{a+x}{a-x} \right|$$

$$\int \frac{xdx}{(a^2-x^2)^2} = \frac{1}{2(a^2-x^2)}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) = \sinh^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{xdx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2}$$

$$\int \frac{x^2 dx}{\sqrt{x^2 + a^2}} = \frac{x\sqrt{x^2 + a^2}}{2} - \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{x^3 dx}{\sqrt{x^2 + a^2}} = \frac{(x^2 + a^2)^{3/2}}{3} - a^2 \sqrt{x^2 + a^2}$$

$$\int \frac{dx}{x\sqrt{x^2 + a^2}} = -\frac{1}{a} \ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right)$$

$$\int \frac{dx}{x^2 \sqrt{x^2 + a^2}} = -\frac{\sqrt{x^2 + a^2}}{a^2 x}$$

$$\int \frac{dx}{x^3 \sqrt{x^2 + a^2}} = -\frac{\sqrt{x^2 + a^2}}{2a^2 x^2} + \frac{1}{2a^3} \ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right)$$

$$\int \sqrt{x^2 + a^2} dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2})$$

$$\int x\sqrt{x^2 + a^2} dx = \frac{(x^2 + a^2)^{3/2}}{3}$$

$$\int x^2 \sqrt{x^2 + a^2} dx = \frac{x(x^2 + a^2)^{3/2}}{4} - \frac{a^2 x \sqrt{x^2 + a^2}}{8} - \frac{a^4}{8} \ln(x + \sqrt{x^2 + a^2})$$

$$\int x^3 \sqrt{x^2 + a^2} dx = \frac{(x^2 + a^2)^{5/2}}{5} - \frac{a^2 (x^2 + a^2)^{3/2}}{3}$$

$$\int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right)$$

$$\int \frac{\sqrt{x^2 + a^2}}{x^2} dx = -\frac{\sqrt{x^2 + a^2}}{x} + \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{\sqrt{x^2 + a^2}}{x^3} dx = -\frac{\sqrt{x^2 + a^2}}{2x^2} - \frac{1}{2a} \ln\left(\frac{a + \sqrt{x^2 + a^2}}{x}\right)$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}}$$

$$\int \frac{xdx}{(x^2 + a^2)^{3/2}} = \frac{-1}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x^2 dx}{(x^2 + a^2)^{3/2}} = \frac{-x}{\sqrt{x^2 + a^2}} + \ln(x + \sqrt{x^2 + a^2})$$

$$\int \frac{x^3 dx}{(x^2 + a^2)^{3/2}} = \sqrt{x^2 + a^2} + \frac{a^2}{\sqrt{x^2 + a^2}}$$

$$\int \frac{dx}{x(x^2 + a^2)^{3/2}} = \frac{1}{a^2 \sqrt{x^2 + a^2}} - \frac{1}{a^3} \ln \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right)$$

$$\int \frac{dx}{x^2 (x^2 + a^2)^{3/2}} = -\frac{\sqrt{x^2 + a^2}}{a^4 x} - \frac{x}{a^4 \sqrt{x^2 + a^2}}$$

$$\int \frac{dx}{x^3 (x^2 + a^2)^{3/2}} = \frac{-1}{2a^2 x^2 \sqrt{x^2 + a^2}} - \frac{3}{2a^4 \sqrt{x^2 + a^2}} + \frac{3}{2a^5} \ln \left(\frac{a + \sqrt{x^2 + a^2}}{x} \right)$$

$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$$

$$\int \cos^2(ax) dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$$

$$\int \sin(ax) \cos(ax) dx = \frac{1}{2a} \sin^2(ax)$$

$$\int \sin(ax) \cos^2(ax) dx = -\frac{\cos^3(ax)}{3a}$$

$$\int \sin^2(ax) \cos(ax) dx = \frac{\sin^3(ax)}{3a}$$

$$\int \sin^2(ax) \cos^2(ax) dx = -\frac{1}{32a} \sin(4ax) + \frac{x}{8}$$

$$\int x \sin^2(ax) dx = \frac{x^2}{4} - \frac{x \sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}$$

$$\int x \cos^2(ax) dx = \frac{x^2}{4} + \frac{x \sin(2ax)}{4a} + \frac{\cos(2ax)}{8a^2}$$

$$\int_0^a \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi x}{a}\right) dx = \frac{a}{2} \delta_{mn}$$

$$\int_0^a \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi x}{a}\right) dx = \frac{a}{2} \delta_{mn} \quad (\text{note: if } m = n = 0, \text{ then integral} = a)$$

$$\int_{-a}^a \cos\left(\frac{m\pi x}{2a}\right) \cos\left(\frac{n\pi x}{2a}\right) dx = a \delta_{mn} \quad (\text{note: if } m = n = 0, \text{ then integral} = 2a)$$

$$\int_0^\pi P_\ell(\cos \theta) P_m(\cos \theta) \sin \theta d\theta = \int_{-1}^1 P_\ell(x) P_m(x) dx = \frac{2}{2\ell + 1} \delta_{\ell m}$$