

The Design of Structured Finance Vehicles

Sanjiv Ranjan Das
Leavey School of Business
Santa Clara University ¹

Seoyoung Kim
Leavey School of Business
Santa Clara University

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Abstract

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We model the design of a structured finance deal, specifically, its capital structure, leverage risk controls, asset quality, and the rollover frequency of senior debt. Instead of providing safety, stringent risk management controls often accelerate failure, and optimal risk management choices depend critically on the rollover horizon of the senior notes. The expected losses on senior notes become increasingly sensitive to pool risk (i.e., spread volatility) when risk controls become more stringent, particularly under shorter rollover horizons. Post the financial crisis, we intend to inform the creation of safer SPVs in structured finance, and we propose avenues of mitigating senior note risk through contingent capital.

Keywords: SPVs; SIVs; special purpose vehicle; structured investment vehicle; structured finance risk controls; structured finance credit ratings; contingent capital.

1 Introduction

Structured finance deals emerged as an increasingly important means of risk sharing and obtaining access to capital prior to the financial crisis of 2008. Market segmentation and an ever-increasing demand for safe debt resulted in a marked shift in the suppliers of safe debt from the commercial banking system to the shadow banking system, which provided an increasing share of “safe” assets via special purpose vehicles (SPVs), overtaking many traditional sources of safe debt (Gorton, Lewellen, and Metrick (2012)). At their peak in 2007, Pozsar, Adrian, Ashcraft, and Boesky (2012) estimate that shadow banking liabilities had grown to nearly \$22 trillion.

Given their scope and magnitude, it is crucial to understand this new and highly complex asset class. For example, before the financial crisis, twenty-nine structured investment vehicles, none of which remain today, held an estimated \$400 billion in assets.¹ Despite their AAA rating, senior notes issued by these SIVs experienced an average 50% loss, and subordinated notes experienced near total loss.²

From these widespread failures, a natural question arises as to whether the design of SPVs was adequate to ensure promised repayment to senior note holders with AAA-level certainty. Did the demise of SPVs in the crisis occur despite careful security design and risk management practices, or because of it? A parsimonious theoretical model of structured finance vehicles developed here shows that the locus of viable SPVs was exceedingly small, and these vehicles were destined to fail with high probability. Our model also suggests a modified, more resilient SPV structure.

A structured finance deal is engineered to tranche investments in an asset pool into prioritized cash-flow claims. The resulting liability structure comprises two broad sources of capital for SPVs: the so-called equity portion, which comprises bonds/notes

¹<http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/5769361/400bn-SIV-market-sold-off-in-two-years.html>

²<http://www.risk.net/risk-magazine/news/1517514/almost-siv-assets-sold-fitch>

commonly denoted as “capital” notes, and the de facto debt portion of the capital structure, known as “senior” notes, which are supported by the subordinated capital notes. The senior notes form the primary source of financing, often accounting for more than 90% of the liabilities. Structured finance deals are designed with the intent to secure a AAA rating for the senior notes, which pay slightly above the risk free rate of interest.

The asset pool of the SPV may comprise a wide range of fixed income and credit assets, such as corporate bonds, residential mortgage-backed securities (RMBS), commercial mortgage-backed securities (CMBS), as well as other asset-backed securities (ABS) collateralized by credit card loans, auto loans, home-equity loans, etc. The quality of assets can range from those rated AAA to below investment grade, and diversification of the pool along various dimensions is intended to reduce the risks inherent in these assets.

Designing a deal that is safe for senior note holders is inherently difficult. Studies have explored the securitization and design of special purpose vehicles in structured finance, and the attendant benefits to investors and issuers. DeMarzo and Duffie (1999) examine the role of information and liquidity costs inherent in selling tranches of a structured finance deal, highlighting the lemons problem in the issuance of asset backed securities. DeMarzo (2005) demonstrates liquidity efficiencies to creating low risk senior notes from the pooling and tranching of asset backed securities.

This paper explores very different aspects of structured finance design than has been considered in the literature so far, and is aimed at determining the optimal design of a SPV, i.e., the capital structure, risk controls, and rollover horizon of debt tranches. Many results are counterintuitive, suggesting that a naive approach to SIV design might exacerbate risk rather than mitigate it. Hence, improperly designed risk controls for the tranching and pooling of securities can offset the risk diversification benefits pointed out in DeMarzo (2005).

We summarize our main results. First, we examine the effect of risk controls on the quality of tranches issued by the SPV, and show that *tightening risk controls may in fact increase risk*. We show that tranche ratings and risks are extremely sensitive to the primary form of risk control—the leverage constraint³ mandated by the SPV—which if violated, leads to “defeasance”, i.e., early termination of the vehicle and a liquidation of the SPV’s assets. Paradoxically, tighter risk controls can increase the ex-ante expected losses of the senior notes because the probability of defeasance increases sharply when leverage controls are tightened, making the senior notes more likely to lose value. This effect is more pronounced under greater fire-sale discounts.

Second, the *rollover horizon of the senior notes is a polarizing factor in determining the most effective level of leverage risk controls*, with the optimal leverage control threshold increasing (i.e., becoming more likely to trigger defeasance) in the rollover horizon. Overall, tight risk controls are beneficial in the case of long rollover horizons, but detrimental for short horizons.

Third, *ratings volatility increases with risk controls*. There are interesting interactions between expected losses (ratings) to the senior note holders and the riskiness of the asset pool (i.e., spread volatility), the size of the senior tranche, and the level of the leverage risk control threshold. Under shorter rollover horizons, the expected losses on senior notes become increasingly sensitive to pool risk as leverage controls become more stringent. Hence, there is a trade-off between reducing credit risk and reducing expected loss exposure to pool risk. That is, if senior-note rollover horizons are shortened and leverage controls are tightened to reduce credit risk, the expected losses on the senior tranche become more sensitive to the spread volatility of the underlying asset pool.

³The leverage constraint usually stipulates a minimum allowable ratio of pool assets to senior notes, i.e., akin to the inverse of a loan-to-value (LTV) ratio. Hence, this leverage trigger is of value greater than 1.

Fourth, *optimal SPV design is highly sensitive to the expected fire-sale discount upon defeasance*, particularly under longer rollover horizons. For instance, under standard spread volatility assumptions and an expected fire-sale discount of 10%, an SPV can issue a senior tranche size of 89.3%, achieving expected losses no greater than 0.01% if they set the minimum leverage threshold constraint (ratio of assets to senior notes) to 1.115. However, if the fire-sale discount were slightly greater at 15%, this senior tranche size / leverage-constraint combination corresponds to an expected loss to senior notes of 5.2%, which is far greater than the 0.01% cutoff required to maintain high-quality ratings.⁴ In fact, under an expected fire-sale discount of 15%, the maximal attainable high-quality senior tranche size is 85%, with the leverage threshold constraint set to 1.0. Thus, we see that the chosen SPV design is very different based on small differences in the expected fire-sale discount upon defeasance.

This consideration is particularly important given the highly uncertain nature of fire-sale discounts in times of distress, whereby one SPV's defeasance may easily trigger that of another SPV, pushing up fire-sale discounts even higher. For instance, Cheyne Finance recovered 44% of par value in initial liquidation rounds;⁵ Sigma Finance recovered 15%.⁶ Thus, stress tests of SPV design must not only account for swings in spread volatility (i.e., extreme pool risk) but must also account for the potentially vast swings in fire-sale discounts under defeasance.

Fifth, *contingent capital in SPVs is pareto improving to all classes of debt*. When the defeasance trigger is first hit, capital note holders are required to infuse capital, paying down a portion of the senior notes and allowing the SPV to continue rather

⁴We define this cutoff based on the *Rating Methodology* guide published by Moody's Investors Service. See "Probability of Default Ratings and Loss Given Default Assessments for Non-Financial Speculative-Grade Corporate Obligors in the United States and Canada," Global Credit Research, August 2006, page 10, Exhibit 2.

⁵<http://www.risk.net/risk-magazine/news/1504163/cheyne-assets-disappoint-in-rescue-auction>. Cheyne was one of the largest SIVs created in 2005 before the onset of the financial crisis. It was the first to include subprime assets in its pool.

⁶<http://www.telegraph.co.uk/finance/newsbysector/banksandfinance/5769361/400bn-SIV-market-sold-off-in-two-years.html>

than defease. This substantively decreases expected loss rates to both senior *and* capital note holders.⁷

Numerical analysis shows that, for many standard SPV parameters, these vehicles were designed to fail, and require careful restructuring in order to provide safe returns to senior note holders. Our findings provide normative prescriptions for the risk management of structured deals, whereby the risk controls, capitalization, and rollover horizon interact in a delicate balance to achieve a viable SPV.

Our paper also relates to the recent literature on the risks of securitization. Coval, Jurek, and Stafford (2009a) discuss the features of securitized pools, where senior tranches are akin to economic catastrophe bonds, failing under extreme situations, but offering lower compensation than investors should require. We provide evidence, in fact, that pervasive risk control design flaws make such bonds more prone to economic catastrophe, demonstrating why their paper finds these bonds are poor investments. In related work, Coval, Jurek, and Stafford (2009b) argue that small errors in parameter estimates of the collateral pool result in a large variation in the riskiness of the senior tranches. Complementing this asset-side result, we find that the liability side matters too: irrespective of the risk parameters of the collateral pool, small changes to risk control level (i.e., the leverage ratio threshold) can cause large changes to the riskiness and value of the senior tranches, and thus, to the rating achieved by these notes.

Whereas these papers suggest that systematic extreme risk was not reflected in senior tranche pricing, they also point out that fire sales matter in increasing corre-

⁷For instance, consider an SPV issuing two-year notes, with a leverage threshold of 1.04, underlying asset spread volatility of 1.05%, and expected fire-sale discount of 10%. In the absence of pre-committed capital infusions, a 92% senior tranche size has an expected loss rate of 2.87%, indicating that a far smaller senior tranche size is required to secure a AAA rating on senior notes. However, with the use of contingent capital, the expected loss to senior note holders decreases to 0.01%, allowing the larger senior tranche size while maintaining a AAA rating. Moreover, the expected loss to capital note holders decreases from 63.27% to 48.13%, thereby benefiting all parties involved.

lations of distressed assets (see Coval and Stafford, 2007; Shleifer and Vishny, 2010; Diamond and Rajan, 2011). Similarly, Cont and Wagalath (2012) model the impact of fire sales in multiple funds on the volatility and correlation of asset returns in a setting with multiple assets. We show that this is a major factor in the assessment of risk controls and has a first-order effect in generating wide swings in expected losses, resulting in lower ratings. In relation to these papers, Hanson and Sunderam (2013) argue that pooling and tranching creates “safe” senior tranches owned by the majority of investors, leading to a dearth of informed investors in good times and resulting in insufficient risk controls, which are shown to be weak when bad times arrive. Our paper argues that stringent risk controls are not always required, and often exacerbate the problems experienced from a drop in pool asset values, highlighting a new form of “neglected” risk (see Gennaioli, Shleifer, and Vishny, 2010).

The rest of the paper proceeds as follows. Section 2 presents the model set up, and Section 3 shows how the probability of defeasance and expected loss varies with risk controls, and examines how the size of the senior tranche and its roll-over frequency impacts risk and ratings. Section 4 examines how the feasible region for high quality senior note ratings is impacted by the issuing SPV’s design. Section 5 discusses mitigating risk for senior notes using contingent capital. Section 6 concludes.

2 Model

2.1 Basic Set Up

We develop a parsimonious model of structured finance and SPV design. The aggregate value of the asset pool at time t is denoted as $A(t)$, and the liabilities are denoted as $B(t)$ for the senior notes, and $C(t)$ for the capital notes and, $A(t) = B(t) + C(t)$. The face values of the capital and senior notes are denoted D_C and D_B , respectively. At inception (i.e., at time $t = 0$), $B(0) = D_B$ and $C(0) = D_C = A(0) - B(0)$.

Investors in senior notes seek safe assets with assured returns, and hence, SPVs are designed with the intent to attain a AAA credit rating for the senior notes, reflecting a 0.01% expected loss from default, i.e., the probability of default times the loss on default is expected to be less than 1/100th of one percent.⁸ The expected loss is critically dependent on the following factors:

1. The credit quality of the asset pool (i.e., its mean spread α and spread volatility σ),
2. The size of the senior tranche (D_B) and subordinated equity tranche (D_C),
3. The time to rollover/maturity of the senior notes T , and
4. The risk controls outlined by the SPV, i.e., the leverage control level K .

The capital note holders retain the residual after repayment of the senior notes and management fees borne by the SPV.⁹ Thus, capital note holders bear first losses on the SPV's assets. The SPV makes a spread between the rate of return on the assets and the rate of interest paid to the senior notes, from which the capital note holders are compensated. This spread arises because the assets held are of longer maturity than that of the senior notes that are periodically rolled over. Hence, the maturity of the senior notes is shorter than that of the capital notes and the weighted average maturity of the assets of the SPV.

To afford the senior notes additional protection, the SPV covenants typically include safeguards pertaining to asset quality in addition to requirements regarding the duration and liquidity of the assets in the collateral pool. In addition to these asset restrictions, the SPV is monitored on an ongoing basis, primarily through a

⁸See Cantor, Emery, and Stumpp (2006).

⁹In case the capital note holders are the founders/managers of the SPV, they also collect management fees.

leverage test, whereby the ratio of collateral value to senior-note obligations must meet a pre-specified cutoff.¹⁰ Failure to meet these requirements forces the SPV into “defeasance” or “enforcement” mode, whereby the assets of the SPV must be sold to wind down operations. The proceeds are used to first pay off the senior note holders, with the residual used to pay off the equity note holders.

We define an SPV’s leverage at any point in time as

$$\frac{A(t)}{D_B} \geq K \geq 1 \quad (1)$$

where K is the lower bound on the ratio of collateral assets, $A(t)$, to senior notes, D_B , that is permitted by the SPV. For example, an SPV with no equity notes has a leverage ratio equal to 1, and an SPV with assets worth 100 and senior notes with a face value of 92, has a leverage ratio of 1.087. Suppose a lower limit of $K = 1.04$ is placed on the leverage ratio. Then the SPV enters defeasance (enforcement) mode when $A(t) = D_B \cdot K = 92 \times 1.04 = 95.68$, i.e., a 4.32% drop in the value of the assets held by the SPV, which results in a 54% loss to the capital note investors assuming a frictionless market with no fire-sale discounts.

In reality, enforcement mode likely occurs when markets are under stress and other financial institutions (including other SPVs) are also attempting to unwind, and assets are unlikely to be sold at full market value but rather at fire sale prices, at some percentage discount δ . Hence, the anticipated recovery is only a fraction $(1 - \delta) < 1$ of the assets’ value at defeasance, i.e., $((1 - \delta)D_B K)$. Since $K \geq 1$, with diffusion processes for the assets $A(t)$, unless defeasance occurs, the senior notes incur no losses. If defeasance occurs, then the losses incurred by senior notes amount to $\max(0, D_B - (1 - \delta)D_B K) = D_B \max(0, 1 - (1 - \delta)K)$. Using the illustrative numbers

¹⁰Other tests are imposed, such as liquidity and pool composition, but these are less likely to trigger defeasance than the leverage test, which is essentially a trigger corresponding to a prespecified fall in assets. Thus, the SPV risk model trigger is akin to a barrier in the Black and Cox (1976) class of structural default models.

above, the senior notes will incur losses if $(1 - (1 - \delta)K) > 0$, i.e., the recovery rate $(1 - \delta)$ is less than $1/K = 1/1.04 = 96.15\%$, or equivalently, if the fire-sale discount $\delta \geq 3.85\%$.

In the ensuing analyses, we make the following parameter choices, which are reasonable average values based on usual market statistics about structured finance deals and the SPVs designed to operationalize them. We choose two sizes of the senior notes tranche, both common usage, i.e., $D_B = \{0.88, 0.92\}$, expressed as fractions of the total asset pool of the SPV. Each of these levels corresponds to equity note levels of 0.12 and 0.08, respectively. We vary the threshold leverage constraint K from 1.0 to $A(0)/D_B$. The expected fire sale discount upon default, δ , is set to 10% and 15%, meaning that the recovery rate on asset sales upon defeasance is $(1 - \delta) = \{0.90, 0.85\}$, respectively, for these two levels. The risk free rate is set to be $r = 0.02$. The senior notes are rolled over at two horizons, 1 year and 4 years.

The value of the asset pool depends on the current spread at which it is trading and we denote this as $s(t)$. Asset spread volatility σ is taken to be 65 basis points and varied up to 105 bps, and in some cases all the way up to 500 bps. For asset-backed securities, normal spread volatility is in the range of 11–57 bps,¹¹ and the values used reflect stressed values of standard deviations, commonly used by rating agencies in their evaluation of SPV structures.

2.2 The State Price of Defeasance

The state price of defeasance is the discounted probability that the SPV will be in defeasance before the rollover date for senior debt. We define $f[\tau]$ as the first passage probability density of the defeasance barrier being hit at time τ (i.e., when the assets drop to defeasance level). The state price of a specific defeasance stopping time τ is

¹¹See Moody's Special Comment titled "The Relationship between Par Coupon Spreads and Credit Ratings in US Structured Finance," December 2005, by Jian Hu and Richard Cantor.

$e^{-r\tau} f[\tau]$. For a part of the expected loss calculation, we need to compute the integral $\int_0^T e^{-r\tau} f[\tau] d\tau$, i.e., the state price of defeasance, which is equal to the present value of a cash-or-nothing binary barrier option that pays \$1 the moment the underlying $A(t)$ touches the barrier $D_B \cdot K$.

Computing expected loss on the senior notes is simplified if we have the state price of defeasance, which depends on stochastic changes in the value of the asset pool, which in turn, depend on the stochastic process for $s(t)$ (the yield increment on the assets), which we characterize shortly. The price of the asset pool $A(t)$ comes from discounting assets at the risk free rate plus asset spread, i.e., the current yield on the assets. The variable $s(t) \geq 0$ is a spread widening factor that drives the asset pool value down from its initial value of $A(0)$ to barrier $D_B \cdot K$; i.e., we define $A(t) = A(0)e^{1-s(t)}$. Hence, we normalize $s(0) = 1$ and then define the barrier level of $\bar{s} > s(0)$, such that

$$A(0)e^{(1-\bar{s})} = D_B \cdot K \quad (2)$$

Inverting this equation we have the level of $s(t)$ at which defeasance is triggered as follows:

$$\bar{s} = 1 - \ln \left[\frac{D_B \cdot K}{A(0)} \right] \quad (3)$$

Assuming a stochastic process for $s(t)$ enables the calculation of the probability of defeasance in closed form using barrier option mathematics. We assume that this asset spread widening factor follows a geometric Brownian motion, i.e.,

$$ds(t) = \alpha s(t) dt + \sigma s(t) dW(t), \quad s(0) = 1 \quad (4)$$

Factor $s(t)$ drives the asset process $A(t)$, where the drift is α , the volatility parameter is σ , and dW is a Wiener increment. The larger the drift α , and volatility σ , the greater the probability of defeasance. So, when $A(t)$ makes a first-passage transition to $D_B \cdot K$, then $s(t)$ transitions to \bar{s} (as per equation (3)), with first passage probability

$f[\tau]$ as defined earlier. Under this process the formula for the discounted first-passage density for $s(0) = 1$ reaching \bar{s} , i.e., the probability of defeasance, is as follows:¹²

$$\int_0^T e^{-r\tau} f[\tau] d\tau = \bar{s}^{\mu+\lambda} N(-z) + \bar{s}^{\mu-\lambda} N(-z + 2\lambda\sigma\sqrt{T}) \quad (5)$$

where

$$z = \frac{\ln(\bar{s})}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}; \quad \mu = \frac{\alpha - \sigma^2/2}{\sigma^2}; \quad \lambda = \sqrt{\mu^2 + \frac{2\alpha}{\sigma^2}}$$

We note that $f[\tau]$ may be written in more detailed form as follows:

$$\begin{aligned} f[\tau] &\equiv f[s(\tau) = \bar{s} | s(0), s(t) < \bar{s}, 0 \leq t < \tau] \\ &\equiv f[A(\tau) = D_B K | A(0), A(t) > D_B K, 0 \leq t < \tau] \end{aligned} \quad (6)$$

and the latter expression will be used for further analysis in the paper using the first passage time density function $f[\tau]$. We turn to the rating process next.

2.3 Expected Losses and Ratings

Rating agencies run simulated models of the SPV's assets to determine the expected losses on the senior notes. Expected losses map into ratings for these notes and hence, it is important to assess the interaction of risk controls K (the leverage trigger), size of the senior tranche D_B , and spread volatility of assets in the pool (σ) in determining these expected losses.

The general expression for the present value of expected losses on the senior notes on defeasance is as follows:

$$\begin{aligned} EL_B &= D_B \max(0, 1 - (1 - \delta)K) \\ &\cdot \int_0^T e^{-r\tau} f[A(\tau) = D_B \cdot K | A(0), A(t) > D_B \cdot K, 0 \leq t < \tau] d\tau \end{aligned} \quad (7)$$

¹²See Haug (2006) for details of the cash-or-nothing binary barrier option that forms the basis for this pricing equation.

The loss on defeasance is a constant, $D_B \max(0, 1 - (1 - \delta)K)$, which is multiplied by the cumulative first passage defeasance probability. This equation is separable in the loss on default and the first passage time probability because the state of defeasance is a constant barrier, dependent on threshold K , and hence the first-passage probability is always triggered at the same state. Here, r is the risk free rate, and $f[A(\tau) = D_B \cdot K | A(0), A(t) > D_B \cdot K, \forall t < \tau]$ is the first-passage probability of $A(t)$ to barrier $D_B \cdot K$, which we denoted above more parsimoniously as $f[\tau]$. Equation (5) presents the expression for $\int_0^T e^{-r\tau} f[\tau] d\tau$, which we can apply directly in equation (7) to calculate the expected loss, EL_B , on the senior notes. As stated earlier, for senior notes to receive a AAA rating, $EL_B \leq 0.01\%$.

Overall, we see that $E(L) > 0$ if $(1 - \delta) < 1/K$. Note that since $K > 1$, defeasance always occurs before default ($A(T) < D_B$). Thus, at maturity, the assets must be sufficient to pay the senior notes else defeasance would have occurred earlier. We now turn to the analysis of various aspects of the SPV structure.

2.4 Stochastic Fire Sale Discount

The model may be extended to accommodate a stochastic fire sale discount δ . This robustness test shows that the results of the paper remain unaffected even when there is uncertainty about the the size of the fire sale discount (numerical results presented in the following section).

We assume a state variable x that drives the fire sale percentage discount δ as follows.

$$\delta(t) = 1 - \frac{1}{2} \left[\arctan(x(t)) \frac{2}{\pi} + 1 \right] \in (0, 1) \quad (8)$$

where $x(t)$ follows the stochastic process:

$$dx(t) = \kappa(\theta - x(t)) dt + \eta \sqrt{x(t)} dZ, \quad dW \cdot dZ = \rho dt \quad (9)$$

$$\theta = \tan \left\{ \frac{\pi}{2} [2(1 - \delta_0) - 1] \right\} \quad (10)$$

Here κ is a mean-reversion rate, η is the volatility of the state variable, and θ is the level of x when $\delta(t) = \delta_0$, i.e., the fire sale discount in a model where it is not stochastic. Here ρ is the correlation between the spread process Wiener shock $W(t)$ (see equation 4) and the fire sale discount stochastic process $\delta(t)$'s Wiener shock $Z(t)$ (see equation 9). The fire sale discount is bounded between $(0, 1)$ as may be seen in the mathematics of equation (8). We note that $E[\delta(t)] = \delta_0$. As $x(t)$ cycles over time, so does the fire-sale discount $\delta(t)$. For larger portfolios, fire-sale discounts tend to be larger, and this may be adjusted by setting δ_0 appropriately.

Under this framework, we recompute the expected losses to senior notes and compare these values to the ones from equation (7). We do so by simulating the bivariate density function for $\{A, \delta\}$ at the defeasance barrier. As we will see in the following section, the difference in results from making the fire sale discount stochastic is not material. Hence, we impose a pre-determined δ upon defeasance, and employ the closed form expressions given in the paper.

3 Analysis

Although it is clear that increases in the leverage constraint K must increase the probability of defeasance, a natural question arises as to how sensitive the probability of defeasance is to changes in leverage threshold K and how these factors ultimately affect expected losses to senior note holders. In the event that fire-sale discounts are high on account of correlated SPV asset pools (see Covitz, Liang, and Suarez, 2013), large changes in the probability of defeasance arising from small changes in the leverage constraint K , will cause wide swings in expected losses, and consequently in credit ratings. We call this sensitivity “design” risk, i.e., small perturbations in SPV

risk control design may lead to substantial neglected risks (Gennaioli, Shleifer, and Vishny (2010)), in this case, high ratings volatility, of which investors are unaware.

Intuitively, we see that leverage controls on an SPV must be implemented with care to prevent excessive or premature defeasance, which in turn leads to a large quantum of systemic risk if all SPVs implement similar risk controls, possibly triggering simultaneous defeasance. The resulting selling cascade may lead to even greater fire sale discounts, which may impose large amounts of risk even on the senior notes. We now proceed to explore how these factors, in tandem, affect ex-ante expected losses to senior note holders.

3.1 Risk Management increases Risk

The primary form of risk control in an SPV is the leverage constraint, i.e., the threshold K of leverage at which defeasance is triggered if the assets fall sufficiently in value. Since the leverage threshold in our model is defined as $K = A(t)/D_B$, we usually expect an SPV to have a threshold of $K > 1$, the reason being that triggering defeasance earlier leaves a greater amount of capital notes to buffer losses that might otherwise be borne by the senior note holders. However, if we set K too high, then defeasance may occur too rapidly and increase the likelihood of incurring deadweight costs on defeasance, which may increase expected losses to the senior tranche. It may even be optimal to set K below 1, since allowing a drop in the value of the portfolio below the value of the senior notes may allow the necessary time in which assets can recover, staving off premature defeasance that would lead to deadweight costs.

In Figures 1 and 2 we implement equation (7), plotting the expected loss on senior notes as a function of the leverage threshold K for a one-year and four-year horizon, respectively. We keep the size of the senior tranche fixed at $D_B = 0.92$ (upper panel); for comparison, we also examine this relation under a senior tranche size of $D_B = 0.88$ (lower panel).

Beginning with Figure 1 (one-year horizon), we observe that the function for expected loss can also be very sensitive to K , i.e., to the stringency of the leverage control imposed on the SPV, depending on the size of the senior tranche (D_B) and the expected fire-sale discount upon default (δ). Moreover, the difference in expected losses under varying fire-sale discounts ($\delta = \{0.10, 0.15\}$) increases very quickly as the defeasance trigger, K , increases.

Specifically, we observe that at a lower fire sale discount rate ($\delta = 0.10$), increasing K does not materially impact the expected losses to the smaller senior tranche size ($D_B = 0.88$), but expected losses begin to increase dramatically with K at the larger senior tranche size ($D_B = 0.92$). When the senior notes comprise 92% of the liabilities, the expected loss at a leverage threshold of $K = 1.04$ delivers a high-quality rating (i.e., low expected loss), but as the leverage threshold exceeds $K = 1.04$, the expected loss increases rapidly. However, we observe that at a higher fire-sale discount rate ($\delta = 0.15$), expected losses are very sensitive to K , increasing dramatically for both the larger and smaller senior tranche sizes. As a test for robustness the results in Table 1 show that the expected losses to senior notes remain virtually the same even when δ is stochastic (see Section 2.4). The level of fire-sale discounts is known to vary with the size of the liquidated portfolio,¹³ and this may be modeled in our setting by varying the mean size of the fire-sale discount through parameter δ_0 , where we choose this parameter to be larger for bigger liquidations.

In contrast, from Figure 2 (four-year horizon), we observe that for the larger senior tranche size of 92%, expected losses increase with K and then begin to decline, though they remain far too high to justify a AAA rating. For the smaller tranche size of 88%, we observe that at very high levels of K , expected losses are low enough to justify a AAA rating, as long as the fire-sale discount is sufficiently low (in this case, 10%).

¹³Variation in the size of the fire-sale discount with portfolio liquidation size is discussed in Chan and Lakonishok (1995), Chan and Fong (2000), Dufour and Engle (2000), and Bouchad, Farmer, and Lillo (2009).

Thus very high values of K are more helpful under longer horizons, where stringent leverage controls can make the senior notes safer and assure a higher rating.

We also explore how incorporating jumps may affect our observations. The results, presented in Figure 3, indicate that even for the smaller tranche size of 88%, expected losses are generally too high to justify a AAA rating under a four-year horizon when we include the possibility of a 40% jump with likelihood 0.001 per annum. If we decrease the jump size to 10%, then we see that for smaller leverage thresholds AAA-rating is feasible, but under larger thresholds, the expected losses remain far too high. Intuitively, with fire-sale discounts, the SPV is better off under leverage constraints with far too low of a probability to be accessed (thereby staving off the fire sale), or with a beginning Asset-to- D_B ratio that is sufficiently high so as to make it improbable to access a leverage constraint (something that is only possible with a sufficiently small senior tranche size).

Overall, we see that the rating of the SPV can be extremely sensitive to leverage risk controls, and this sensitivity hinges critically on the expected fire-sale discount upon defeasance, highlighting a fundamental flaw in SPV design: i.e., the insufficient attention paid to the magnitude of fire-sale discounts on defeasance. Indeed many of the models used by rating agencies in practice did not explicitly account for fire sale discounts, calling into doubt the ex-ante AAA ratings assigned prior to the 2008 crisis.

Ultimately, in some cases, it is more beneficial to set the defeasance point K sufficiently high so as to ensure an adequate capital notes buffer for the senior notes. In other cases, it is more beneficial to set the defeasance point low enough so as to allow sufficient time for the assets to recover. The designated prescription changes substantially with the fire sale discount, since setting stringent risk controls, i.e., high defeasance triggers, is especially risky during poor economic times when fire-sale discounts tend to be high. In short, *risk controls fail to do their job in precisely those*

states of the economy when they are needed the most. As experienced in the 2008 financial crisis, stringent controls trigger were tripped for many SPVs simultaneously, exacerbating this problem. Thus, it may in fact be better to set very low thresholds or no leverage constraint at all. In the final section of the paper, we recommend a solution using contingent capital calls.

3.2 Sensitivity to Pool Risk

With recent notable failures of structured finance deals, much blame has been placed on the poor quality of assets held by SPVs (e.g., subprime mortgages). The extant ratings process for SPVs revolves around determining pool risk level σ for rating agency simulation models. If estimation risk for this parameter is high, then ratings are likely to be noisy measures of senior note credit quality. Thus, natural questions arise as to how much expected losses are affected by underlying pool risk, whether this was indeed a primary source of risk, and what are the factors either mitigating or exacerbating its importance.

We examine these issues in two ways. First, we contrast the results from Figures 1 and 2 with changing pool risk, i.e., we vary parameter σ in order to understand how expected losses vary as this parameter changes. Second, we examine whether this relation is attenuated or exacerbated by changes to the leverage threshold. If sensitivity to pool risk increases with the tightening of leverage constraints, then risk controls are especially tenuous just when they are most needed. Finally, we examine how these sensitivities are affected by varying combinations of senior-tranche size and rollover horizon.

In Figures 4, 5, 6, and 7, we plot percentage expected losses on senior notes against σ under varying tranche sizes, horizons, and thresholds. For both smaller and larger senior tranche sizes, we observe that under the shorter one-year rollover horizons (Figures 4 and 5), the expected losses on senior notes become increasingly

sensitive to pool risk as leverage controls become more stringent. That is, under greater levels of K , expected losses increase even more with increases in σ . Under the longer four-year horizon, we observe that expected losses under the larger senior tranche size (Figure 6) are not increasingly sensitive to pool risk as K is increased, but overall, expected losses are too high to sustain a AAA rating. Under the smaller senior tranche size (Figure 7), we observe an initial increase in the sensitivity to pool risk as K is increased, then the sensitivity to pool risk begins to decline at even greater values of K (by which point, the expected losses become too high to support a AAA rating).

Overall, we find that increasing asset pool risk (σ) usually (but not always) makes it less likely to achieve viable top quality ratings for the senior notes. Therefore asset quality is material, even if risk management is appropriately tuned. This problem is exacerbated if risk controls (K) are tightened, complicating the intuition that adequate risk controls are helpful in managing credit risk in SPVs, because stringent risk controls make expected losses higher in the presence of fire sale discounts (δ), in addition to making expected losses more sensitive to asset pool risk. These issues suggest that efficacious structured finance and SPV design is surprisingly hard to achieve, and offer insights into the dramatic failure of structured finance deals during the recent financial crisis.

4 Locus of Acceptable Ratings

The preceding analyses have demonstrated that the structured vehicle design process is akin to being between a rock and many hard places, where small changes in design lead to large changes in expected loss and volatile ratings. In this section, we examine the region of viable top quality (AAA) ratings for senior notes given the design parameters of the SPV, i.e., a choice of size of senior tranche (D_B), leverage risk control

(K), and rollover horizon (T). The SPV design problem, from the viewpoint of the originator / capital note holders, is to select the leverage constraint K and rollover horizon T that allows us to maximize the size of the senior tranche D_B while keeping the expected loss sufficiently small (i.e., less than or equal to 0.01%) to deliver a AAA rating on the senior notes.

$$\max_{K,T} D_B, \quad \text{s.t.} \quad \frac{EL_B}{D_B} \leq 0.01\% \quad (11)$$

Due to market segmentation, the senior notes tranche must be large to benefit the originators / capital note holders. We examine this issue by plotting the locus of pairs of $\{D_B, K\}$ (keeping T fixed) that deliver expected percentage losses no greater than 0.01% under six sets of spread volatility and fire-sale discount configurations: $\sigma = \{0.0065, 0.0105, 0.0500\} \times \delta = \{0.10, 0.15\}$, keeping the riskless rate fixed at $r = 0.02$. All parameters are within economically reasonable ranges, barring the high-stress case of $\sigma = 0.05$.¹⁴ Typical structured finance rating procedures undertaken by rating agencies call for stressing the one standard deviation move in asset spreads by a factor of 2x to 5x, and 500 basis points is a typically high stress case.

In Figure 8, we present the results based on a rollover horizon of $T = 1$ year, and in Figure 9, we present the results based on a rollover horizon of $T = 4$ years. We make the following observations:

1. Both Figures 8 and 9 show a decreasing acceptable size of the senior notes tranche (D_B) as we increase the spread volatility (σ) of the asset pool (reading down the columns in the figure). The range of viable SPV structures also decreases as we increase the fire-sale discount δ (reading across the rows in the figure), and this difference becomes starker as we increase the rollover horizon.

¹⁴We also explored the case of $\sigma = 0.02$ (untabulated) that is qualitatively similar to that of $\sigma = 0.0105$ under both the shorter and longer rollover horizons.

Strikingly, at a longer rollover horizon of $T = 4$ years, the larger senior tranche sizes are viable under very strict leverage thresholds (K) when the fire-sale discount is $\delta = 10\%$, but at a greater fire-sale discount of $\delta = 15\%$, we observe that larger senior tranche sizes are not viable even when pushing up the leverage threshold K . Thus, optimal SPV design is highly sensitive to even small changes in the fire-sale discount, particularly when the rollover horizon is long.

2. We note that in the financial crisis, the effective fire sale discount was in the range of $\delta = 50\%$ to 75% , and our examples are provided for cases of $\delta = \{10\%, 15\%\}$, i.e., highly conservative discounts more in line with one-off SPV failure, rather than the systemic failures evidenced in the recent past. Even at these low levels of fire sale discounts, for reasonable levels of risk, the maximal senior note tranche is $D_B = 90\%$ approximately, somewhat lower than the levels seen in practice (for example, the famous Cheyne deal that failed in August 2007, had a senior note tranche size of 92%). Thus, at reasonable stressed levels of pool risk, it is difficult to justify senior tranche sizes that were observed in practice.
3. Overall, under the shorter rollover horizon of $T = 1$ year, optimal SPV design is not sensitive to the fire-sale discount under more common ranges of spread volatility (i.e., under $\sigma = \{0.0065, 0.0105\}$), yielding the same decision in the maximal senior tranche size that can accommodate an expected loss no greater than 0.01% . However, under the high-stress case of $\sigma = 0.05$, optimal SPV design is highly sensitive to even small changes in the expected fire-sale discount upon default, whereby the maximum possible senior tranche size is close to 0.90 if the fire-sale discount is $\delta = 10\%$ but is closer to 0.80 if the fire-sale discount is $\delta = 15\%$. Furthermore, at $\delta = 10\%$, the optimal design requires a high level of K , whereas at $\delta = 15\%$ the optimal design mandates a very low level of

K . Hence, for small changes in δ , the optimal risk control changes from one extreme to the other. This again highlights the vast difficulty in designing an SPV optimally, especially if uncertainty regarding the fire sale discount δ is very high.

4. Under a longer rollover horizon of $T = 4$ years, the optimal SPV design is sensitive to the expected fire-sale discount even under the more likely ranges of spread volatility (i.e., under $\sigma = \{0.0065, 0.0105\}$), yielding slight differences in the maximal senior tranche size that can accommodate an expected loss no greater than 0.01%. This difference becomes even more dramatic under the high-stress case of $\sigma = 0.05$, with a maximal senior tranche size close to 0.90 if the fire-sale discount is $\delta = 10\%$, but a maximal senior tranche size slightly greater than 0.60 if the fire-sale discount is $\delta = 15\%$. Once again the prescriptions for risk controls for the two fire sale discount values entail polar opposite outcomes, once again affirming that, irrespective of rollover horizon, small changes in the expected fire sale discount imply vastly different risk management protocols.

5 Repairing SPVs

The preceding critique of SPVs suggests that static risk controls with large losses on defeasance offer no soft landing for senior note holders in times of financial crises, indicating that senior note holders should be further compensated for the “neglected risks” they bear, see Gennaioli, Shleifer, and Vishny (2010). These additional benefits may be dynamic and state-dependent, resulting in a value transfer from capital notes to senior notes in poor states of the world.

A simple prescription for risk buffering the senior note holders is to build in contingent capital, whereby capital note holders (or the issuer/manager) are required to

provide additional capital to buy back a sufficient quantity of senior notes at par, thus returning the leverage ratio to a safe level. This dynamic risk management approach is similar to a margin call, whereby the extra infusion/buyback is analogous to the variation margin. Ex-ante, this built-in capital call discourages the creation of an SPV that the issuer/manager knows to be too risky, and promotes an equilibrium where only viable SPVs are created.¹⁵ Payment of contingent capital could be ensured by credit or liquidity guarantees, much in the way that conduit sponsors of fully supported ABCP (asset backed commercial paper) guarantee payment (Acharya, Schnabl, and Suarez (2013)); though, the contingent capital we prescribe below would be far less than full support.

We now examine the net benefits of capital calls. Our chosen method requires that when the defeasance trigger is hit at stopping time τ , i.e., when the assets fall to a level such that $A(t)/D_B = K$, then we have a capital infusion by the capital note holders (or SPV originator/underwriter) that pays down D_B to some level $D'_B < D_B$ such that $A(t)/D'_B > K$. This remedy recapitalizes the SPV, replacing senior notes with capital notes, with no change in the assets of the SPV.

Hence, the senior notes are reset with fresh horizon $(T - \tau)$ and for a lower principal $D'_B(\tau)$, when the leverage constraint is violated, i.e., $A(\tau) = D_B \cdot K < 1$. This remedy effectively postpones the liquidation of the SPV, and postpones or possibly eliminates the deadweight fire sale losses on defeasance, allowing time for the SPV's assets to recover. The capital note holders also stand to benefit when deadweight costs are delayed or possibly even eliminated if the prices of the underlying assets rise, but they bear the costs of this waiting period since they are the ones providing the additional capital. Thus, although senior note holders are definitively better off

¹⁵Basel III envisages requiring contingent capital in addition to Tier 1 capital from banks. While this is not yet under consideration in the US, in September 2011, the Independent Commission on Banking recommended that UK banks be required to hold an additional 7-10% of contingent capital, also known as bail-in capital, where debt would automatically convert to equity on breaching a trigger level of leverage.

when the SPV's covenants require contingent capital infusions, the question remains as to whether the capital note holders are better off as well.

We now proceed to evaluate expected losses to the capital note holders, assuming first that no restructuring is permitted. The expected loss comprises two components: (a) the expected loss on defeasance prior to rollover date, and (b) the expected loss when there is no defeasance until the rollover date. The present value of total expected loss can be expressed as follows:

$$\begin{aligned}
EL_C &\equiv L[A_0, D_B, D_C, K, \delta, r, T] & (12) \\
&= [D_C - \min(0, D_B K(1 - \delta) - D_B)] \int_0^T e^{-r\tau} \times f[A(\tau) = D_B K] d\tau \\
&\quad + e^{-rT} \int_{D_B K}^{A_0} [A_0 - A(T)] \cdot \Pr[A(T)|A(t) > D_B K, \forall t] dA(T) \\
&= [D_C - \min(0, D_B K(1 - \delta) - D_B)] \int_0^T e^{-r\tau} \times f[A(\tau) = D_B K] d\tau \\
&\quad + e^{-rT} A_0 \int_{\bar{s}}^1 [1 - e^{1-s(T)}] \cdot \Pr[s(T)|s(t) < \bar{s}, \forall t] ds(T)
\end{aligned}$$

where, as before, $f[A(\tau) = D_B K]$ is a more concise expression for the first-passage probability of assets to defeasance boundary $D_B K$. The first line of this equation pertains to expected loss on defeasance prior to T , and the second line calculates expected loss at T . $\Pr[A(T)|A(t) > D_B K, \forall t]$ is the conditional probability of stochastic terminal value $A(T)$, given no defeasance prior to maturity. The last line comprises a change of variable from $A(t)$ to $s(t)$.

We now proceed to evaluate expected losses to the capital note holders, assuming a pre-commitment to infuse capital, rather than defease, when the leverage threshold is hit. We denote this new level of expected losses as EL'_C . The original first-passage probability to defeasance was based on a senior note level of D_B , but is now based on the level $D'_B(\tau) < D_B$ instead. Hence, the revised expected present value of losses

to capital note holders is given by the nested equation:

$$\begin{aligned}
EL'_C &= (D_B - D'_B(\tau)) \int_0^T e^{-r\tau} \cdot f[A(\tau) = D_B K] d\tau \\
&+ \int_0^T L[A(0), D'_B(\tau), D_C, K, \delta, r, T - \tau] \cdot e^{-r\tau} \cdot f[A(\tau) = D_B K] d\tau \\
&+ e^{-rT} \int_{D_B K}^{A(0)} [A(0) - A(T)] \cdot \Pr[A(T)|A(t) > D_B K, \forall t] dA(T)
\end{aligned} \tag{13}$$

where the function $L[\cdot]$ comes from equation (12). The first line in the expression above accounts for the expected infusion by capital note holders in the amount $D_B - D'_B$, the second line accounts for the expected loss should defeasance occur (after infusion), and the third line accounts for the expected loss to capital note holders when there is no defeasance.

We employ a fast simulation approach for computing EL_C and EL'_C . Overall, the net gain to capital note holders from restructuring the SPV is as follows:

$$\Delta EL_C = EL_C - EL'_C \tag{14}$$

The sign of this term is indeterminate and depends on the parameters of the SPV; i.e., the leverage threshold K , the initial size of the senior tranche D_B , the expected fire-sale discount δ , the spread volatility σ , the risk-free rate r , and the investment horizon T , as well as the pre-committed infusion amount upon triggering the leverage constraint. If positive, then the scheme is Pareto optimal; otherwise, it represents wealth transfer from capital note holders to senior note holders. As fire sale discounts δ increase, we expect that the scheme is more likely to be Pareto optimal.¹⁶

In Table 2, we present examples of contingent-capital commitments that transform an unviable SPV, of unit asset value, to one with AAA-rated senior notes. Specifically,

¹⁶This computation ignores the possible increase in capital requirements for the SPV sponsor when required to keep contingent capital. Hence, it is an upper bound on the savings to capital note holders from creating a SPV with remediation by contingent capital.

under an expected fire-sale discount of $\delta = 10\%$, the given SPV parameters support a senior tranche size of $D_B = 0.88$, which corresponds to an expected percentage loss to senior note holders of 0.01%. Under a senior tranche size of $D_B = 0.90$, this expected loss jumps up to 0.39%. However, with a pre-committed infusion of 0.02 upon hitting the leverage threshold, the expected loss to senior note holders drops to 0.01%, and the expected loss to capital note holders also decreases, from 40.13% to 38.32%. Moreover, the probability of defeasance decreases from 6.18% to 0.13%. Under a senior tranche size of $D_B = 0.92$, we see an expected loss of 2.87% in the absence of contingent capital. Thus using contingent capital results in both senior notes and capital notes becoming better off.

In addition to these recapitalizing approaches, we can attempt other remedies as well. For example, senior notes can be paid a rate of interest that is indexed to their rating or to that of the pool. As the rating falls, higher rates can be paid based on a predetermined menu, whereby the capital to make such payments come from the management fees first, and then from cash flows to capital note holders. Ex-post, this approach compensates the senior notes for bearing additional risk, though payment of the higher rate is contingent on the SPV's survival. More importantly, this approach discourages the SPV's issuers ex-ante from constructing an SPV they know to be fragile.

Alternatively, we can restore the value of the asset pool by requiring the SPV issuer/manager to purchase the lower rated assets that have dropped in value and to replace them with top quality assets, bringing the leverage ratio $A(t)/D_B$ back to being greater than K . In effect, the capital note holders write credit protection (i.e., a spread option) on the pool's lower rated assets, that is triggered when the value of the asset pool drops below the leverage threshold, but does not lead to liquidation of the SPV. Once again, this ex-ante prevents knowingly fragile structured finance deal from being created.

Ultimately, in any of these remedies, “own” risk of the capital note holders is certainly an issue. However, given that such SPVs are usually issued by deep-pocket financial firms (or too-big-to-fail banks), own risk is less of a factor.

6 Concluding Comments

We develop a parsimonious model to analyze the design of structured finance deals and the special purpose vehicles (SPVs) created to operationalize them. Our findings are as follows: First, tightening risk management controls can dramatically increase ex-ante expected losses to senior note holders. Second, the presence of fire-sale discounts, i.e., deadweight costs of defeasance, leads to fragile structures that are not designed to sustain the levels of risk, or to ensure repayment of principal to senior note holders commensurate with a top quality credit rating. Third, optimal risk management choices qualitatively depend on the rollover horizon of the senior notes. Fourth, senior note ratings are very sensitive to leverage controls. Fifth, expected-loss sensitivity to pool risk (i.e., spread volatility of the underlying assets) increases dramatically with increases in leverage risk controls under shorter rollover horizons. The confluence of these design characteristics suggests that SPVs were not as resilient to economic shocks as they should have been, and features of the design contributed to the demise of many SPVs.

Overall, the fact that ratings and risks are highly sensitive to the fire-sale discount, pool volatility, and risk controls suggests that the senior tranche should be much smaller, and that more capital notes (i.e. a larger equity tranche) are required to sustain high-quality ratings on senior notes, as suggested by Admati and Hellwig (2013), and by Hanson and Sunderam (2013). We recommend using contingent capital remedies that make the senior notes safer, while also improving the quality of the capital notes. Whereas contingent capital offers remediation ex-post to SPV

vulnerabilities, it also likely prevents unviable SPVs from being created ex-ante, since some of the ex-post costs are borne by the creators of the SPV through their holdings of capital notes.

The financial crisis of 2008 was largely marked by the failure of structured finance in general, and special purpose vehicles in particular. This analysis of the pathology of the crisis and recommendations for remediation provides guidance for the future evolution of the shadow banking sector.

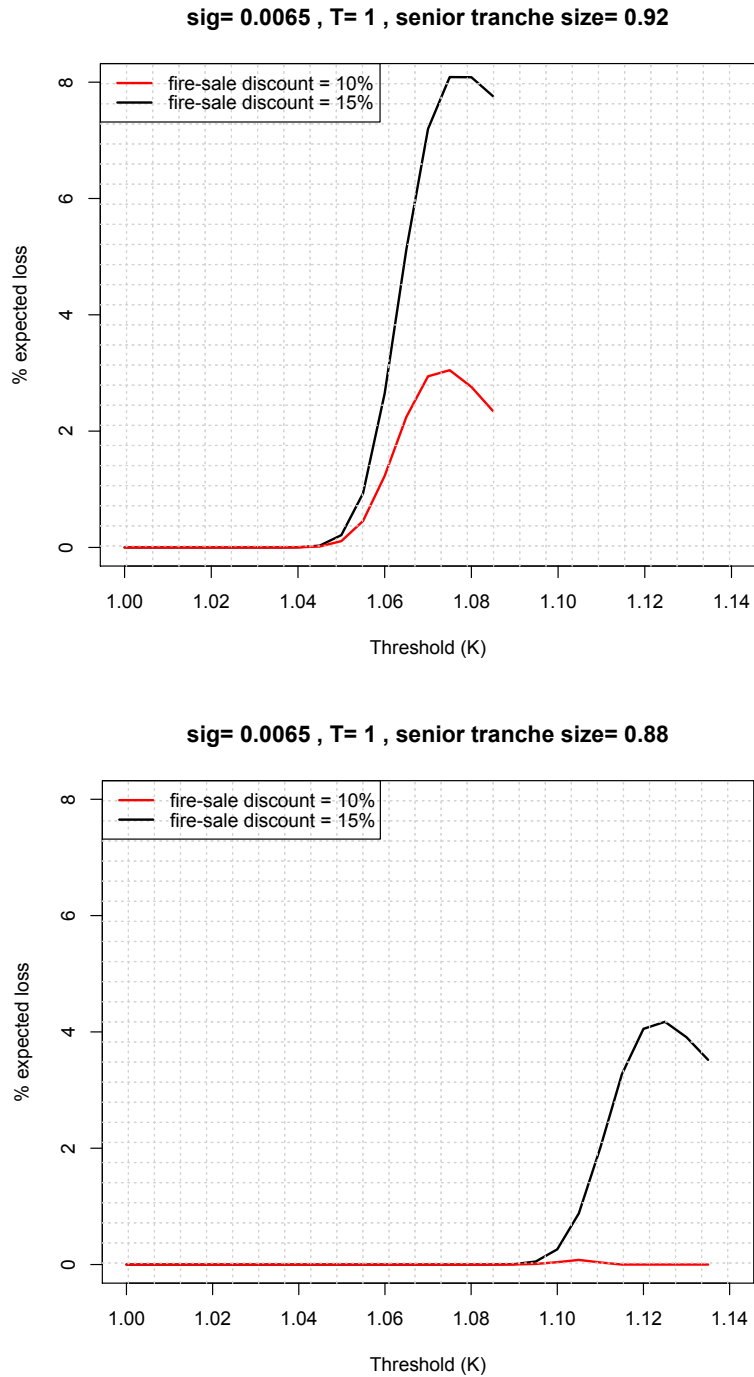


Figure 1: Percentage expected loss plotted against the leverage threshold, K . The base case parameters used are: $A(0) = s(0) = 1$, $T = 1$ years, $\alpha = r = 0.02$, $\sigma = 0.0065$, $\delta = \{0.10, 0.15\}$. The two panels consider varying sizes of the senior tranche, i.e., $D_B = \{0.92, 0.88\}$.

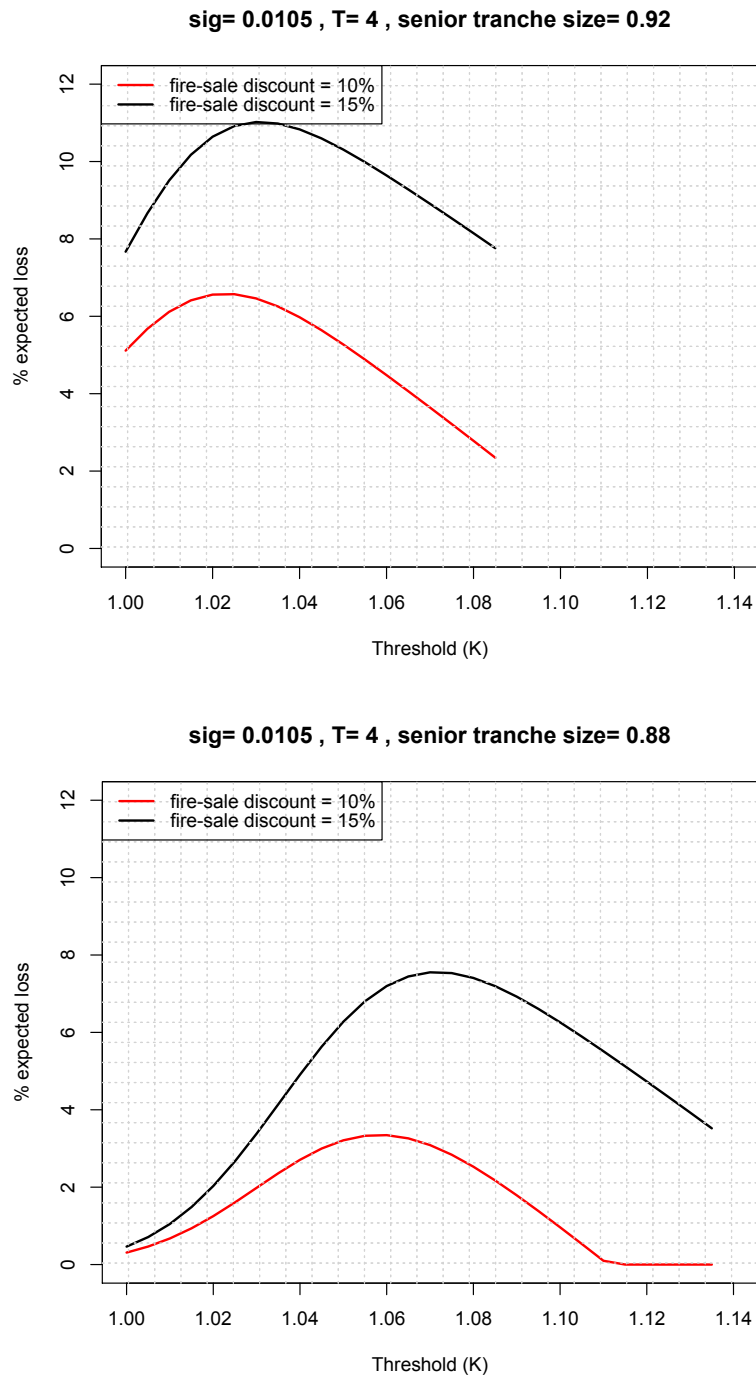


Figure 2: Percentage expected loss plotted against the leverage threshold, K . The base case parameters used are: $A(0) = s(0) = 1$, $T = 4$ years, $\alpha = r = 0.02$, $\sigma = 0.0105$, $\delta = \{0.10, 0.15\}$. The two panels consider varying sizes of the senior tranche, i.e., $D_B = \{0.92, 0.88\}$.

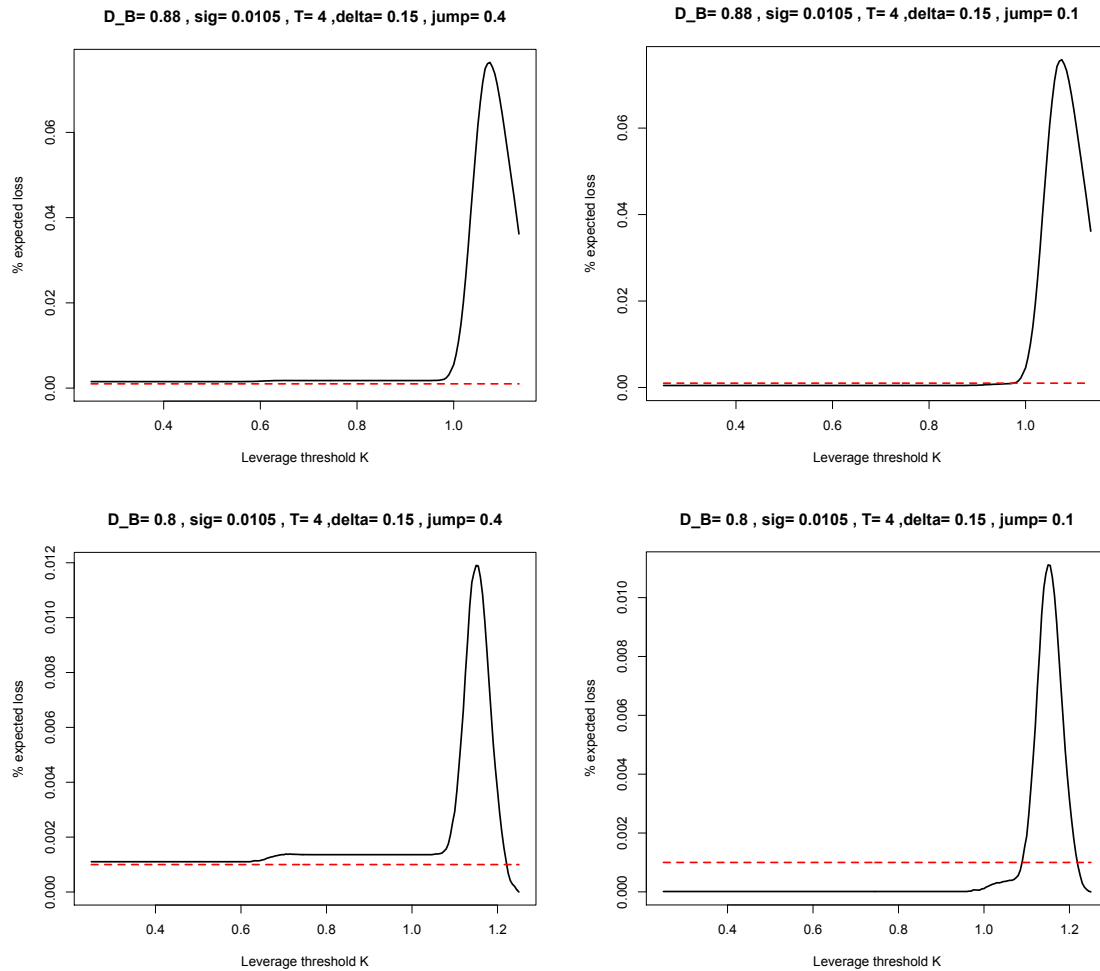


Figure 3: Percentage expected loss plotted against the leverage threshold, K , when incorporating jumps $j = \{0.10, 0.40\}$. The base case parameters used are: $A(0) = s(0) = 1$, senior tranche $D_B = \{0.88, 0.80\}$, $T = 4$ years, $\alpha = r = 0.02$, $\sigma = 0.0105$, $\delta = 0.15$. Jumps occur at a likelihood of 0.001 per annum. The red horizontal line indicates the 0.01% threshold required to maintain a AAA-rating.

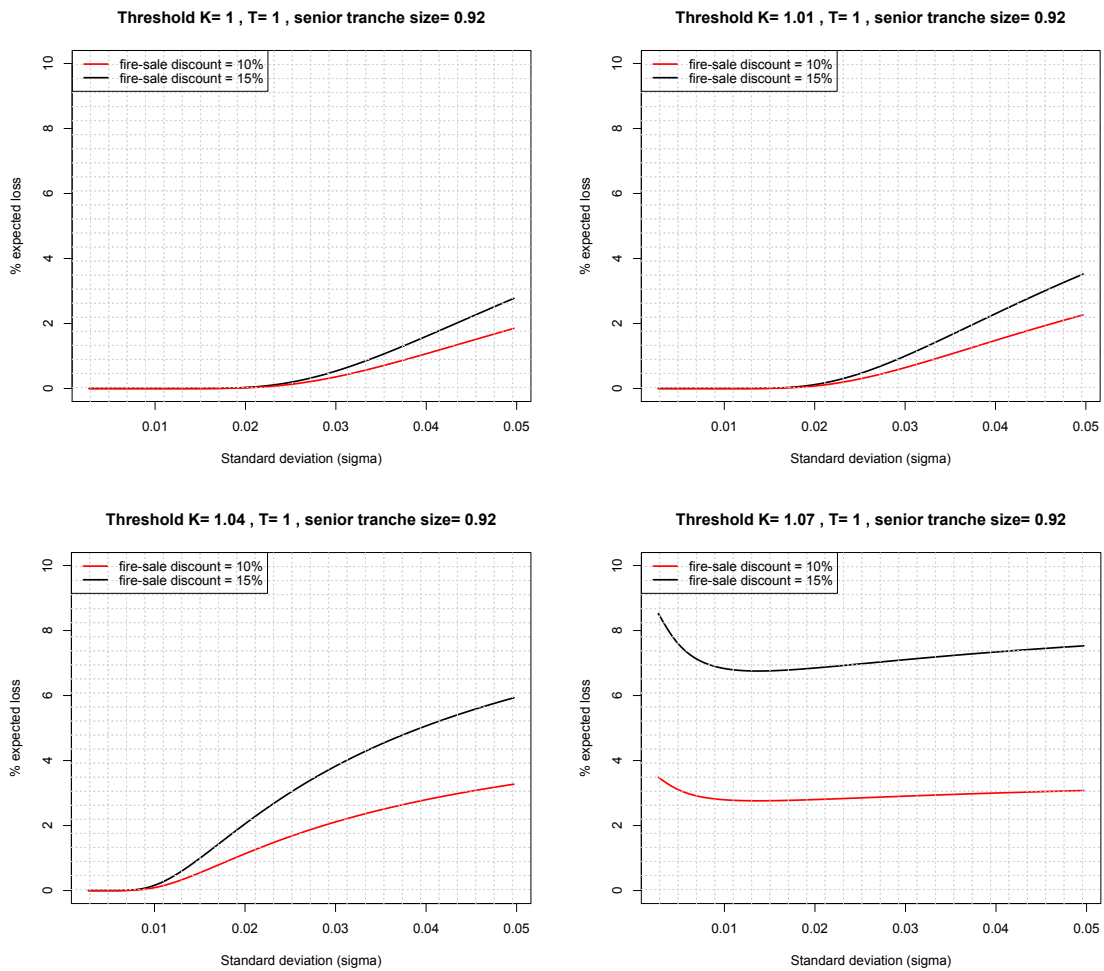


Figure 4: Percentage expected loss plotted against spread volatility, σ . The base case parameters used are: $D_B = 0.92$, $A(0) = s(0) = 1$, $T = 1$ year, $\alpha = r = 0.02$, $\delta = \{0.10, 0.15\}$. The four panels consider varying threshold leverage constraints $K = \{1.00, 1.01, 1.04, 1.07\}$.

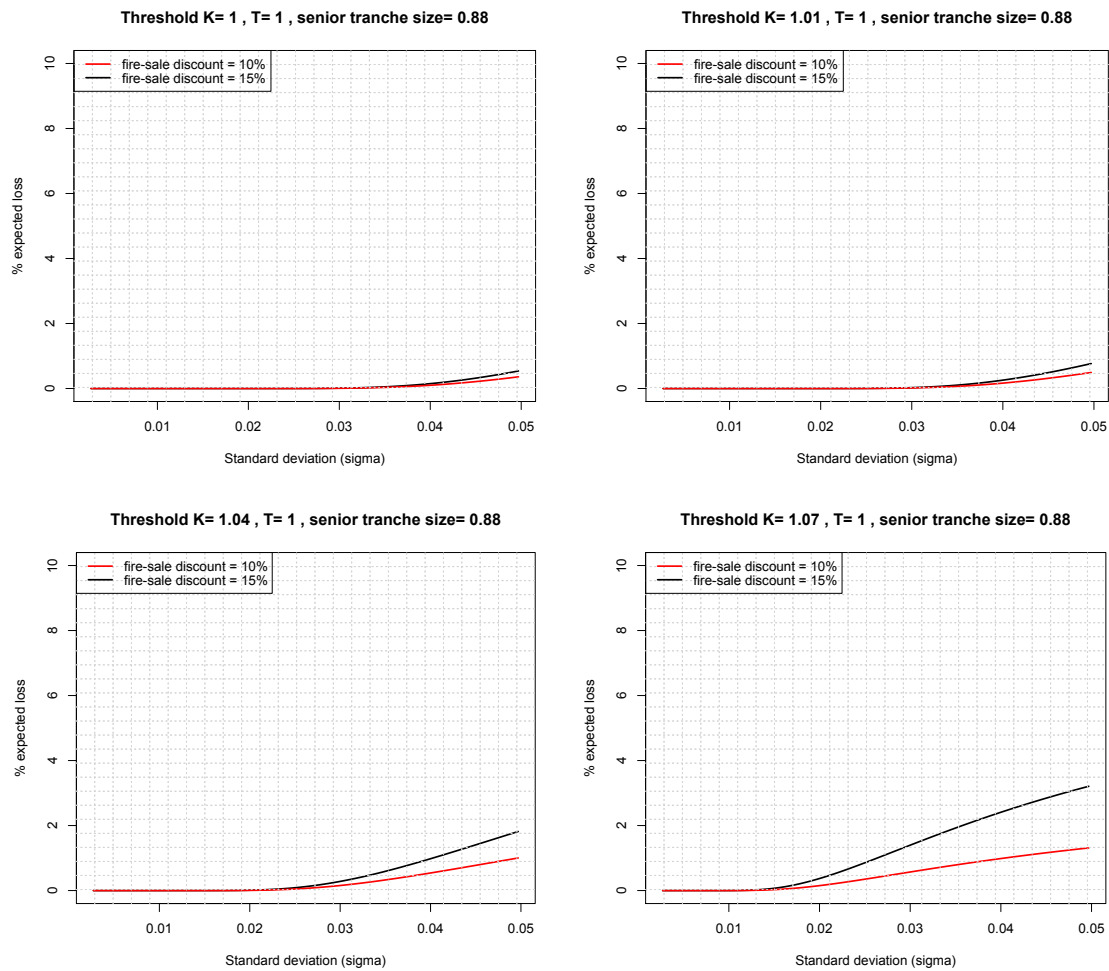


Figure 5: Percentage expected loss plotted against spread volatility, σ . The base case parameters used are: $D_B = 0.88$, $A(0) = s(0) = 1$, $T = 1$ year, $\alpha = r = 0.02$, $\delta = \{0.10, 0.15\}$. The four panels consider varying threshold leverage constraints $K = \{1.00, 1.01, 1.04, 1.07\}$.

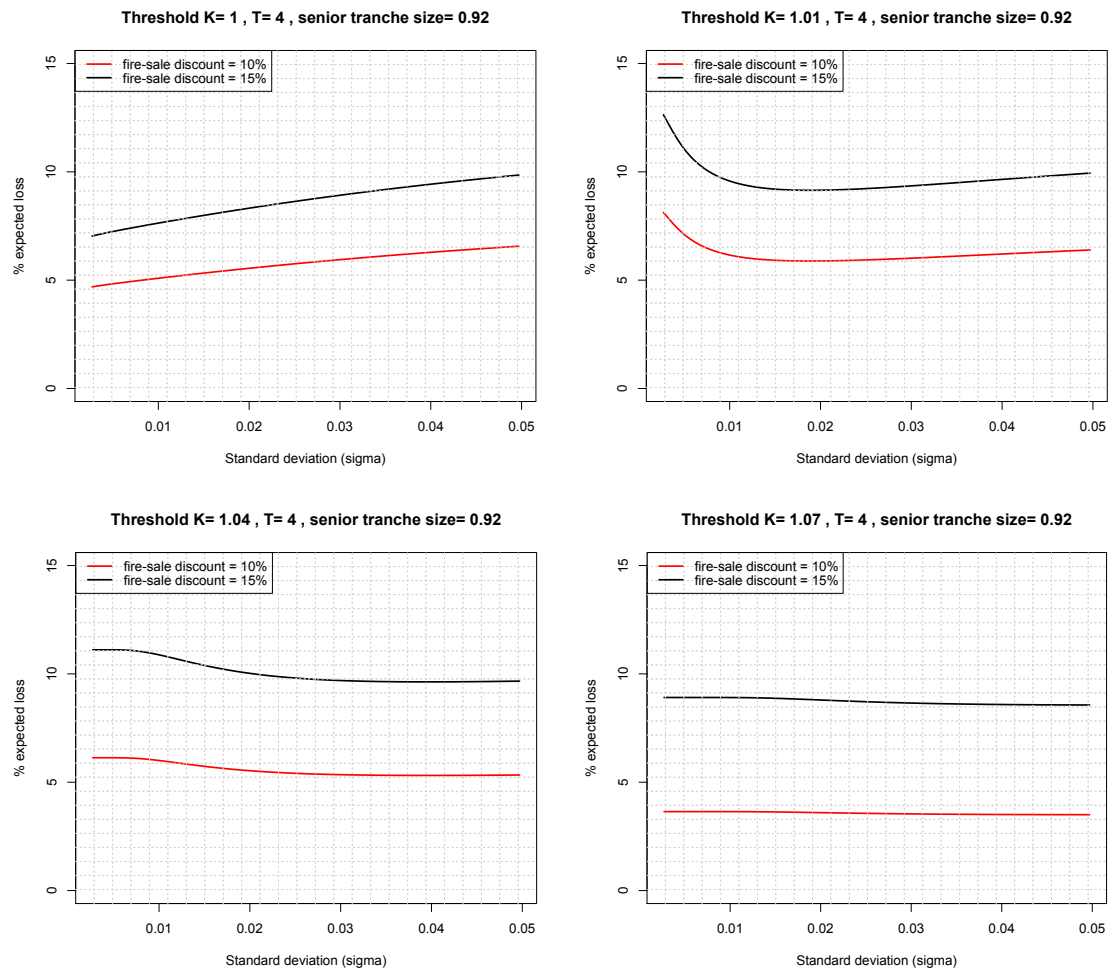


Figure 6: Percentage expected loss plotted against spread volatility, σ . The base case parameters used are: $D_B = 0.92$, $A(0) = s(0) = 1$, $T = 4$ years, $\alpha = r = 0.02$, $\delta = \{0.10, 0.15\}$. The four panels consider varying threshold leverage constraints $K = \{1.00, 1.01, 1.04, 1.07\}$.

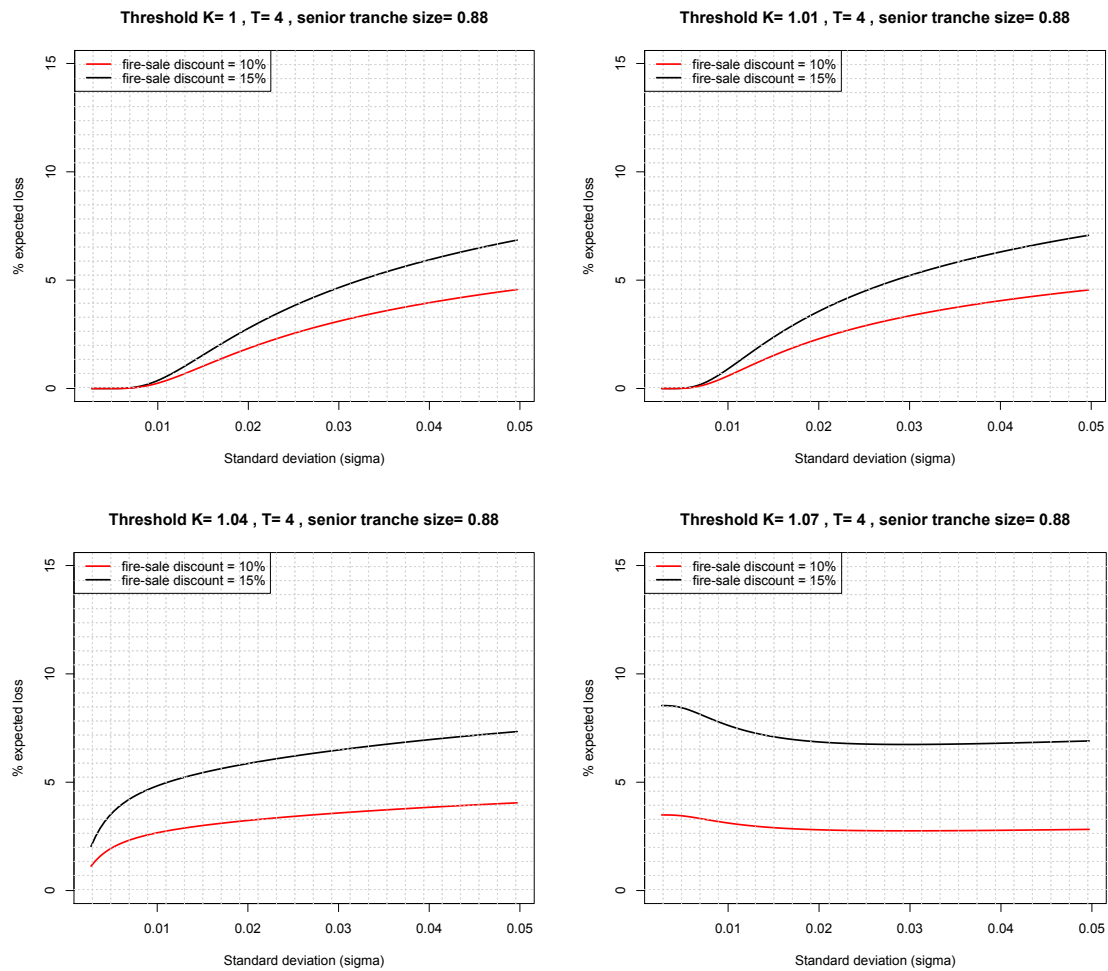


Figure 7: Percentage expected loss plotted against spread volatility, σ . The base case parameters used are: $D_B = 0.88$, $A(0) = s(0) = 1$, $T = 4$ years, $\alpha = r = 0.02$, $\delta = \{0.10, 0.15\}$. The four panels consider varying threshold leverage constraints $K = \{1.00, 1.01, 1.04, 1.07\}$.

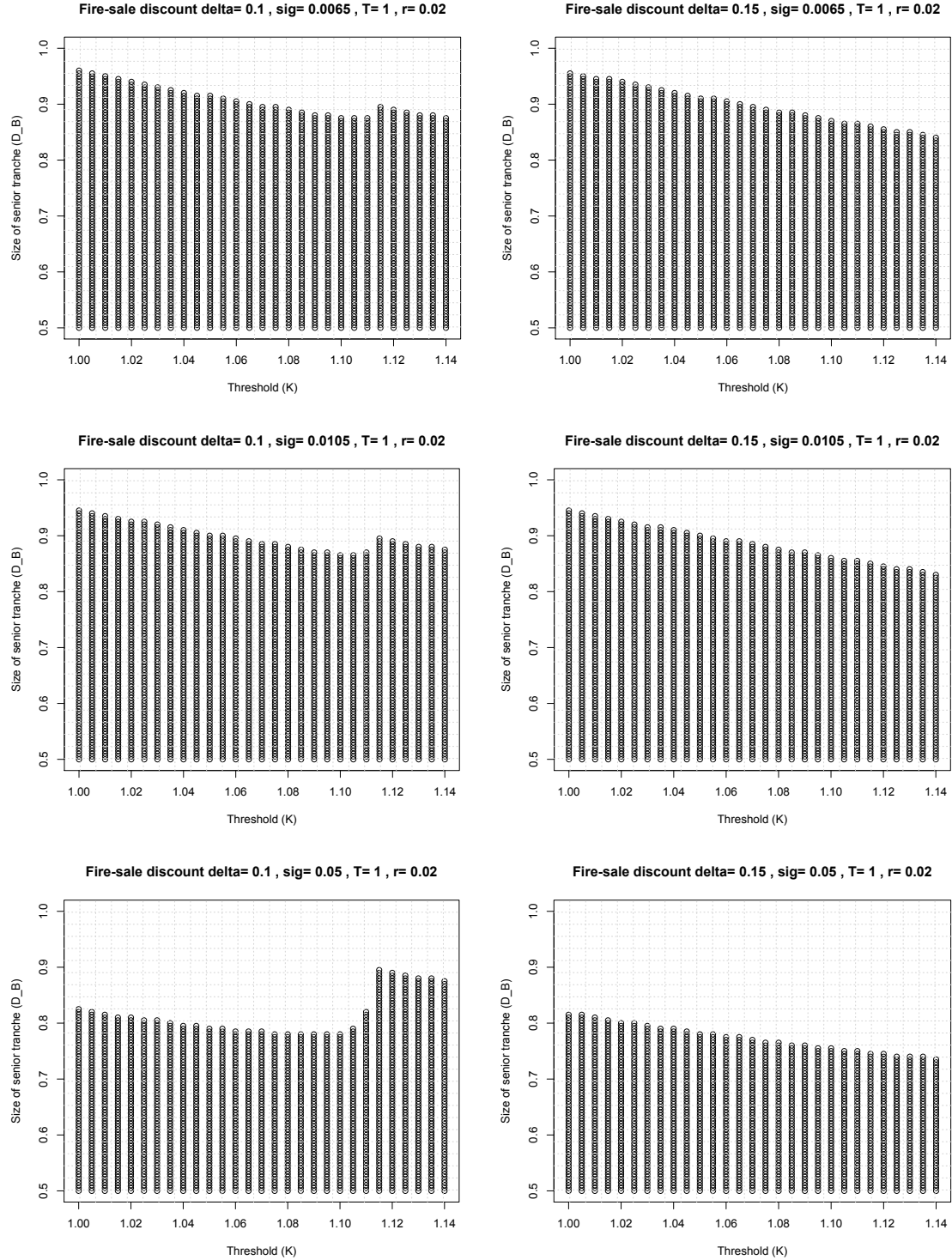


Figure 8: Locus of the size of the senior tranche, D_B , and leverage threshold, K , resulting in a percentage expected loss of less than or equal to 0.01%. The base case parameters are: $A(0) = s(0) = 1$, $T = 1$ year, $\alpha = r = 0.02$, with spread volatilities of $\sigma = 0.0065$ (Row 1), $\sigma = 0.0105$ (Row 2), and $\sigma = 0.05$ (Row 3). Column 1 uses a fire-sale discount of $\delta = 0.10$, and Column 2 uses a fire-sale discount of $\delta = 0.15$.

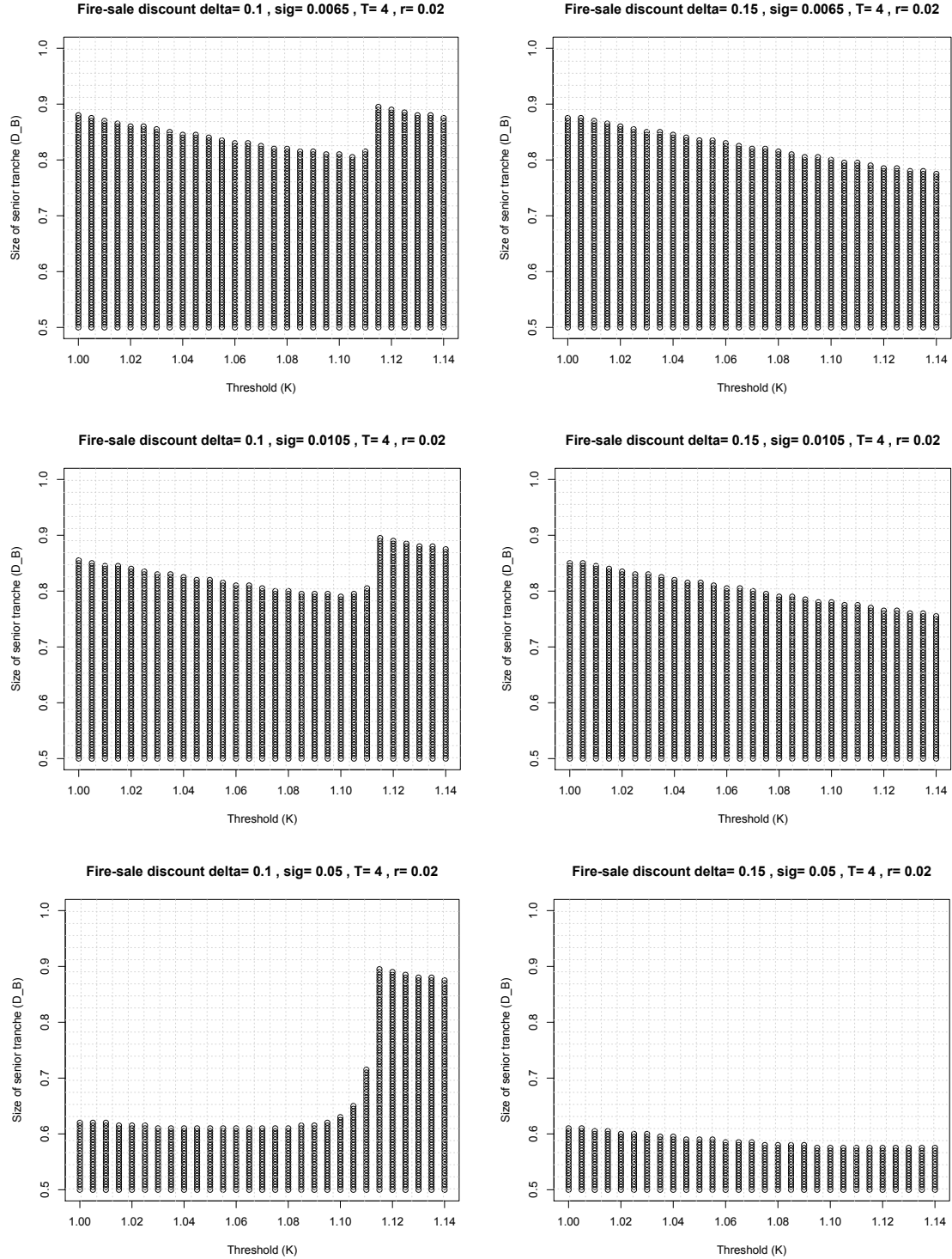


Figure 9: Locus of the size of the senior tranche, D_B , and leverage threshold, K , resulting in a percentage expected loss of less than or equal to 0.01%. The base case parameters are: $A(0) = s(0) = 1$, $T = 4$ years, $\alpha = r = 0.02$, with spread volatilities of $\sigma = 0.0065$ (Row 1), $\sigma = 0.0105$ (Row 2), and $\sigma = 0.05$ (Row 3). Column 1 uses a fire-sale discount of $\delta = 0.10$, and Column 2 uses a fire-sale discount of $\delta = 0.15$.

Table 1: Percentage expected losses to senior note holders when the fire sale discount is stochastic. The parameters used are: $A(0) = s(0) = 1$, $T = 2$ years, $K = 1.04$, $\alpha = r = 0.02$, $\delta_0 = 0.15$, $\sigma = 0.0105$. The senior note tranche size is $D_B = 0.90$. Related to Section 2.4 the parameters are: $\kappa = \{0.8, 0.4\}$, $\eta = \{0.01, 0.05, 0.10, 0.20\}$, i.e., a range of parameters is used. The correlation between the Wiener processes dW (equation 4) and dZ (equation 9, $dW \cdot dZ = \rho dt$) is varied for $\rho = \{0.0, 0.25, 0.50\}$ in the three panels below. The stochastic process for the fire sale discount is bounded between $(0, 1)$, and is based on the following equations: (a) $\delta(t) = 1 - \frac{1}{2} \left[\arctan(x(t)) \frac{2}{\pi} + 1 \right] \in (0, 1)$, (b) given $x(t)$ follows the stochastic process, $dx(t) = \kappa (\tan \{ \frac{\pi}{2} [2008(1 - \delta_0) - 1] \} - x(t)) dt + \eta \sqrt{x(t)} dZ$. Note that the values in the tables below are percentage expected losses in decimal form. Note that $E[\delta(t)] = \delta_0$.

$\rho = 0$		κ	
η	0.4	0.8	
0	0.006271	0.006271	
0.01	0.006271	0.006271	
0.05	0.006276	0.006276	
0.1	0.006296	0.006290	
0.2	0.006383	0.006345	

$\rho = 0.25$		κ	
η	0.4	0.8	
0	0.006271	0.006271	
0.01	0.006245	0.006251	
0.05	0.006145	0.006173	
0.1	0.006035	0.006084	
0.2	0.005858	0.005935	

$\rho = 0.50$		κ	
η	0.4	0.8	
0	0.006271	0.006271	
0.01	0.006218	0.006230	
0.05	0.006016	0.006071	
0.1	0.005785	0.005886	
0.2	0.005383	0.005556	

Table 2: Percentage expected losses to senior versus capital note holders, denoted EL_B and EL_C , respectively. This table compares the expected losses under a structure that does not allow capital infusions to the expected losses under a structure that does. The base case parameters used are: $A(0) = s(0) = 1$, $T = 2$ years, $K = 1.04$, $\alpha = r = 0.02$, $\delta = 0.10$, $\sigma = 0.0105$. D_B denotes the initial senior tranche size, D_C denotes the capital-note tranche size, and $Pr(def)$ denotes the probability (%) of defeasance. *infuse* denotes the amount to be infused, if any, upon triggering the leverage threshold K ; i.e., capital-note holders purchase a fixed amount of debt back from senior-note holders, thereby allowing the SPV to continue rather than defease.

D_B	D_C	infuse	$EL_B(\%)$	$EL_C(\%)$	Pr(def)
0.88	0.12	—	0.01	31.93	0.13
0.90	0.10	—	0.39	40.13	6.18
0.90	0.10	0.02	0.01	38.32	0.13
0.92	0.08	—	2.87	63.27	45.61
0.92	0.08	0.04	0.01	48.13	0.13

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