Solutions Homework 1

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Determine the Concept The slope of an $x(t)$ curve at any point in time represents the speed at that instant. The way the slope changes as time increases gives the acceleration. If the slope becomes less negative or more positive as time increases (as you move to the right on the time axis), then the acceleration is positive. If the slope becomes less positive or more negative, then the acceleration is negative. The slope *of the slope* of an $x(t)$ curve [that is, the slope of a $v(t)$ curve] at any point in time represents the acceleration at that instant.

The slope of curve (a) is negative and becomes more negative as time increases.	Therefore, the velocity is negative and the acceleration is negative.
The slope of curve (b) is positive and constant and so the velocity is positive and constant.	Therefore, the acceleration is zero.
The slope of curve (c) is positive and decreasing.	Therefore, the velocity is positive and the acceleration is negative.
The slope of curve (d) is positive and increasing.	Therefore, the velocity and acceleration are positive. We need more information to conclude that a is constant.
The slope of curve (e) is zero.	Therefore, the velocity and acceleration are zero.

$$\boxed{(d)\text{ best shows motion with constant positive acceleration.}}$$

Now see if you can immediately do problem P13, which revolves around the same concepts.

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Determine the Concept Yes. If the velocity is changing the acceleration is not zero. The velocity is zero and the acceleration is nonzero any time an object is *momentarily* at rest. If the acceleration were also zero, the velocity would never change; therefore, the object would have to remain at rest.

Note that when both the acceleration and the velocity have the same sign, the speed increases. On the other hand, when the acceleration and the velocity have opposite signs, the speed decreases.

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Picture the Problem Since the slope of the $v(t)$ curve in the given intervals is constant, the acceleration is also constant. The value of the acceleration is found using $a = \Delta v/\Delta t$ for the three intervals of constant acceleration shown on the graph.

(*a*)
Find a_{av} for the interval AB:

$$a_{AB} = \frac{15\,\text{m/s} - 5\,\text{m/s}}{3\,\text{s}} = \boxed{3.33\,\text{m/s}^2}$$

Find a_{av} for the interval BC:

Here the line is horizontal. This means that the velocity is contant, hence the acceleration is zero.

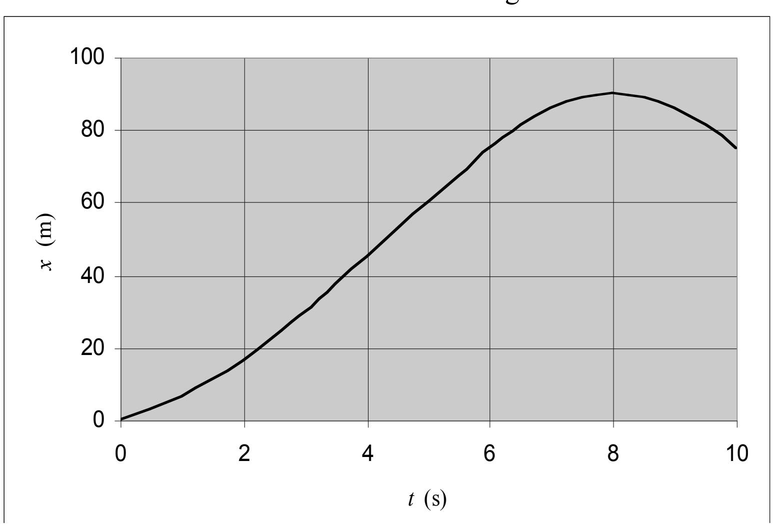
Find a_{av} for the interval CE:

$$a_{CE} = \frac{-15\,\text{m/s} - 15\text{m/s}}{4\,\text{s}} = \boxed{-7.50\text{m/s}^2}$$

(*b*) The distance traveled is the integral of the velocity. Use the formulas for the areas of trapezoids and triangles to find the area under the graph of v as a function of t.

$$\begin{aligned}\Delta x &= (\Delta x)_{A\to B} + (\Delta x)_{B\to C} + (\Delta x)_{C\to D} + (\Delta x)_{D\to E} \\ &= \tfrac{1}{2}(5\,\text{m/s} + 15\,\text{m/s})(3\,\text{s}) + (15\,\text{m/s})(3\,\text{s}) + \tfrac{1}{2}(15\,\text{m/s})(2\,\text{s}) + \tfrac{1}{2}(-15\,\text{m/s})(2\,\text{s}) \\ &= \boxed{75.0\,\text{m}}\end{aligned}$$

(*c*) Consider just the initial segment AB. The velocity there is $v = v_0 + a_{AB}\,t$, with $v_0 = 5$ m/s. Initially, the displacement grows as $x = 0 + v_0\,t + a_{AB}\,t^2/2$, a curved curve that is a piece of a parabola. The displacement then changes to growing linearly when the velocity becomes constant, and so on. The graph of displacement as a function of time is shown below. In the region from B to C the velocity is constant and the x- versus-t curve is a straight line.



(*d*) Reading directly from the figure, we can find the time when the particle is moving the slowest.

$\boxed{\text{At point D, } t = 8\text{ s, the graph crosses the time axis; therefore, } v = 0.}$

Note also that at that time, as can be seen from the above graph, the displacement stops growing and the particle turns around, as expected.

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Picture the Problem In the absence of air resistance, the acceleration of the bolt is constant, given by g, the acceleration caused by gravity at the surface of the earth. Let's choose a coordinate system in which upward is positive and the origin is at the bottom of the shaft ($y = 0$). [Remember that this is a choice. You could also choose a coordinate system with downward positive and the origin somewhere else. The aim here is to use a coordinate system that is *comfortable* and leads to easy calculations.]

(*a*) Write down the facts first. Don't worry about the direct question that the homework asks. You will find the answer from the facts that you write down, later.

Using a constant-acceleration equation, relate the position of the bolt to its initial position, initial velocity, and fall time:

$$\begin{aligned} y_{\text{bottom}} &= 0 \\ &= y_0 + v_0 t + \tfrac{1}{2}(-g)t^2 \\ &= y_0 + v_0 t - \tfrac{1}{2}gt^2 \end{aligned}$$

Solve for the position of the bolt when it came loose:

$$y_0 = -v_0 t + \tfrac{1}{2}gt^2$$

Substitute numerical values and evaluate y_0:

$$\begin{aligned} y_0 &= -(6\,\text{m/s})(3\,\text{s}) + \tfrac{1}{2}(9.81\,\text{m/s}^2)(3\,\text{s})^2 \\ &= \boxed{26.1\,\text{m}} \end{aligned}$$

(*b*) Using a constant-acceleration equation, relate the speed of the bolt to its initial speed, acceleration, and fall time:

$$v = v_0 - gt$$

Substitute numerical values and evaluate $|v|$:

$$v = 6\,\text{m/s} - (9.81\,\text{m/s}^2)(3\text{s}) = -23.4\,\text{m/s}$$

and

$$|v| = \boxed{23.4\,\text{m/s}}$$

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Picture the Problem In the absence of air resistance, the object's acceleration is constant. Choose a coordinate system in which downward is positive and the origin is at the point of release. In this coordinate system, y_{bottom} = 120 m at the bottom of the fall.

Express the distance fallen in the last second in terms of the object's position at impact and its position 1 s before impact:

$$\Delta y_{\text{last second}} = 120\text{ m} - y_{\text{1 s before impact}} \quad (1)$$

Using a constant-acceleration equation, relate the object's position at any given time to its initial position, initial velocity, and fall time. Then substitute the position and time you are interested in.

$$y = y_0 + v_0 t + \tfrac{1}{2} g t^2$$

or, because $y_0 = 0$ and $v_0 = 0$,

$$y = \tfrac{1}{2} g t^2$$

or, $y_{bottom} = \tfrac{1}{2} g t_{fall}^2$

Solve for the fall time:

$$t_{\text{fall}} = \sqrt{\frac{2y_{bottom}}{g}}$$

Substitute numerical values and evaluate t_{fall}:

$$t_{\text{fall}} = \sqrt{\frac{2(120\,\text{m})}{9.81\,\text{m/s}^2}} = 4.95\,\text{s}$$

We now know that, one second before impact, the object has fallen for $t_{\text{fall}} - 1$ s = 3.95 s. Using the same constant-acceleration equation, calculate the object's position $t_{\text{fall}} - 1$ s into its fall:

$$y(t_{\text{fall}} - 1\text{s}) = \tfrac{1}{2} g (t_{\text{fall}} - 1\text{s})^2$$

$$= \tfrac{1}{2} g \left(\sqrt{\frac{2y_{bottom}}{g}} - 1\text{s}\right)^2 = \left(\sqrt{y_{bottom}} - \sqrt{\frac{g}{2}}\, 1\text{s}\right)^2$$

$$= \left(\sqrt{120\text{ m}} - \sqrt{\frac{9.81\text{ m}}{2}}\right)^2$$

$$= \left(\sqrt{120} - \sqrt{\frac{9.81}{2}}\right)^2 \text{ m} = 76.4\text{ m}$$

or

$$y(3.95\text{ s}) = \tfrac{1}{2}(9.81\text{ m/s}^2)(3.95\,\text{s})^2$$

$$= 76.4\,\text{m}$$

Substitute in equation (1) to obtain:

$$\Delta y_{\text{last second}} = 120\,\text{m} - 76.4\,\text{m} = \boxed{43.6\,\text{m}}$$

Remark: In the last calculation above, I tried to show you a trick: If you do not substitute numbers until the end, but instead go on as far as possible just with algebra, it is easier to check that everything works out as it should and one is not dependent on previous numerical results. In most cases going algebraically until the end also gives more information about what is going on, or delivers a simpler expression (this is not the case for this example, though.)