TWO-DIMENSIONAL DARK SPATIAL SOLITONS

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ABSTRACT

The existence of two-dimensional odd dark spatial soliton is predicted. The exact one-dimensional hyperbolic-tangent profile of the 1D odd dark solitons seems a reasonable first-order approximation in the 2D case. Numerical and experimental results are presented demonstrating the existence of two-dimensional even dark spatial solitons. Results on mutual 2D dark soliton attraction/repulsion are discussed.

1. INTRODUCTION

Generally, the nonlinear evolution of a beam/pulse should be described by the (3+1)-dimensional Schrödinger equation. In some specific cases the dimensionality could be lowered to (1+1) (e.g. temporal evolution of pulses in optical fibers, propagation of elliptical beams in planar waveguides).

The optical solitons form a specific class of pulses/beams, for which the linear effects (group-velocity dispersion, diffraction) are canceled by the temporal/spatial self-phase modulation[1,2] Self-defocusing nonlinear media could support dark spatial solitons only, while bright and dark temporal solitons could both exist. The 1D fundamental black spatial/temporal soliton is an anti-symmetric function of space/time with an abrupt phase shift (phase step) of π and zero intensity at its center[3]. This soliton is denoted as 'black odd soliton', in contrast to the 'even dark soliton', which do not have an initial phase shift. As a consequence, independent from the background intensities the even input formations split at least into two odd dark solitons with a reduced contrast, each one with its own phase shift (less than π).

Recently, 1D dark spatial solitons are experimentally observed in the form of dark soliton stripes[4]. The influence of backgrounds of finite extent is studied in 5). The analysis[6] on the existence of stable black 2D self-supported beams stimulated this work. The goal of the present analysis is to show that 2D dark spatial solitons (2D DSS) do exist. This statement is proven by numerical simulations and by an experiment for 2D even dark spatial solitons (EDSS). Interactions, known in the 1D case, are observed to be qualitatively similar in the case of 2D odd dark spatial solitons.

2. TWO-DIMENSIONAL ODD DARK SPATIAL SOLITONS

The 2D DSS should be stable solutions of the (2+1)-dimensional NLSE[7]

$$i\partial E/\partial z + \beta\,(\partial^2/\partial x^2 + \partial^2/\partial y^2)E + kn_2|E|^2 E = 0 \quad . \qquad (1)$$

The numerical procedure used is a 2D generalization of the Split-Step Fourier Method. One dimensional versions of this procedure are frequently used for modeling nonlinear optical processes in optical fibers.

In order to ensure adequate initial conditions for the formation of a 2D ODSS the 2D spatial phase distribution, shown on Fig.1, should be imposed on a plane wavefront. Characteristic of this spiral-phase-delay is the π phase shift in each radial direction (localized at the dip center). This distribution, predicted intuitively by adding a rotational symmetry to the transverse phase-distribution of an 1D odd dark soliton[3] was proven numerically to be adequate in two spatial dimensions. We analyzed the nonlinear evolution of an input intensity dip of the form

$$E(r,\varphi,z{=}0) = A\,B(r)\tanh(r/r_0)\exp(iF(r,\varphi)) \quad , \quad r = (x^2 + y^2) \qquad (2)$$

with a phase distribution

$$F(r,\varphi) = m\varphi \qquad , \varphi \in (0\,,\,2\pi) \qquad (3)$$

imposed on a super-Gaussian background beam

$$B(r) = \exp\{-(r/[15r_0])^{16}\} \qquad (4)$$

In Eqs.(2-4) r and φ are the radial and azimuthal coordinate, respectively, and m is an odd number. In order to avoid a dip-to-background interaction, the width of the background beam is chosen 15 times higher than the width of the dark formation.

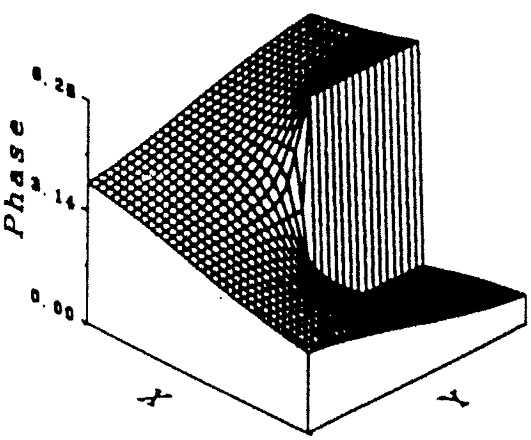


Fig. 1

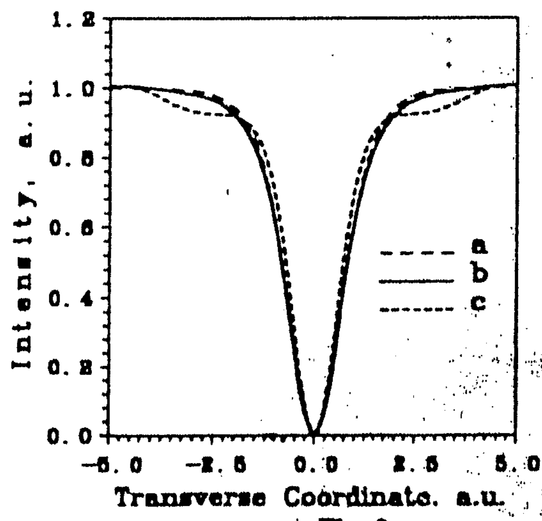
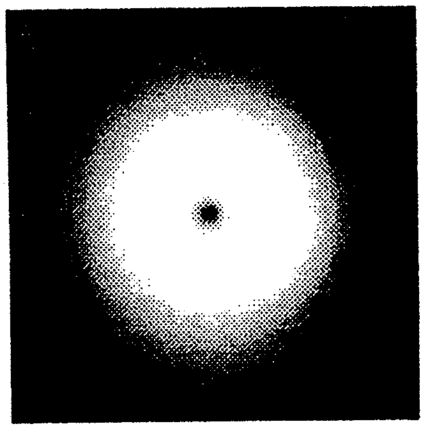


Fig. 2

Fig 2 shows a radial cross-section of the input (z=0) 2D dark odd formation (long dashed line) The solid curve demonstrates the 2D dark beam shape at five nonlinear lengths The shape of the resulting 2D dark formation as well as its phase portrait closely reproduce the initial distributions, given by Eqs.(2-4). The smoother wings of the 2D formation indicate, that the input 2D hyperbolic-tangent profile slightly differs from the exact one for the 1D case (dashed line). In the absence of exact analytical and approximate numerical results in 2D, this approximation seems reasonable. It is interesting to note that the background intensity required for obtaining a 2D ODSS is found to be $\sqrt{2}$ times higher than the respective value in the 1D case.



Exceeding more than twice the background intensity for the fundamental 2D ODSS results in an evolution of the input single odd dark formation into an on-axis fundamental 2D ODSS and a diverging 'gray' fringe (Fig. 3). This behavior, known for the 1D ODSS[3)] strongly supports the statement, that 2D ODSS should really exist. Mutual 2D dark soliton attraction and repulsion, known for 1D bright solitons, will be discussed in Sec. 4.

Fig. 3

3. TWO-DIMENSIONAL EVEN DARK SPATIAL SOLITONS

We modeled numerically and observed experimentally the evolution of 2D EDSS. The input conditions are described by Eqs.(2-4) at F=const. 2D EDSS are easily produced experimentally (Fig.4) by imposing 2D amplitude masks in front of a thermal nonlinear medium[8,4)] (ethanol slightly dyed in red). For the purpose of a comparative experimental analysis, the masks consisted of pairs of reflecting dots and stripes of equal diameters/widths ranging from 50μm to 250μm. A copper vapor laser source (P=4W) was used to produce the background signal. This technique is routinely used for generating of 1D dark spatial solitons (dark soliton stripes). The evolution of the input dark formations is recorded by a CCD camera and a frame-grabber.

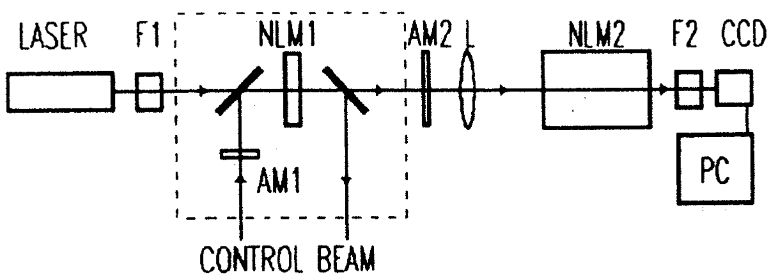


Fig. 4

The dashed square indicates a section of the experimental setup, in which we intend to produce the desired spiral phase distribution (Fig.1) by a second control beam being intensity modulated with a circular gradient neutral-density filter. The physical mechanism of this phase modulation is the induced-phase modulation in a separate nonlinear medium (NLM1).

A strong evidence of the soliton nature is the transverse velocity of the dark spatial soliton pairs[4,8,9)] Fig.5a plots the EDSS transverse velocity related to the divergence angle between the two gray odd solitons of the same pair as a function of the normalized width/radius of the input 1D/2D input dark formations. The solid curve shows the result of the 1D theory of Zakharov and Shabat[9], whereas the triangles denote our numerical results in 1D case. The little bit higher deviation from the theory at small stripe widths is a result of the computer-limited discretization and has no physical meaning. The squares denote our results in the 2D case. As seen, the transverse splitting velocity is always higher in the 2D case and, qualitatively, has the same behavior.

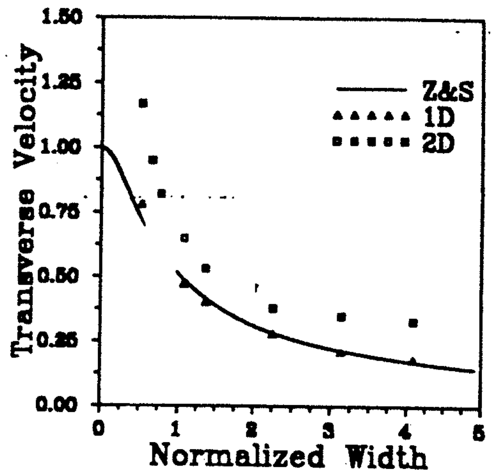


Fig. 5a

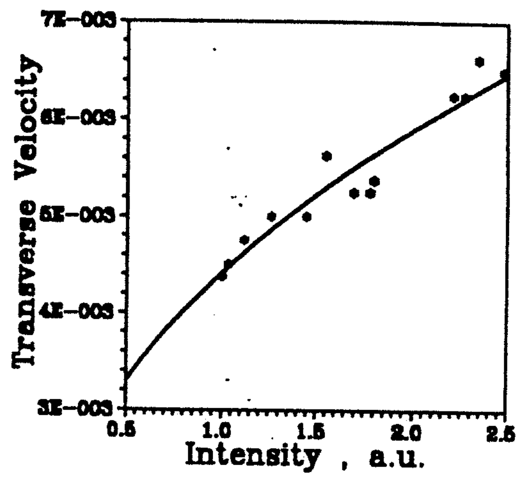


Fig. 5b

Fig. 5b plots the intensity dependence of the transverse velocity at a fixed value of the input 2D dark beam diameter. The dots present the experimental values. The first six of them are extracted from the recorded gray-scale images shown on Fig. 6. The solid line is a numerical result.

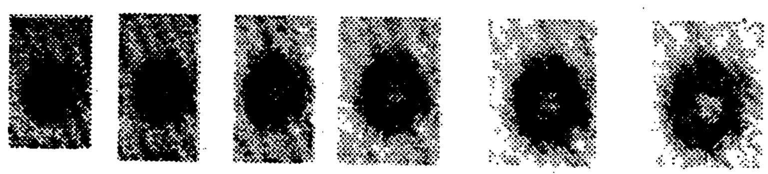


Fig. 6

Some characteristic pictures observed experimentally by increasing the background intensity are shown on Fig.7 - 2D dark formation with a diffraction compensated by the nonlinearity (Fig.7a), 2D ODSS-splitting resulting in a diverging dark ('gray') ring with a center intensity higher than the background level (Fig.7b), dark ring with a central dark dot (Fig.7c). The interference lines resulted from quvette input and output faces and were difficult to avoid. Nevertheless, the results from Fig.7 are indicative for the modulation stability of the 2D ODSS.

Our numerical simulations (Fig.8a-c), confirm that, after the first dark ring formation, the center intensity should be expected to be higher than that of the background. In a qualitative agreement with the results of other authors, the numerical results obtained demonstrated the existence of phase jumps (less than π) across the 2D dark ring formation. The longitudinal reduction (along the nonlinear medium) of the dark ring contrast, however, is a serious difference in comparison to the splitting of 1D EDSS into a 1D dark soliton pair.

Fig.7

(a) (b) (c)

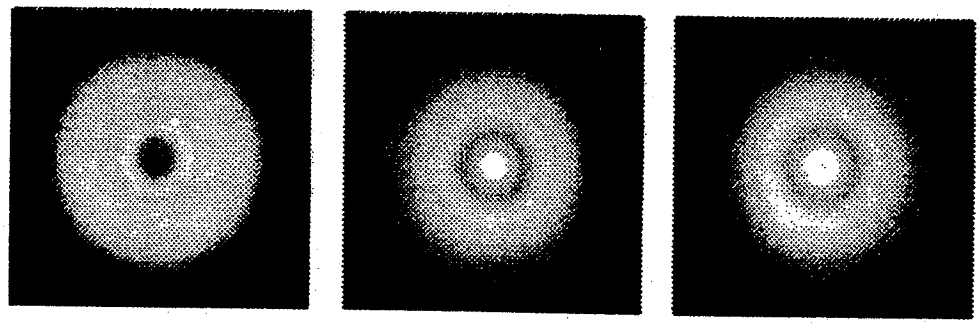


Fig. 8

4. 2D ODD DARK SOLITON INTERACTIONS

Under the slowly-varying envelope approximation, the spatial self-action and the interaction of 2D optical beams along a nonlinear medium are described by the incoherently coupled nonlinear Schrödinger equations[10)]

$$i\partial U/\partial\xi \;-\; (1/2)\Delta U + R_U\left\{|U|^2 + \sigma\,|V|^2\right\}U = 0 \qquad , \qquad (5)$$

$$i\partial V/\partial\xi \;-\; (1/2)\Delta V + R_V\left\{|V|^2 + \sigma\,|U|^2\right\}V = 0 \qquad , \qquad (6)$$

where $U(x,y,\xi)$ and $V(x,y,\xi)$ are the beam envelopes, $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $R_{U,V}$ denotes the ratio between the nonlinear and diffraction lengths for each beam, and σ is the cross-phase modulation (XPM) coefficient.

The 2D interactions described are critically assessed in view of their qualitative agreement with 1D results of other authors. Particularly, we proved the possibility a XPM-assisted 2D vector dark solitons to be achieved[10)] Once again, the hyperbolic-tangent approximation was found to be a reasonable one in 2D.

Further, we modeled the evolution of two axially offset (δ=0.5) 2D odd dark spatial solitons. Obviously, this beam misalignment breaks the radial symmetry of the problem. Despite the background's self-defocusing and shape changes, two 2D beam trappings are observed near ξ=3 and ξ=9. This 2D beam attraction and repulsion (ξ=6) is visualized on Fig.9a (central slices along the interaction axis). Qualitatively, the process seems analogous to the evolution of 1D dark/bright beams/pulses[11)]. Two characteristic grayscale images of both beams are shown on Fig. 9b at ξ=3. The radial interaction-asymmetry, however, leads to an interesting side-effect.

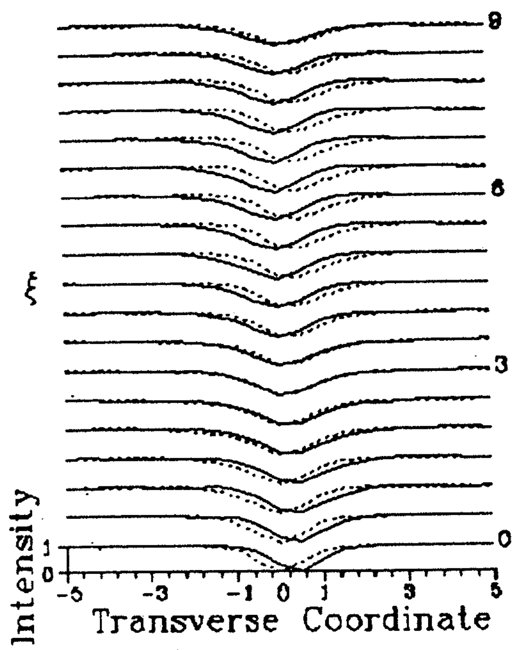


Fig.9a

Two spiral structures are formed on the backgrounds around each of the dark beams. Moreover, they form themselves and continuously rotate in opposite directions along the nonlinear medium.

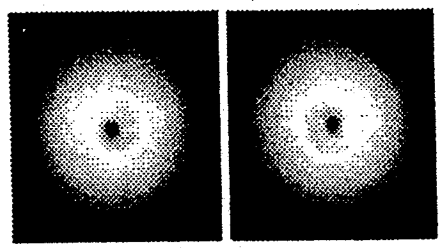


Fig.9b

5. CONCLUSION

We presented numerical and experimental results supporting the statement, that 2D odd and even dark spatial solitons do exist. The data obtained seem fundamentally important since no analytical solutions of the 2D NLSE are known. Two interaction configurations of 2D ODSS are analyzed numerically. A good qualitative agreement with 1D results of other authors is found. The results may appear practically important, for instance, if multiple 2D ODSS are used to guide synchronously (in time) pulses of information.

6. ACKNOWLEDGMENTS

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