Robust Power Generation Planning Under Demand Uncertainty

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Introduction

An electric company faces the problem of generating power supply at minimum cost in presence of demand uncertainty. We present a robust optimization approach to this problem, where we model the random demand as an uncertain parameter belonging to a known range [1]. We further add constraints, called budget-of-uncertainty constraints. This allows the company to adjust for its level of risk aversion. Our objective is to minimize the worst-case production cost.

Problem Statement

- How do we incorporate demand uncertainty in power generation planning?
- Which power generators should be used in each period in order to minimize the total daily cost?

- Daily electricity demands (in megawatts)

Time period	0am-6am	6am-9am	9am-12pm	12pm-2pm	2pm-6pm	6pm-10pm	10pm-12am
Nominal demand	12000	32000	25000	36000	25000	30000	18000

- Types of power generators available

Generator Type	Available number	Min output (MW)	Max capacity (MW)	Fixed cost ($/h)	Add. MW cost ($/h)	Start-up cost ($)
1	10	750	1750	3024.23	2.7	6720.50
2	4	1000	1500	2419.38	2.2	2150.56
3	8	1200	2000	5040.38	1.8	3225.84
4	3	1800	3500	6451.68	3.8	1612.92

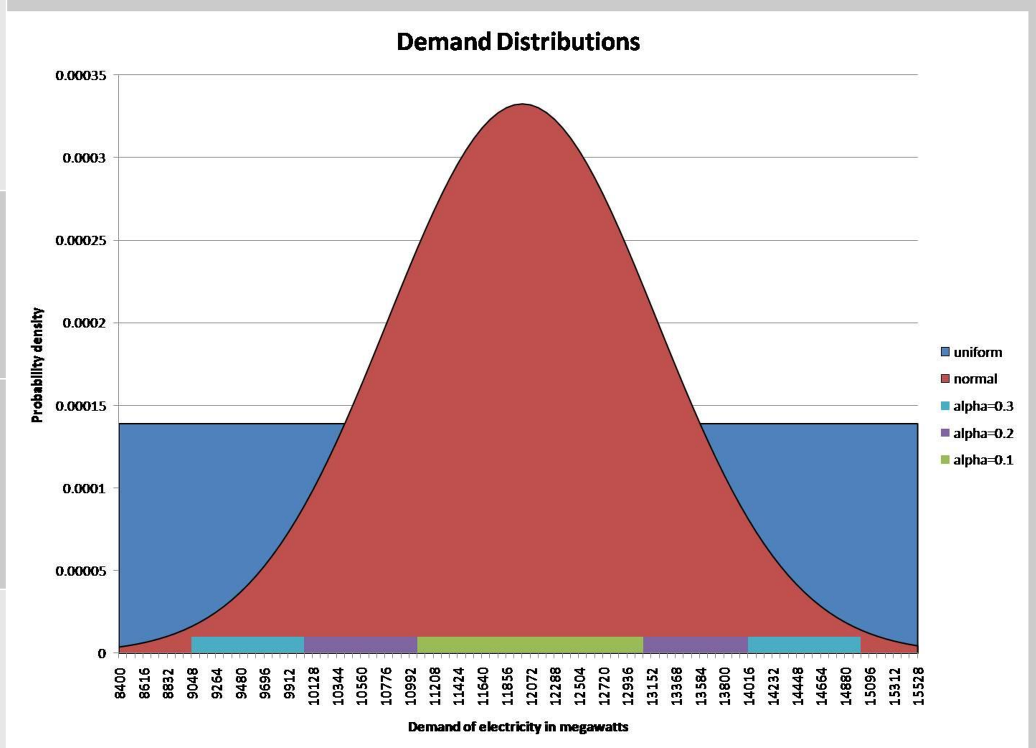
Motivation

- Insufficient capacity can lead to blackouts, while over-capacity increases costs.
- Demand for electricity is volatile and is hard to estimate beforehand.
- Traditional methods rely on a probabilistic description of uncertainty, which suffers from tractability issues.
- Probability distributions tend o have complicated formulas that might be time-consuming for managers to fully understand.

Methodology

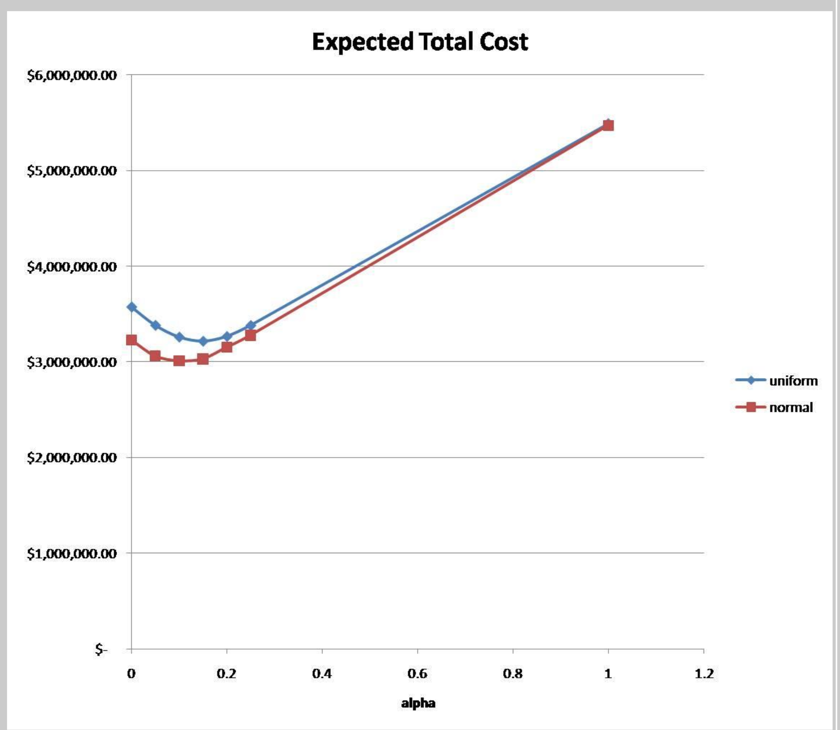
1. Formulate mathematical model	• Minimize production cost • Subject to constraints of the generators
2. Apply robust optimization	Add budget-of-uncertainty constraints, which limit the maximum scaled deviation of the aggregate demands to a parameter (the budget of uncertainty) selected by the decision-maker, up to the current time period. • Demand constraint: $\sum_{p=1}^{gen}(pmin_p * y_{pt} + u_{pt}) \geq d_t, \forall t$ • Aggregate demand: $\sum_{s=1}^{t}\sum_{p=1}^{gen}(pmin_p * y_{ps} + u_{ps}) \geq \sum_{s=1}^{t} d_s, \forall t$ • Random demand: $\sum_{s=1}^{t} d_s = \sum_{s=1}^{t}(\overline{d_s} + \widehat{d_s} * z_s), \forall t$ $\Downarrow$ $\sum_{s=1}^{t} d_s = \sum_{s=1}^{t}\overline{d_s} + \alpha * A_t, \forall t$
3. Simplify formulation	The mathematical model must be solvable by existing solvers
4. Analyze solutions	• Generate “actual” demands based on a uniform and a normal distribution • Compare solutions of the robust approach to the nominal case
5. Perform numerical experiment	• Run 50 test cases • Find expected total cost

Graph below shows the demand distributions used in generating actual demands. The alpha values correspond to scaled deviations from the mean demand. Ex: alpha = 0.1 means that the demand falls within 10% of the mean.

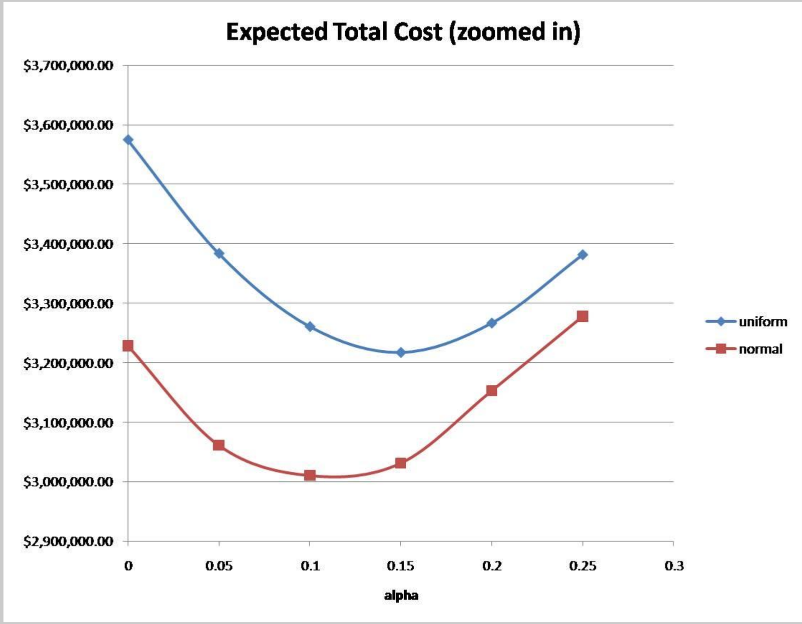


Results

This graph shows the expected total cost for each alpha value.



The Expected-Total-Cost graph is zoomed in below to show the minimum values.



By applying robust optimization, we see significant cost reduction.

	Uniform	Normal
Cost reduction comparing to the nominal case	$ (268,198.55)	$ (163,351.59)
Percentage change with respect to the nominal case	-10%	-7%

Conclusion

Our research makes two important contributions to the literature.

1. From a modeling standpoint, robust optimization has previously shown poor performance for the type of problems we are considering because of the problem structure [2]. We enforce problem constraints for the nominal demand and add aggregate demand constraints for which we apply the robust optimization approach.

2. From an implementation perspective, our results indicate that the width of the range forecast can be chosen to capture the shape of the demand distribution. Numerical experiments show that our robust optimization approach reduces expected cost while protecting the system against demand uncertainty.

Future Work

Add constraints to the optimization model that force the generators to be up or down for a minimum number of periods because it takes time for generators to cool down.

Acknowledgement

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References

1. D. Bertsimas and A. Thiele. A Robust Optimization Approach to Inventory Theories. (2006) Operations Research 54(1):150-168.
2. D. Bertsimas and M. Sim. The Price of Robustness. (2004) Operations Research 52(1): 35-53.