What is calculus?

Calculus is a branch of mathematics that deals with rates of change. Its roots go back as far as Ancient Greece and China, but calculus as we know it today began with Newton and Leibnitz in the 17th century. Today it is used extensively in many areas of science, as well as in economics and business. Basic ideas of calculus include the ideas of limit, derivative, and integral. The derivative of a function is its instantaneous rate of change of the output, with respect to the input. Thus, the derivative of height (with respect to position) is slope; the derivative of position (with respect to time) is velocity; and the derivative of velocity (with respect to time) is acceleration.

The integral of a function can be thought of as giving the area under its graph of the given function, or as a sort of weighted total over time. Thus, the integral of slope is (up to a constant) height; the integral of velocity is, up to a constant, position; and the integral of acceleration (with respect to time) is velocity. As you may have guessed, integrals and derivatives are related, and are in a sense, opposites.

Now, many functions (though not all) can be represented by algebraic expressions. For instance, the area of a circle is related to its radius by the formula $A = \pi r^2$ and the distance that a body falls in a time $t$, starting at rest, is given by $x = \frac{1}{2}at^2$. Given such an expression, calculus allows us to find expressions for the integral and derivative of the function, when they exist.

Why is calculus important?

In the sciences, many processes involving change, or related variables, are studied. If these variables are linked in a way that involves chance, and significant random variation, statistics is one of the main tools used to study the connections. But, in cases where a deterministic model is at least a good approximation, calculus is a powerful tool to study the ways in which the variables interact. Situations involving rates of change over time, or rates of change from place-to-place, are particularly important examples.

Physics, astronomy, mathematics, and engineering make particularly heavy use of calculus; it is difficult to see how any of those disciplines could exist in anything like its modern form without calculus. However, biology, chemistry, economics, business, computing science, and other sciences use calculus too. Many faculty, therefore require a calculus course from all their students.

It should be understood that there is more to mathematics than calculus. Linear algebra, probability, and geometry are just some of the other branches of mathematics, introduced in your earlier education, that are important at the university level. And problem-solving skills, which cut across all branches of mathematics, allow the mathematics to be applied to other subjects.
Additional Statements on the Importance of Calculus:

Business without calculus is like baseball without the ball. Yes you have some of the equipment to play. You can run the bases, swing the bat, and wear the uniform, but you really aren’t playing the game. There is no game winning hit or game saving plays without the ball. In business, you can count your sales and account for your profit or loss; but it is mathematical concepts from calculus that allow you to understand and manage your risks. How much will sales go up with a change in price is the derivative of sales with respect to price. What is the optimal amount of goods to be produced is found by taking a derivative and setting it equal to zero. Making good decisions, gaming winning plays requires an understanding of the risks businesses face and to understand the risks, it sure helps to understand calculus.

David Myers, Lehigh University, Professor of Finance

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On the importance of math (particularly calculus) for undergraduate business majors:

Students who elect to enroll in an undergraduate business program do so (presumably) with an eye to a future career in a related field. With that in mind, the curriculum is tailored to providing graduates with the skills necessary to excel in a business environment. Admittedly, the term “business” is rather broad; however, the same basic skill-sets are required for all fields, from finance to marketing, economics to management.

One skill that is lacking is an understanding of basic calculus (and algebra) concepts. A familiarity with the basic principles of functions, limits, derivatives, integrals, and infinite series should be the (mathematical) foundations upon which students build their curriculum. The logic and problem solving skills that come from such an understanding will help students in their future pursuits, regardless of the “field” upon which they decide.

As someone who deals in financial derivative structuring and trading, my calculus (and calculus-based statistics) classes and been invaluable tools that I use daily at work. In the seven months I’ve been working, I’ve been asked to explain basic calculus concepts (as they apply to risk modeling) to both my peers and senior management.

Having an understanding of basic math (calculus and probability) is something all undergraduate students should be able to claim upon receipt of their degree.

-Jim Beattie, Lehigh ’07, B.S. Finance, B.S. IBE Information Systems Engineering

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Lehigh University business and economics students should learn calculus not only because it is one of the major intellectual achievements of world civilization, but also because understanding calculus makes studying economics much easier. One fundamental microeconomics question is whether a seller’s price increase leads to increased profits. With profit as a function of price, answering this simple question amounts to calculating the derivative of profit with respect to price. Another straightforward microeconomics pricing issue is calculating the annual fee that a health club should charge its members. The answer to this question involves integral calculus, or summing over health club visits, a member’s willingness-to-pay.

Economists use calculus to “model” many economics issues that involve supply and demand, investors making portfolio choices, firms interacting strategically with each other, and industry growth and decline. Economists use these math-based models to analyze the world, identify the most important factors that affect different economic outcomes, and make predictions.
These models are more easily mastered by students who can think abstractly and have a working knowledge of calculus. Thus, for students of economics and business, calculus is a tool, much like a hammer or a screwdriver. James Dearden and Mary E. Deily, Lehigh University, Professors of Economics

What background do I need?

You should have taken Grade 12 pre-calculus mathematics, or an equivalent course, and understood the material. You do not need a high school calculus course, however; if you have taken one, you should not assume that you can skip classes, or not study, during the first part of the course. University calculus goes deeper than most courses in high schools do, even if they appear to cover the same material. If you have been lucky enough to have a calculus course in high school, consider it as a preview of the upcoming feature that you are now about to see! And remember to keep using your algebra and other mathematical skills, to keep them sharp.

What is taking university calculus like?

You will probably find that university calculus is faster-paced than your high-school courses. At most universities the lecture sections will be bigger and the professor will not be able to slow the class to the pace of the slowest students. You — and you alone — will be responsible for handing work in on time, and for being present for classes, tests, and exams. This is not a course that can be passed by just memorizing everything; you have to understand it. This won’t happen instantly, and the instructor cannot make it happen; you have to do that yourself, and be an active participant in the course. If you make this effort consistently through the term, you will find that it pays off.

The course material consists of a rather small number of big ideas, and a moderate number of formulae you will need to know and learn to apply, not hundreds of short cuts and special rules. A common mistake, especially with word problems, is trying to learn one “plug-in” rule for each different kind of problem that you might encounter. Don’t do that; instead, try to understand the underlying patterns.

Here are the stages by which you will learn and master a new idea in calculus (or any other college course).

• First, you should read ahead in the textbook, so that you have at least a rough idea of what the instructor is going to say. This will help you follow the lecture and make note-taking easier.

• In the lecture, the instructor will introduce the new idea, give some examples, and perhaps explain how it fits in with other ideas or give some problem-solving tips.
At this point, a few students think the process is over till the final exam and that if they don’t know everything then, it will be the instructor’s fault. Not so; the ball is back in your court! Work on problems, do the assignments (including additional study problems if you need to, whether assigned or not), and make sure that you understand what you are doing. It’s better to work for an hour or so several times a week than in one killer session. This is the heart of the learning process; the lecture is just to help you get started.

If you don’t understand something, decide what it is that you don’t understand and go for help. Go prepared to tell the instructor (or friend, tutor, learning center person, etc.) what it is that you don’t understand. (“Everything” is not helpful.) On the other hand, do not go along demanding that the instructor tell you how to answer Question 6.11 and refusing to listen to any explanation of why it’s done that way.

Finally, your knowledge will be tested on quizzes, midterms, or final examinations. The exams will probably make up the majority of your grade.

If you have handed in copied, half-understood assignments, you will lose more marks at exam time than you gained in the short term. But if you kept up with the lectures and assignments, making sure you understood each bit before going on, you will probably do well.

If you find yourself falling behind, you will have to catch up. Don’t panic - catching up is not impossible. There are various resources to help.

- **Yourself.** If you are falling behind and not putting plenty of time in studying - say five or six hours outside class per week for each course - the solution may be as near as your desk and textbook.

- **Your textbook** contains hundreds of worked examples, and thousands of problems. Usually, about half of the problems have answers in the back of the book; and you may be able to buy a study guide that shows the working of those problems in more detail. You can also get other books such as “Schaum’s Outline of Calculus” containing more worked problems.

- Your instructor will have **office hours** during which you can go for help. Try to figure out ahead of time what you need help with; you will get more out of the visit.

- **Discuss** your problem with your classmates. This is not the same thing as copying their assignments, of course! University penalties for cheating are severe, and can include expulsion.

- You might want to get a **tutor.** Do not try to get the tutor to do your assignments for you; you will not be able to bring the tutor into the exam with you! Get the tutor to make sure you understand the material.

- Our university has a **math learning center,** group tutoring sessions, and other resources that you can use. Take advantage of them!

- Our university has a Center for Academic Success where they are prepared to assist you with study skills, effective note-taking, exam nerves, etc. There is also counseling available for other problems that might interfere with your study.
HELP! I have to take a placement test!

We have found that incoming students are not uniformly well-prepared for calculus. Many students are very well prepared, and ready to start the usual calculus curriculum; but some are not. As a result, a placement test is required. The result of that exam should be used to guide you to the appropriate level of math course. This test is designed to stop students from starting calculus without adequate preparation and then falling behind and failing. Experience has shown that most students who start calculus without adequate preparation do fail.

HELP! They want to take my calculator away!

In our first-year calculus courses, calculators are not permitted. To be exact, they do not permit them in the pretest, tests, or exams; you can certainly have one in the lecture if you want, or use one for your homework.

• Since our calculus courses have a no-calculator policy, the questions will be designed so that a calculator is not needed. It would be unreasonable for the professor to ask you to compute \((1.42433x + 2.4577)^3\) without a calculator. But (s)he won’t. You might be asked to compute \((x+2)^3\); the calculator will not be needed here.

• In a situation where calculators are not permitted, any expression that does not have a well-known simplification may always be left as it is. Thus, for instance, while you are expected to know that \(\sqrt{25} = 5\), and that \(\log_{10} 1000 = 3\), you may always leave (eg) \(\sqrt{17}\) or \(\ln(1000)\) in those forms. Now, that is even easier than using the calculator! Do note, however, that you would generally be expected to simplify (eg) \(\sqrt{17}/\sqrt{17}\) to 1, should such an expression arise!

The use of algebra (as opposed to arithmetic) to simplify expressions is common in our courses. Conventions as to what should and should not be simplified will be made clear during the course.

• Even though our calculus course does not allow calculators, you will often be required to give, as an answer, an algebraic expression involving integers. For instance, you might be asked: “Give the answer as an expression of the form \(\sqrt{b/c}\) where \(b\) and \(c\) are integers.” It is far more important, for the purposes of calculus, to know that \(\sin (\pi/4) = \sqrt{2}/2\), and why, than to know that it is approximately equal to 0.707.

When you get into courses where there is a reason for test questions to involve significant amounts of arithmetic, you will be allowed — and encouraged — to use a calculator, perhaps even a computer. In the meantime, if you are not allowed to use a calculator in tests, you can be fairly sure that the arithmetic will be kept simple.

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