Galois Groups of CM Fields in Degrees 24, 28, and 30

Alex Borselli

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The study of CM Fields arose from a generalization of the theory of complex multiplication in elliptic curves. A CM field is a totally imaginary number field K, which is a quadratic extension of a totally real field K_0. As totally imaginary number fields, K can be embedded in $\mathbb{C}$, and complex conjugation on $\mathbb{C}$ induces an automorphism of K. By Galois theory and complex conjugation, the Galois group, G, of a degree $2n$ CM field, K, is a transitive permutation group with an even order center. A CM type, Φ, is a set of n non-conjugate embeddings of K in $\mathbb{C}$. We can thus find the orbits of Φ under the action of G and classify the pair (K, Φ) as degenerate if there is an orbit of rank less than $n+1$. I will first present some elementary computations in degree 8 by Dodson, and then I will give my results in degrees 24, 28, and 30 that were motivated by the work of Dodson and Zoller.