An Introduction to Galois Groups of CM Fields

Alex Borselli

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The study of CM Fields arose from a generalization of the theory of complex multiplication in elliptic curves. A CM field is a totally imaginary number field K, which is a quadratic extension of a totally real field K_0. As totally imaginary number fields, K can be imbedded in $\mathbb{C}$. So, complex multiplication on $\mathbb{C}$ induces an automorphism of K. The Galois group of a degree m Galois extension of $\mathbb{Q}$ is always a transitive permutation group, which is just a subgroup of the symmetric group on m elements, S_m. When looking at Galois groups of CM fields, because of the induced automorphism by complex conjugation, these will be transitive permutation groups with even order centers. In this talk, I will discuss permutation groups and give a basic introduction to CM fields. Then, I will describe some important types of transitive permutation groups, called minimally transitive permutation groups, and their analog under the consideration of even order centers due to their importance with respect to CM fields.