Capabilities and Limitations of Infinite-Time Computation

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4/14/2014

Traditional "finitary" computability studies which problems can be resolved algorithmically in finitely many steps. My research in infinitary computability entertains the broader question of which problems can be handled via "supertask algorithms," which allow infinitely many steps of computation. This field has enjoyed a surge of interest recently, in part because of an ongoing debate among physicists as to whether supertask machines are indeed realizable.

The component of my research which has produced the most polished and definitive results to date develops several new objects of study, prominent among which are infinite-time analogues of Rado's famous busy beaver functions from finitary computability. Much as the classical busy beaver functions furnish natural examples of functions which cannot be computed via finitary algorithms, I have shown that my infinite-time busy beaver functions cannot be handled by even infinitary algorithms, and in fact, can be proven (in a certain formal sense) to be just as difficult to compute as it is to resolve a certain infinite-time analogue of the Halting Problem. These results serve as a partial characterization of some of the limits of infinite-time computations, and also have yielded a slew of open questions which I seek to resolve as part of my dissertation work.

This talk serves to outline a small, but illustrative, portion of my current results, as well as some closely related topics of future study. While no previous knowledge of infinitary computability is assumed here, some exposure to classical finitary computability, especially the Turing machine model, might prove helpful.