Economic Cycles: Asymmetries, Persistence, and Synchronization

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Abstract. Marking upswings and downswings for a time series $\{y_t\}$ provides insights that are not immediately obvious, but may be meaningful to academics, policymakers, and the general public. The mean and standard deviation of durations, as well as the amplitude and steepness of a given phase, yield fruitful insights about cycle asymmetries and persistence. Expansions and contractions in one series can then be compared to those in another to determine whether their respective cycles are synchronized. However, our primary focus here is on classical nonparametric methods for the analysis of duration, dating back to the seminal work of Burns and Mitchell (1946), Cutler and Ederer (1958), Bry and Boschan (1971), and Cox (1972). Although our specific application is to unemployment cycles, the ideas and techniques discussed in the chapter apply to a wide variety of both micro- and macroeconometric studies.

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1. Introduction

An event history is a longitudinal record of specified events. Examples at the individual level include dates of schooling, marriage, birth of children, job change, illness, promotion, retirement and migration. Examples at the aggregate level include dates of militarized disputes, riots, revolution, economic expansions and contractions, bull and bear markets, and massive layoffs. Regardless of the underlying activity, an event consists of a qualitative change that marks a new phase for either the individual or collective. Consider that the beginning of a bull market marks the end of a bear market and that the beginning of a militarized interstate dispute marks the end of placid relations between states with diplomatic ties.

A pertinent issue is whether the probability of exiting a phase, or *state*, depends on its duration. For instance, is a four-hour riot more likely to end within the next hour than a one-hour riot? Does the likelihood of changing jobs decrease with the time invested in the current job? Is a young economic expansion more robust to failure than an old one? If so, we anticipate the end of an old expansion, but are surprised by the end of a young one.

The demographer sees each of the above questions answered best by a life table analysis, the biostatistician by a survival analysis, and the engineer by a reliability or failure-time analysis. The economist tends to view the individual-specific questions on job change as somehow different from the aggregate-level question on business cycles, with the question on riots fitting perhaps somewhere in between. In fact, these questions are generally handled by various sub-disciplines, as the microeconometrician examines individual employment while the macroeconometrician examines the overall business cycle. Despite the initial segregation, however, it is now understood that life-table, survival, and reliability analyses are intellectually very similar, and the analysis of business cycles can be handled in much the same way as that involving promotion and job change.[1]

2. Marking time

One important distinction between microeconometric and macroeconometric event studies lies in the complexity of marking time. For individual level or microeconometric events such as marriage, job change, promotion, birth and death, it is relatively easy to pinpoint when the qualitative change occurs. In contrast, for national unemployment, economic expansions and contractions, and bull and bear markets, the timing of change is less certain. Consider the Business Cycle Dating Committee of the National Bureau of Economic Research (NBER). The committee is comprised of experts who mark time - that is, mark the *turning points* - in economic activity by consensus. They examine trends in real GDP, real income, employment, industrial production and wholesale-retail sales, in conjunction with their collective reasoning that an economic contraction, or *recession*, must last more than a few months.[2]

The NBER derives its method from the graphically oriented approach of Burns and Mitchell (1946), who define the *classical cycle*. The graphical approach is based on marking turning points for several specific cycles, then aggregating that information to form a reference cycle. In formalizing the graphical approach, however, Harding and Pagan (2002) observe that Burns and Mitchell would have *preferred* to use a single series, namely GDP, had that been available to them. The single series approach seems especially appropriate for low frequency data.

Harding and Pagan (2002) thus employ a modified version of the Bry and Boschan (BB) (1971) algorithm to mark turning points for quarterly GDP observations. Their BBQ algorithm again leads to the so-called classical cycle, though defined in a more rigorous, non-qualitative manner.[3] Harding and Pagan (2006) use a similar nonparametric approach to codify the methods for deriving the NBER reference cycle. Regardless of whether single or multiple series are employed to mark time, Harding and Pagan observe that using algorithms is largely immune from deleterious compositional effects of dating committees, such as the NBER Dating Committee. Consider that Artis, Marcellino and Proietti (2004) employ a modified version of BBQ for use in their study of European business cycles.

Parametric models complement the nonparametric analysis. Pagan (1997) examines some simple linear statistical models and shows they are capable of replicating the observed phase *durations* of classical cycles in Australia, the United Kingdom and the United States. He demonstrates that realistic linear statistical models of national output have 1) deterministic trend growth, but of low magnitude, if any; 2) near-unit-root behavior in the deterministically detrended data, if not exactly unit-root behavior; and 3) innovations of a certain magnitude. Economic

[1] For the reader interested in a concise discussion focused on measuring business cycles, see Harding and Pagan (2008).

[2] The NBER URL is http://www.nber.org/cycles.html

[3] The Bry-Boschan (BB) program was originally designed for monthly data. James Engel, Don Harding and Mark Watson have each written or modified GAUSS programs that are similar to BB. The "Q" in the moniker BBQ stands for "quarterly" intervals even though BBQ easily accommodates either monthly or quarterly intervals. Our BBQ program is a derivative of the one written by Don Harding and is available upon request.

models with final equations for output that match these specifications are observationally equivalent, and this explains why King and Plosser (1994, p. 436) find it difficult to distinguish between the Klein-Goldberger model and a neoclassical real business cycle model. Despite potential identification problems, however, it is only by melding quantitative analysis with economic theory that one can hope to distill the relative importance of monetary, real, expectational, and international shocks. To do so, Harding and Pagan (2000) emphasize the production of statistics that directly address the ability of a model to replicate business cycle characteristics.

2.1. Reasons for marking time. Let $y_t = \log(GDP_t)$, and consider mapping $\{y_t\}$ to a binary time series, $\{S_t\}$, with $S_t = 0$ for recessions and $S_t = 1$ for expansions. Pagan (2004) and Harding and Pagan (2007) offer the following rationale for the reader's interest in $\{S_t\}$:

(1) *S_t frequently emphasizes features of y_t that are not immediately obvious.* Observing the behavior of y_t over different phases affords us a better understanding of its features.
(2) *S_t may be more meaningful to decision makers than y_t.* Reaction to an economic downturn is often strong among the electorate, and it is of interest to determine whether the probability of exiting a downturn depends on how long one has been in it and whether such exit probabilities, or *hazards*, have changed in fundamental ways in recent history.
(3) *S_t may be the object of interest if questions are asked about the synchronization of cycles across sectors or countries.* If S_t is derived from many underlying series, it is often more convenient to compare the representative S_t values than to compute a large number of correlations from the underlying series.
(4) *S_t is generally more robust than y_t to relatively unimportant short-lived shocks.* Such shocks may substantially affect statistics based on the GDP growth rates but have little impact on overall trends. In contrast, S_t emphasizes the qualitative trend, up or down.

2.2. Techniques for marking time. In short, mapping $\{y_t\}$ to $\{S_t\}$ yields fruitful insights about cycle asymmetries, persistence, and synchronization. Some nonparametric rules for marking time are exceptionally simple. For yearly aggregate data, Neftci (1984) and Cashin and McDermott (2002) employ the calculus rule, $S_t = 1(\Delta y_t > 0)$. For quarterly data, one often-used rule in the popular press is the extended Okun rule. The rule states that, for a recessionary phase, termination is signified by two successive quarters of positive growth, $(\Delta y_{t+1} > 0, \Delta y_{t+2} > 0)$. Similarly, for an expansionary phase, termination is signified by two successive quarters of negative growth, $(\Delta y_{t+1} < 0, \Delta y_{t+2} < 0)$. The simplicity of such rules is very attractive, and any limitations of such rules are quickly revealed by visual inspection of the observed series. In fact, regardless of the rule employed, the constructed turning points should visually coincide with those apparent in a plot of the observed time series, $\{y_t\}$.

2.2.1. *BBQ.* To locate turning points in the *level* of GDP, the BBQ algorithm first determines a potential set of local peaks and troughs. Time t is a local peak if

$$(y_t - y_{t-2} > 0, y_t - y_{t-1} > 0, y_t - y_{t+1} > 0, y_t - y_{t+2} > 0)$$

with the inequality reversed for troughs. The algorithm ensures that peaks and troughs alternate, so that an expansion is immediately followed by a contraction, and vice versa. Finally, the algorithm considers combining phases, or creating new phases, according to a set of predetermined rules. For instance, a censoring rule for business cycles is that either a contraction or expansion must last a minimum of 2 quarters and complete cycles must last a minimum of 5 quarters.

BBQ can also mark turning points for other types of series. For example, to locate a potential peak for the *growth* cycle in GDP, replace the requirement that $y_t - y_{t-2} > 0$ with the requirement that $\Delta y_t - \Delta y_{t-2} > 0$, and so on. Pagan and Sossounov (2003) modify BBQ to factor in the *magnitude* of growth rates in financial series. Other applications of nonparametric methods to mark the turning points include Lunde and Timmermann (2004) and Ohn, Taylor and Pagan (2004), who investigate bull and bear markets; Cashin, McDermott and Scott (2002), who investigate booms and slumps in commodity markets; Eichengreen, Rose and Wyplosz (1995), who investigate exchange rate crises; and Ibbotson, Sindelar and Ritter (1994), who examine hot and cold IPO markets.

2.2.2. *Markov chain models.* Hamilton's (1989) innovative Markov chain switching-regime model can also be employed to mark time. For the parametric switching-regime model, a latent random variable, s_t^*, governs the state or *regime* with, say, $s_t^* = 0$ indicating low or negative average growth, and $s_t^* = 1$ indicating high or positive average growth. Two states, signifying negative and positive average growth rates, are adequate to mark the turning points since $\Delta y_t < 0$ indicates a downswing and $\Delta y_t > 0$ indicates an upswing in the level of GDP.

Consider a simple latent-structure model for GDP:

$$\Delta y_t - \mu_{s_t^*} = \phi(\Delta y_{t-1} - \mu_{s_{t-1}^*}) + \varepsilon_t \tag{2.1}$$

The mean of the growth-rate process switches between "low" and "high," with $\mu_0 < \mu_1$. For each date t in the sample, Hamilton shows how to obtain an estimate of $P(s_t^* = 0|F_T)$, where F_T contains the past, or even the *complete*, sample history of growth rates, $\{\Delta y_t\}_{t=1,\ldots,T}$.

Define $S_t^* = 1[0.5 - P(s_t^* = 0|F_T)]$, so that $S_t^* = 1$ during projected high-growth phases, and $S_t^* = 0$ during low-growth phases. The observed binary series, $\{S_t^*\}$, can be used in a survival, or *duration*, analysis to determine whether the probability of remaining in a given phase, either contraction or expansion, depends on how long one has been in it. An important consideration is that Hamilton's (1989) model is such that s_t^* evolves with the probability of remaining in a given phase independent of its duration, so that contractions and expansions are assumed to be *duration independent*. Of course, this makes $\{S_t^*\}$ less than ideal to represent phases that may actually be *duration dependent*.

Although Durland and McCurdy (1994), Filardo (1994), Diebold, Lee and Weinbach (1994), Macheu and McCurdy (2000) and Jensen and Liu (2007) generalize Hamilton's assumptions in various directions, no parametric model matches the flexibility and transparency of BBQ to mark time. In particular, for the purpose of solely *marking* the turning points in an observed series such as GDP, it is unnecessary to consider a latent-structure model with or without covariates that helps *predict* the turning points. That is, there is no need to proxy $\{S_t\}$ with $\{S_t^*\}$ for the purpose of a duration analysis.

This is not to say, however, that Markov-switching (MS) models describing fluctuations in y_t are uninformative. In fact, MS models are alternatives to the linear models emphasized by Pagan (1997).[4] Hamilton (2005) argues that linear models are incapable of replicating the cyclical pattern in key economic aggregates, and he devises a simple nonlinear model for unemployment. In particular, linear models cannot capture the fact that the unemployment rate rises more quickly than it falls over the business cycle. Although technology, the labor force, and the capital stock are all key determinants of long-run growth, the forces that contribute to a business downturn can be quite different; and they typically introduce asymmetric behavior that necessitates a nonlinear dynamic representation. Harding and Pagan (2002) also note the deficiency of linear models for replicating the *shapes* of expansions in the business cycle. The point that is often lost is that a duration analysis complements the empirical results from either linear or nonlinear models of y_t; and for the purpose of a duration analysis, it is unnecessary to specify the model for y_t.

2.3. Detrending the series. Likewise, there is generally no need to detrend the series to obtain $\{S_t\}$. Cooley and Prescott (1995) first remove the trend prior to marking the turning points. The trend is typically thought of as a permanent effect, and the remainder as a temporary effect. Unfortunately, confusion is likely to ensue when one attempts to separate permanent from temporary effects, because not all temporary components measure the same thing.

For example, consider decomposing aggregate output so that $y_t = P_t + z_t$, where y_t is the logarithm of GDP, P_t is the permanent effect and z_t is the temporary effect. The permanent effect captures slow-moving low-frequency movements in y_t, and the temporary effect captures the faster-moving high-frequency movements. The term P_t is typically an integrated or I(1) stochastic series, but it can just as easily be defined as some type of deterministic trend. The interpretation of z_t, either as an output gap or some function of growth rates, depends on how P_t is defined.

2.3.1. *Output gaps versus growth rates.* Consider first defining the permanent component as the deterministic trend, $P_t = a + bt$. The temporary effect is the output gap, $z_t = y_t - a - bt$, and the time trend captures steady increases in capital and labor that feed into the aggregate production function. In other words, the output gap defines the difference in actual and potential GDP. Marking time by the sign of z_t determines phases of output above or below the trend. On the other hand, if we define the permanent component as $P_t = y_{t-1}$, the temporary effect is now the growth rate since $z_t = \Delta y_t = y_t - y_{t-1}$. Marking time by examining positive and negative values of z_t defines the classical business cycle.

We make a distinction here between growth cycles and gap or *deviation* cycles. For growth cycles, we seek turning points in Δy_t; there is no reason to specify a trend curve. For gap cycles, however, we seek deviations from a specified trend curve. In contrast, Zarnowitz and Ozyildirim (2006), among others, classify a gap cycle as a special type of growth cycle. For their gap analysis, Zarnowitz and Ozyildirim recommend that the trend curve be determined by the classical nonparametric phase-average-trend (PAT) algorithm of Boschan and Ebanks (1978). Zarnowitz and Ozyildirim then argue that a gap analysis is more informative than a direct growth-rate analysis in the study of national output. First, they argue that growth

[4]Teräsvirta (2006) provides an overview of MS and other types of univariate nonlinear time series models.

rates over short time spans are very erratic and must be smoothed with complex moving averages that potentially distort patterns. Second, they find that the timing of growth rates is very different from that of the corresponding level series. Of course, an alternative interpretation of the second finding is that the growth cycle is providing different information than is the classical business cycle.

2.3.2. *Filtering procedures.* The specification of any trend curve is somewhat arbitrary. However, Zarnowitz and Ozyildirim (2006) find that the PAT algorithm produces a nonlinear trend curve that smoothly transits from higher to lower growth. They also find that the trend from the PAT algorithm fits as well as a log-linear trend, the stochastic Beveridge and Nelson (1981) trend, the local linear trend of Harvey (1989), the Hodrick-Prescott (1997) trend, and Rotemberg's (1999) heuristic trend.

Intuitively, different types of trends produce different types of temporary components. For instance, King and Rebelo (1993) show that the temporary component from the Hodrick-Prescott (HP) filter is a two-sided weighted average of growth rates. However, if the data-generating process is the pure random walk, $y_t = y_{t-1} + \epsilon_t$, Harding and Pagan (2005) show that the HP temporary component is well represented by a weighted average of current and lagged values of the growth rates with slowly declining weights. In contrast, the Beveridge-Nelson temporary component for the random walk is degenerate since the permanent component is y_t.

The point is this. It is easy to think that all temporary components are measuring the same thing as long as each is a stationary process; however, this is not the case. For business cycles, the litmus test appears to be whether the turning points match well with those of the NBER. Consider that, for quarterly observations on GDP, Hamilton (1989) compares his estimated latent-structure probabilities with turning points in aggregate activity. He demonstrates that his estimate of $P(s_t^* = 0|F_T)$ is generally greater than 0.5 during recessions and less than 0.5 during expansions. On the other hand, the very flexible BBQ algorithm of Harding and Pagan (2002) also yields turning points that accord well with those of the NBER.[5]

3. The discrete-time hazard function

Duration data for economic cycles are invariably discrete since data are collected at discrete intervals of time, e.g., weekly, monthly, quarterly, or yearly. Although data for markets such as housing or financial markets are available at short intervals, the more interesting questions about such cycles are typically best captured by intervals of at least a month. Consider also that a discrete-time duration analysis can be viewed as an approximation to a continuous-time analysis, or vice versa, and general notions about one apply to the other.[6] A discrete-time framework has the advantage of being more natural for the types of data encountered

[5]Our list of techniques to mark time is not exhaustive. Boldin (1994) reviews five techniques to mark time for business cycles: the NBER business cycle dating committee; GDP growth rules; the Commerce Department's Bureau of Economic Analysis (BEA) indicators; Stock and Watson's (1989, 1991) experimental business-cycle indices; and a Markov-Switching model for unemployment.

[6]A nice introduction to continuous-time duration analysis is Greene (2006, pp. 710-712). Another good overview of duration techniques, either discrete or continuous, is the chapter on transition data in the microeconometrics text by Cameron and Trivedi (2005, pp. 573-608).

at the aggregate level; the framework is inherently semi-parametric and is easy to understand and implement. Since formal statistical inference depends on the framework, it is best to adopt that of discrete-time if data are measured at long intervals.

Consider a random subsample of n observations $(T_1, T_2, ..., T_n)$ from a discrete, cumulative distribution F, such that $F(a) = 0$ for $a < 0$.[7] The probability-distribution function, or *density function*, is $f(t) = P(T = t)$, and the discrete-time *hazard function* is

$$h(t) = P(T = t|T \geq t) = f(t)/G(t), \tag{3.1}$$

where $h(t)$ is the hazard function, and $G(t) = P(T \geq t)$ is the *survival function.* The density function, $f(t)$, gives the probability that a duration will last *exactly* t periods, the survival function, $G(t)$, gives the probability that a duration will last *at least* t periods, and the hazard function, $h(t)$, gives the conditional probability that a phase will terminate in period t, given that it has lasted t *or more* periods. If rising or falling, the hazard provides useful information about the likelihood of a change in phase. Using over 100 years of annual data, Mills (2001) finds several instances of non-constant hazards in the business cycles of 22 countries.

The hazard may also be useful in the assessment of general market conditions. For instance, Diebold and Rudebusch (1990) and Ohn, Taylor and Pagan (2004) observe that post-WWII contractions are more prone to revert to expansion than pre-WWII contractions. One explanation for this finding is that policymakers are now much better able to manage potential economic crises. A second explanation is that individuals and firms are better able to smooth shocks due to innovation and financial deregulation. On the other hand, Watson (1994) finds that, for most individual sectors of the economy, the average contraction and expansion durations for the prewar and postwar periods are similar; and, more recently, Stock and Watson (2003) suggest that favorable market conditions in the modern era are more likely due to good luck than to good management or innovation.[8] This is especially true for recent times as there have been relatively few long-lived supply disruptions since the 1970s.

3.1. Hazard plots. It is generally informative to plot the hazard function, with hazard rates easily computed by the nonparametric life-table method of Cutler and Ederer (1958). The computer package LIMDEP 7.0 constructs such life tables, with an approximate, but intuitive, explanation of the procedure as follows:

- *Place the contractions in ascending order by length.*
- *Construct the hazard rate at time "t" as the ratio of the number of contractions terminating in month "t" over the number of contractions lasting at least "t" months.*

[7] If the length of an economic contraction is influenced by the length of the preceding expansion, *and* vice versa, the assumption of statistical independence is violated. Likewise, an economic contraction caused by an especially bad harvest could behave very differently from a contraction that occurs during the normal operation of modern market economies. These problems can be handled when modeling with covariates. Another way to help ensure homogeneity across spells of expansion and contraction is to segment the time line into distinct sampling periods with the same underlying probability distributions; see, for example, Diebold and Rudebusch (1990).

[8] Watson's (1994) prewar sampling period ranges from roughly 1860 through 1929, and his postwar sampling period ranges roughly from 1947 through 1990.

Sample information is thus used to estimate $P(T = t)/P(T \geq t)$ in a straightforward manner. A contraction that terminates is said to *exit* the sample, while those contractions lasting at least t months are said to be still *at risk*. Of course, the pool of contractions still *at risk* decreases with t. This implies that the effective sample size for estimating the hazard rates for relatively long contractions is less than for short contractions. Formal statistical tests are thus necessary to avoid spurious conclusions from inspecting the graphs alone.

3.2. Benchmark hazards. Nevertheless, graphs convey important information about the general shape of the hazard function. Hollander and Proschan (1975) and Hollander and Wolfe (1999) provide some important benchmarks:

(1) *Constant Failure Rate: CFR*
(2) *Increasing (Decreasing) Failure Rate: IFR, DFR*
(3) *Increasing (Decreasing) Failure Rate on Average: IFRA, DFRA*
(4) *New Better (Worse) than Used: NBU, NWU*

Figure I depicts hazard functions from CFR, IFR, IFRA, and NBU *life distributions.*[9] Because there is a one-to-one relationship between the hazard function and the probability density function, a comparison of hazard rates is a natural way of analyzing the nature of exit probabilities, more so than a comparison of density functions. In particular, the CFR hazard is almost always given special consideration in any duration analysis.

<Figure I about here>

CFR hazards correspond to the geometric density. New economic expansions are no more or less likely to terminate than mature ones. In contrast, if expansions are IFR, the hazard, or *failure,* rate is never decreasing, and our illustrated IFR hazard implies an ever more likely chance of termination, or *mortality.*

IFR hazards are not the only ones that have a tendency to rise. Although the depicted IFRA hazard has periods of decline, IFRA has the same overall upward trend. For example, militarized interstate disputes initially have a decreasing hazard rate for a short period, but then exhibit increasing hazards over most of the duration due, perhaps, to a more concentrated effort to either negotiate or impose settlement.

In contrast to IFRA, the depicted NBU hazard function rises above and then falls back to the initial value. There is no overall trend in either direction. Because the hazard function never falls below the initial value, new phases have the greatest chance of surviving an *additional* week or month. An NBU hazard may arise due to seasonal effects such as holidays or inclement weather, and IFRA hazards are special cases of NBU hazards. A completely analogous situation holds for DFR, DFRA and NWU distributions.

The graphical approach of Cutler and Ederer (1958) is nonparametric and avoids some of the dangers of relying too heavily on parametric methods. For instance, in the emerging area of forensic economics, Bonanomi, Gaughan and Taylor (1998) use the flexible nonparametric life-table method in the estimation of lost profits when the plaintiff claims lost customers due to an alleged transgression. However, in the general economics literature, Sichel (1991) advocates using parametric methods to increase the power of tests for duration dependence. In

[9] A realization from a so-called life distribution cannot be negative. The term "life distribution" is coined from the study of mortality, where many of these distributions were first employed.

the political science literature, Bennett (1999) and Zorn (2000) observe that the parametric Weibull model is the most widely-used form. Bennett also cautions that Cox's semiparametric proportional hazards model does not allow for *precision* concerning the hazard function.

Precisely wrong results, however, are hardly helpful. Ohn et al. (2004) show that the Weibull model is insufficient to capture the richness of economic contractions and expansions, and Taylor (2007) shows that the Weibull model is highly misleading for militarized interstate disputes. Attempts to apply more flexible continuous-time methods have met with mixed success. Diebold, Rudebusch and Sichel (1993), for example, apply their nonlinear exponential linear model to business cycle data that mostly duplicates the results from the Weibull model. Zuehlke (2003) applies the nonlinear model of Mudholkar, Srivastava and Kollia (1996) that allows hazards to be monotonically increasing, monotonically decreasing, U-shaped or inverted U-shaped. However, Ohn et al. (2004) observe that none of these shapes adequately describes the hazard functions for pre-WWII contractions and expansions. Even for the small samples encountered in the business cycle literature, nonparametric analysis provides valuable insights that parametric analysis fails to uncover.

4. Testing for duration dependence

The direction of duration dependence is easily obtained from a sample of durations. First, if the mean duration equals the sample standard deviation, there is no evidence of duration dependence; that is, there is *a constant hazard*. Second, if the mean duration is greater than the sample standard deviation, there is evidence of *positive* duration dependence, or a generally increasing hazard. Finally, if the mean duration is less than the sample standard deviation, there is evidence of *negative* duration dependence, or a generally decreasing hazard.

4.1. The nature of duration independence. Duration independence is considered the neutral case. Long expansions have no greater chance of ending than short expansions; long bear markets have no greater chance of ending than short ones; and long housing slumps have no greater chance of ending than short slumps. The duration of the phase has no predictive power in determining the end of the phase. Because of neutrality, the constant hazard is the standard benchmark, and a graph of the hazard function is frequently employed to see if the hazard function appears roughly constant. If so, there is *duration independence,* and the hazard function does not depend on t. In other words, the null hypothesis is:

$$H_o : h(t) = p \quad \text{for some } 0 < p < 1 \quad \text{and all} \quad t > 0 \tag{4.1}$$

The density *must* be geometric for constant hazards, and the above null hypothesis is equivalent to

$$H_o : f(t) = P(T = t) = (1 - p)^t p \text{ for } 0 \leq t \leq \infty \tag{4.2}$$

In other words, testing for duration independence is equivalent to testing whether the durations follow the geometric density. A *direct*, or *strong-form*, test for the geometric density is the usual chi-square goodness-of-fit test employed by Ohn et al. (2004).

Finally, for the geometric density, $E(T) = (1-p)/p$ and $V(T) = (1-p)/p^2$, leading to a third null hypothesis for duration dependence:

$$H_o : V(T) - [E(T)]^2 - E(T) = 0 \tag{4.3}$$

4.2. Weak-form tests. Pagan (1998) and Mudambi and Taylor (1991,1995) devise tests for duration dependence based on the *consistency relationship* as in (4.3). Such tests are called *weak-form* tests and have close links with continuous-time tests for the *exponential* density. Although the exponential density is simply the continuous-time equivalent of the discrete-time geometric density, there is one rather important difference between the two. The consistency relationship for the exponential density is $V(T) - [E(T)]^2 = 0$ rather than $V(T) - [E(T)]^2 - E(T) = 0$, and thus the choice between discrete-time and continuous-time tests is important except when p is either close to 0 or 1. For when $p \approx 0$ or $p \approx 1$, $V(T) \approx [E(T)]^2$.

4.2.1. *The GMD test.* Mudambi and Taylor (1995) devise a generalized-method-of-moments (GMM) estimator based on the moment condition, $V(T) - [E(T)]^2 - E(T) - \gamma = 0$. For the geometric density, $Ho : \gamma = 0$, and they determine whether GMD $= [(1/n) \sum_{i=1}^{n} (T_i - \overline{T})^2 - \overline{T}^2 - \overline{T}\,]$ is statistically significant from zero. Once normalized, GMD is asymptotically $N(0,1)$, and should be especially sensitive to IFRA and DFRA alternatives since it is completely analogous to a continuous-time test based on $V(T) - [E(T)]^2 = 0$ that is designed for such alternatives. Because GMD is highly skewed in finite samples, however, simulations are necessary to obtain finite-sample critical values.

4.2.2. *The SB test.* Closely related to the GMD test is the *state-based* SB test proposed by Pagan(1998), with a number of positive attributes:

1. *SB focuses directly on the conditional probabilities.*
2. *SB involves a regression and so is easy to explain to a nonspecialist.*
3. *SB can be used to examine prediction issues.*
4. *SB can be used to study how the exit probabilities have changed over time since the parameters can be recursively estimated.*
5. *SB can be easily modified to estimate discrete-time hazard functions, with or without covariates.*

Although SB can be applied to whole cycles, it is best to apply the test to the separate half cycles, or phases. In other words, one should apply SB first to contractions and then to expansions. In fact, Mudambi and Taylor (1991) show that it is incompatable for expansions, contractions, and whole cycles to *all* follow a constant-hazard geometric distribution. So, if expansions and contractions are duration independent, it is certain that whole cycles are duration dependent. Likewise, if whole cycles are duration independent, there is necessarily some form of statistical dependence in the half-cycle components.

Consider a small sample of *contractions* observed at monthly intervals. In the example below, there are gaps in the time line because most months of expansion are excluded from the sample. In fact, the only included months of expansion are those that correspond to a turning point.

	Jan	Feb	Mar	Apr	...	Sep	Oct	Nov	Dec	Jan	Feb	...	Jun	Jul
S_t	0	0	0	1	...	0	0	0	0	0	1	...	0	0
d_{t-1}	0	1	2	3	...	0	1	2	3	4	5	...	0	1

$S_t = 0$ indicates a month of economic contraction, and $S_t = 1$ indicates a month of economic expansion. For instance, the first complete contraction began in January and ended in March, with April as the turning point. The string of S_t values represents two complete contractions and one incomplete, or *censored,* contraction. The durations of the contractions are $T_1 = 3, T_2 = 5$, and $T_3 = 2$. Any censored observations will invariably be at the beginnning and/or end of the string, and such incomplete observations are dropped from the sample. For the sub-sample of expansions, we consider instead $1 - S_t$, so that $(1 - S_t) = 1$ marks the turning point of an economic expansion.

An economic expansion or contraction is a *phase* of the business cycle. Define d_t as the number of months in a given phase. The above table demonstrates values for lagged d_t. Now drop those observations from the sample if $d_{t-1} = 0$, and define m as the number of remaining S_t values. For our example, $m = 8$ since we drop two observations because $d_{t-1} = 0$ and we drop the last contraction due to censoring. A straightforward test for duration dependence in contractions is obtained by testing the null hypothesis, $H_o : \beta_1 = 0$, in the simple regression equation

$$S_t = \beta_0 + \beta_1 d_{t-1} + \epsilon_t \tag{4.4}$$

where $E_{t-1}(\epsilon_t) = 0$.

For constant hazards, $\beta_1 = 0$ and $\beta_0 = p_{01}$, where $p_{01} = P(S_t = 1|S_{t-1} = 0)$. The term d_{t-1} captures autonomous changes in the hazard function. The resulting model can be written as

$$S_t = p_{01} + v_t \tag{4.5}$$

Hamilton (1994, p. 684) shows how to write such an equation if one is considering complete cycles rather than half-cycles. For the purpose of a duration analysis, however, it is only necessary to consider half-cycles: expansions or contractions, bull or bear markets, upswings or downswings.

For non-constant hazards, $\beta_1 \neq 0$, and the termination probability depends on d_{t-1}, the length of time in the specified phase. Ohn et al. (2004) show that the SB t-test is appropriate for testing $H_o : \beta_1 = 0$. We argue here the same point, but from a different approach. In particular, by the definition of the binary indicator, S_t, the duration of an expansion must be at least one month. The geometric density is thus left-censored at unity. The censored probability function is $P(T = t) = (1 - p)^{t-1}p$ for $1 \leq t \leq \infty$, with $E(T) = 1/p$ and $V(T) = (1 - p)/p^2$. Let n be the number of turning points. Then $\overline{T} = (1/n)\sum_{i=1}^{n} T_i$ is a consistent estimator for $E(T)$, $\tilde{\sigma}^2 = (1/n)\sum_{i=1}^{n}(T_i - \overline{T})^2$ is a consistent estimator for $V(T)$, and $\overline{S}$ is a consistent estimator for p.

The least-squares estimator of β_1 is[10]

[10]Our notation for the time indices slightly abuses notation since there are gaps in the time line for a phase analysis of either contractions or expansions.

$$\widehat{\beta}_1 = \frac{\frac{1}{m}\sum_{t=1}^{m} (S_t - \overline{S})d_{t-1}}{\frac{1}{m}\sum_{t=1}^{m} (d_{t-1} - \overline{d})^2} \tag{4.6}$$

where $\overline{S} = \frac{1}{m}\sum_{t=1}^{m} S_t = n/m$ and $\overline{d} = \frac{1}{m}\sum_{t=1}^{m} d_{t-1}$.

Our goal is to show that $p\lim \widehat{\beta}_1 = 0$ under the null hypothesis that durations follow the geometric density. To do so, consider the numerator of the least-squares estimator

$$\frac{1}{m}\sum_{t=1}^{m} (S_t - \overline{S})d_{t-1} \tag{4.7}$$

$$= \frac{1}{m}\sum_{t=1}^{m} S_t d_{t-1} - \overline{S}\,\overline{d} \tag{4.8}$$

$$= \frac{n}{m}\overline{T} - (\frac{n}{m})^2 \frac{1}{n}\sum_{i=1}^{n} \frac{(T_i+1)T_i}{2} \tag{4.9}$$

$$= \overline{S}\frac{1}{2}\{2\overline{T} - \overline{S}[\widetilde{\sigma}^2 + \overline{T}^2 + \overline{T}]\} \tag{4.10}$$

Since for the geometric distribution, $p\lim \overline{T} = 1/p$ and $p\lim \widetilde{\sigma}^2 = (1-p)/p^2$, it follows that

$$p\lim \overline{S}\frac{1}{2}\{2\overline{T} - \overline{S}[\widetilde{\sigma}^2 + \overline{T}^2 + \overline{T}]\} = p\frac{1}{2}\{2\frac{1}{p} - p[\frac{(1-p)}{p^2} + \left(\frac{1}{p}\right)^2 + \frac{1}{p}]\} = 0 \tag{4.11}$$

An immediate implication is that $p\lim \widehat{\beta}_1 = 0$ for a constant-hazard function. On the other hand, just as with GMD, the distribution of SB is skewed right in finite samples, and thus it is necessary to use simulations to obtain finite-sample critical values.

In spite of their skewed distributions, GMD and SB are asymptotically pivotal; that is, asymptotically they do not depend on unknown parameters. For asymptotically pivotal statistics, the bootstrapped critical values are generally more accurate than those based on first-order asymptotic theory. Horowitz (2001) and Davidson and MacKinnon (2006) explain why it is desirable to use pivotal statistics when bootstrapping.

4.3. Strong-form tests. Diebold and Rudebusch (1991) and Ohn et al. (2004) employ the chi-square goodness-of-fit test to determine if durations follow the geometric density. The test statistic is $\chi^2 = \sum_{j=1}^{K}[(O_j - E_j)^2/E_j]$, where O_j is the observed frequency in the jth bin and E_j is the expected frequency in the jth bin. The expected frequency is derived under the null distribution, in this case the geometric density. A well-known rule-of-thumb is that the expected frequency, E_j, should be at least 5 for all bins; see Hoel (1954). To be on the safe side, Ohn et al. (2004) use 6 instead of 5. If one adheres to the rule-of-thumb, long-term experience suggests that χ^2 approximately follows its asymptotic chi-square distribution with K-1 degrees of freedom. Nonetheless, both Diebold and Rudebusch (1991) and Ohn et al. (2004) employ simulation to obtain finite-sample critical values.

5. Modeling with covariates

Most researchers using aggregate-level data segment the time line to control for heterogeneity of the exit probabilities. For example, Edwards, Biscarri and de Gracia (2003) segment the time line for Latin American and Asian countries to determine the effect of financial liberalization on stock-market cycles. Models that employ covariates have not been as popular and the studies that do use them generally assume duration independence. For instance, Estrella and Mishkin (1998) employ a discrete-time analysis to examine various financial variables as predictors of U.S. recessions, and Chin, Geweke and Miller (2000) use a similar analysis to predict turning points in the civilian unemployment rate. Conditional upon the right-hand-side variables, however, both studies assume the hazard function is independent of time.[11]

The strong assumption of duration independence in models with covariates is defensible in some circumstances. In the political science literature, Bennett (1999, p. 262) goes so far as to argue that including covariates to effectively eliminate duration dependence is a laudable goal:

> Unless we can anthropomorphize and assume that the phenomenon we are examining truly has a life of its own, then the pattern or covariation over time that we observe is somehow, somewhere, driven by a variable or set of variables that characterizes the world. If the causal factor driving duration dependence is measured and included in the model as an independent variable, then unexplained duration dependence ... may disappear.

Bennett's view aligns with the concept of *probabilistic reduction* that Spanos (1995) traces back to the biometric tradition of Galton and Pearson; see also Spanos (2006) and Hoover (2006). In the regression framework, probabilistic reduction implies that a complete theory must induce white noise disturbances in the model. In practice, whether covariates can account for the observed duration dependence in any binary series is surely an empirical question that cannot be assumed away for the model at hand. Complete theories are ideal but rare.

Another reason that duration dependence is frequently ignored for discrete-time analysis is that many researchers apparently believe that it is not possible to incorporate such dependence. For instance, Bennett (1999, p. 259) argues that the logit model is insufficient for the analysis of duration data because "it assumes that no duration dependence exists." Below we show that a slightly modified logit model is suitable to capture autonomous changes in the discrete-time hazard as well as changes due to covariates.

Probit models are alternatives to logit models. There is no need, however, to adopt any type of latent structure for either probit or logit. In other words, it is neither necessary nor desirable to insist that there exists some type of latent variable, y_t^*, such that $S_t = 1$ if $y_t^* > 0$, and $S_t = 0$ if $y_t^* \leq 0$. Although such an assumption *is* desirable in the discrete-choice literature, where y_t^* is interpreted as a utility function, it only unnecessarily complicates a duration analysis. In fact, to

[11] For instance, Chin, Geweke and Miller (2000) use covariates to capture drift in the hazard function. However, conditional on the x values, their hazard function is constant since it does not explicitly depend on the duration of the phase.

mark time for economic cycles, we frequently map an *observed* series,$\{y_t\}$, to $\{S_t\}$. Thus, the unobservables of true interest in either the logit or probit probability models are the estimable parameters controlling the termination probabilities of $\{S_t\}$.[12]

5.1. The Logit model. Following Allison (1984), let $P(t)$ represent the discrete-time hazard function. We use $P(t)$ here rather than $h(t)$ because it is more natural to do so for logistic regression. For the logit-hazard model,

$$\log(P(t)/(1-P(t)) = a(t) + \beta_1 x_1 + \beta_2 x_2(t) \tag{5.1}$$

where $a(t)$ represents a set of dummies, one for each of the observed exit periods, that account for autonomous changes in the exit probabilities; x_1 represents a set of covariates that do not change over the course of a given contraction; and $x_2(t)$ represents a set of covariates that do change over the course of a given contraction. In other words, $a(t)$ allows for non-constant hazards, conditional upon the x values. For constant hazards, one should substitute a single intercept parameter, a, for the time varying $a(t)$.

5.1.1. *The LSB test.* Define d_t as the number of consecutive months (i.e., duration) spent in a contraction up and *through* time t, and consider a very simple model with just one $x(t)$, namely d_{t-1}, as defined for the SB test:

$$\log(P(t)/(1-P(t)) = \beta_0 + \beta_1 d_{t-1} \tag{5.2}$$

Drop observations from the sample if $d_{t-1} = 0$. A restriction from the assumption of duration independence is $H_o : \beta_1 = 0$, with the test statistic computed by the corresponding asymptotic t-ratio. The constant-hazard test from logit regression, call it LSB, is obviously closely related to Pagan's regression-based SB test. The potential advantages of using SB are that it is very straightforward, the least-squares algorithm is very stable, and many computer packages recursively estimate least-squares (but not logit) coefficients.

Stability is an important consideration for small samples since the number of observations with $S_t = 1$ is usually small for macroeconomic data. Consider, for example, that there are only about ten post-WWII economic expansions in the American business cycle. For these expansions, the number of observations with $S_t = 0$ is large because of a low termination probability, hence the proportion of observations with $S_t = 1$ is small. With a very low proportion of observations with $S_t = 1$, small samples can be especially problematic for logistic regression.

For sufficiently large samples, however, the logit model is preferred to the linear model. For the linear model, the predicted probabilities can lie outside of the unit interval from 0 to 1, and it is well-known that least-squares estimators are not efficient if the dependent variable is binary. In contrast, logit probabilities are always bounded on the unit interval; and since logit estimation is based on maximum likelihood, the estimators from logit are asymptotically efficient.

[12] The transition probability for the latent state variable in the Durland and McCurdy (1994) Markov switching model has logit form. But $\{S_t\}$ is not the same as $\{S_t^*\}$, and there is no reason to consider the latter for the purpose of a duration analysis.

5.1.2. *A comparison with Cox's model.* Thompson (1977) presents another compelling argument in favor of logit estimation. As the discrete-time intervals become smaller and smaller, the logit model converges to Cox's (1972) continuous-time proportional hazards model. Cox's model can be written as

$$\log h(t) = a(t) + \beta x \tag{5.3}$$

where $h(t)$ is the continuous-time hazard rate, similar to $P(t)$, and x represents a set of covariates that do not vary over time. Like the discrete-time logit model, $a(t)$ can be any function of time. The term *proportional hazard* comes from the fact that, for any t and any two individuals i and j,

$$h_i(t)/h_j(t) = \exp(\beta x_i)/\exp(\beta x_j) = \exp(\beta(x_i - x_j)) \tag{5.4}$$

This function does not vary with time because the autonomous time-varying term, $a(t)$, cancels out. Cox's model, however, is not just limited to proportional hazards, since the model is no longer proportional if some of the x values vary with time. Some computer packages allow for time-varying covariates while others do not. Fortunately, this is not a concern to us since it is always possible to allow for time-varying covariates in logistic regression.

Cox's proportional-hazards model is semiparametric because the autonomous time-varying term, $a(t)$, does not have to be specified. Although the estimators of β are asymptotically unbiased and normally distributed, they are not *fully* efficient because the exact functional form of $a(t)$ is not specified. However, Efron (1977) shows that the loss of efficiency is typically so small that it is not of practical concern. The importance of Cox's model for duration analysis is well-summarized by Allison (1982, p. 35):

> It is difficult to exaggerate the impact of Cox's work on the practical analysis of event history data. In recent years, his 1972 paper has been cited well over 100 times a year in the world scientific literature. In the judgment of many, it is unequivocally the best all-around method for estimating regression models with continuous-time data.

In practice, time is always measured in discrete intervals, although the intervals may be irregular for individual histories. Consider also that, if two or more individuals experience events at the same time, that is, if we observe a *tie*, then the model proposed by Cox (1972) is the logit model. Therefore, although some authors have argued that continuous-time methods are preferable to discrete-time methods on theoretical grounds, or have completely ignored discrete-time methods altogether, the lack of attention to the logit model seems rather unfortunate. The preference for continuous-time models is largely based on computational grounds, but this is certainly less of an issue today than it was twenty-five years ago!

5.2. Predetermined variables and unobserved heterogeneity. An important issue is the number and choice of covariates. Consider, for example, economic expansions. If the length of the prior expansion and/or contraction influences the exit probability of the current expansion, then the length of the prior expansion and/or contraction should be included in the set of explanatory variables. The usual asymptotic t-ratio can then be employed to determine if such effects are important. Thus, with a set of appropriate explanatory variables, either predetermined or strictly exogenous, one can account for certain types of dependence that cannot be handled by simply segmenting the time line.

Unobserved, or neglected, heterogeneity is inherently a problem of omitted variables. As a practical consideration, however, the sample sizes encountered in the study of economic cycles are generally too small to consider many explanatory variables. Explanatory variables are used to control for heterogeneity in the exit probabilities. Bover, Arellano and Bentolila (2002) develop logistic discrete hazard models that can accommodate unobserved heterogeneity. In their microeconometric study of about 27,000 unemployment spells of Spanish men, they explicitly control for unemployment benefits, age, education, head of household status, and dummies for economic sector and year of unemployment. A second model is estimated with time-varying macroeconomic variables, such as the growth rate in GDP and sectoral unemployment rates, substituting for the sectoral and time dummy variables. Autonomous shift dummies, $a(t)$, are included in both models to capture flexible additive duration dependence. A dummy variable is included for each possible exit time, with time marked at monthly intervals.

Bover et al. (2002) treat the length of unemployment benefits as predetermined, though not strictly exogenous, since knowledge about future benefits can influence job choice and especially the decision to re-enter the labor market. The distinction between predetermined and strictly exogenous variables is fairly unimportant unless there is unobserved heterogeneity. In that case, predetermined variables are effectively endogenous, and it is necessary to maximize the joint mixture likelihood for the unemployment and benefit durations. This is accomplished, in part, by introducing a discrete unobserved random variable with finite support. Additional parameters are thus included to model the unobserved heterogeneity.

In the macroeconometrics literature on business cycles, however, postwar sample sizes of about 10 spells preclude the possibility of estimating models that are rich in parameters. As a partial solution, segmenting the time line into prewar and postwar periods is sufficient to control for omitted variables that vary across, but not within, the segments. Since any residual heterogeneity induces *negative* duration dependence, expansions and contractions may appear to be self-perpetuating even if this premise if false. On the other hand, Diebold and Rudebusch (1990) and Ohn et al. (2004) find evidence of *positive*, not negative, duration dependence in the American business cycle. Thus, at least the *direction* of such duration dependence cannot be caused by residual, or neglected, heterogeneity.

6. The shape of cycles

Duration dependence concerns the shape of the hazard function. If the hazard function slopes upward, there is positive duration dependence; if downward, there is negative duration dependence; if flat, there is no duration dependence. However, the shape of the hazard function is but one example of the types of shapes associated with cycles. In particular, Harding and Pagan (2002) and Pagan and Sossounov (2003) discuss the typical shapes of phases, either contractions or expansions. They address the following issues:

(1) *Amplitudes of phases,*
(2) *Cumulative movements within phases,*
(3) *Asymmetric behavior of phases.*

After marking the turning points, the binary series, $\{S_t\}$, is employed along with the observed underlying series, $\{y_t\}$, to describe the shape of a phase. For example, during economic expansions, GDP is observed to rise quickly at first and then slows its ascent before finally reversing direction, thus marking the beginning of a contraction.

In Figure II, we present a stylized economic expansion.

<Figure II about here>

The y axis represents log(GDP), or *amplitude*, and the x axis represents time spent in an expansionary phase, or *duration*. On the time axis, time A represents the first turning point, the trough, and time B the second turning point, the peak. The amplitude of the expansion is the vertical distance between points A and B, measuring the change in GDP from trough to peak. In this instance the amplitude is $\log(GDP_B) - \log(GDP_A)$, where $\log(GDP_B)$ is the value of gross domestic product at time B. The hypotenuse of the triangle is a benchmark representing a constant increase in amplitude, with increases in amplitude proportional to the time spent in the expansionary phase.

Descriptive measures of interest include the average duration and average amplitude of the expansions in the sample, measures of the variability in durations and amplitudes, and a measure to show how closely growth in GDP adheres to the hypotenuse depicted in Figure II. For our sample of n expansions, we observe the durations, $\{T_1, T_2, ..., T_n\}$, and the amplitudes $\{A_1, A_2, ..., A_n\}$.

6.1. Durations. Assume that durations and amplitudes constitute random samples. If so, the $T_i's$ follow identical distributions, $T_i \backsim D(\mu_T, \sigma_T^2)$, with T_i statistically independent from T_j for $i \neq j$. Further, $\overline{T} = \frac{1}{n}\sum_{i=1}^{n} T_i$ is a consistent estimator for μ_T and $\overline{T} \overset{a}{\backsim} N(\mu_T, \sigma_T^2/n)$. This assumes, of course, that the time line has been successfully segmented to ensure that each of the $T_i's$ follows the same distribution, that is, with no mixing of distributions.

Small sample inference, however, is particularly problematic for durations. Consider that the $T_i's$ rarely come from the normal distribution. In fact, for the discrete case with constant hazards, the $T_i's$ follow the geometric distribution, $T_i \backsim GEOM(\mu_T, \mu_T^2 - \mu_T)$, with $\mu_T = 1/p$, where p is the constant hazard. However, since the geometric distribution is considerably right-skewed, for very small samples it is generally unwise to construct confidence intervals on μ_T by employing the t-distribution, since normality of the durations is one of the assumptions supporting its use. Simulations by Pagan and Sossounov (2002) also suggest that $\overline{T}$ is significantly skewed in small samples.

6.2. Amplitudes. Amplitudes are less problematic. For the ith expansion, the amplitude is calculated as $A_i = y_{T_i} - y_0 = \sum_{j=1}^{T_i} \Delta y_j$, with $\Delta y_1 = y_1 - y_0$, $\Delta y_2 = y_2 - y_1$, and so on. In Figure II, y_0 equals $\log(GDP_A)$ and y_{T_i} equals $\log(GDP_B)$. The amplitude of the expansion is thus the sum of the growth rates from time A to B. Although Harding and Pagan (2002) convincingly argue that GDP growth rates are not statistically independent, their sum may be approximately normally distributed by either Gordin's (1969) or Hannan's (1973) Central Limit Theorems (CLT) for stationary ergodic processes; see also White (1984, Ch. V). Thus, approximate normality for the amplitudes, A_i, is more plausible than normality for the durations, T_i, provided that expansions are sufficiently long to allow the CLT to work for each A_i.

6.3. Cumulative gain. Another measure of interest is the *cumulative* gain from trough to peak, that is, the area under the curve that describes the actual path of GDP. An approximation to this gain is obtained by adding together the area in rectangles of unit length and height equal to $y_j - y_0$. The approximation, however, is too large since each such rectangle overstates the actual area by approximately $(y_j - y_{j-1})/2$. Correcting for the overstatement, Harding and Pagan (2002) approximate the cumulative gain for the *ith* expansion by

$$F_i = \sum_{j=1}^{T_i} [(y_j - y_0) - (y_j - y_{j-1})/2] \tag{6.1}$$

$$= [\sum_{j=1}^{T_i} (y_j - y_0)] - A_i/2 \tag{6.2}$$

where $A_i = y_{T_i} - y_0$. A benchmark for F_i is the cumulative area under the hypotenuse depicted in Figure II. The area of this right triangle is easily calculated as $AREA_i = (T_i \cdot A_i)/2$. Taking the difference, $F_i - AREA_i$, and then dividing by the duration of the *ith* expansion, one obtains

$$E_i = (F_i - AREA_i)/T_i \tag{6.3}$$

A positive value of E_i indicates that growth rates generally increase at a decreasing rate over the life of the expansion, and a negative value of E_i indicates that growth rates generally increase at an increasing rate over the life of the expansion. A positive value for $\overline{E} = \frac{1}{n}\sum_{i=1}^{n} E_i$ indicates that most of the growth occurs at the beginning of the typical expansion, and a negative value of $\overline{E}$ indicates that most of the growth occurs at the end of the typical expansion. If $\overline{E} = 0$, then neither characterization is accurate; instead, the actual path for y tends to oscillate around the hypotenuse. $\overline{E}$ is thus a useful descriptive measure of the average shape, or curvature, of expansions. A similar interpretation holds for $\overline{C}$, the average shape of contractions. In the literature on business cycles, Sichel (1994) documents the rapid recovery of an expansion that leads to a positive value for $\overline{E}$. In the financial literature, Edwards et al. (2003) observe that the excess index (equation 6.3) is particularly useful in characterising stock market behavior.

7. Synchronization of cycles

The cyclic characteristics of a single series, $\{y_t\}$, are of interest because they yield insights about the underlying series. Consider also that the cyclic relationship between *two* underlying series, $\{y_{1t}\}$ and $\{y_{2t}\}$, is of like interest to both academics and policymakers. For example, do short periods of financial crisis influence the business cycle? Are cycles in foreign economies closely tied to the American business cycle? Are cycles in national unemployment related to cycles in GDP? Finally, are cycles in oil prices related to the world economic or political (dis)order? Each of these questions can be answered, in part, by examining the observed binary time series, $\{S_{1t}\}$ and $\{S_{2t}\}$, that respectively correspond to $\{y_{1t}\}$ and $\{y_{2t}\}$.[13]

7.1. The coincidence indicator. One way to measure the correspondence between $\{S_{1t}\}$ and $\{S_{2t}\}$ is to employ the *coincidence indicator* of Harding and Pagan (2002)

$$\widehat{I} = 1/T \sum_{t=1}^{T} [S_{1t}S_{2t} + (1 - S_{1t})(1 - S_{2t})] \tag{7.1}$$

where T is the total number of periods in the sample interval, regardless of phase. Consistent with the notation of Harding and Pagan (2002, 2006), we use T in this section to denote the sample size rather than duration. It follows that $\widehat{I}$ is the fraction of periods that $\{S_{1t}\}$ and $\{S_{2t}\}$ are synchronized. Harding and Pagan (2006) note that there is *perfect positive synchronization* between $\{S_{1t}\}$ and $\{S_{2t}\}$ if $\widehat{I} = 1$, and there is *perfect negative synchronization* if $\widehat{I} = 0$. Beyond the literature on national output, Edwards et al. (2003) observe that financial synchronization or *concordance* among Latin American countries has substantially increased after financial liberalization.

7.2. Correlation analysis. The sample correlation coefficient between S_1 and S_2, call it r_s, conveys similar information to $\widehat{I}$. If $r_s = 1$, there is evidence in favor of the null hypothesis, $H_0 : \rho_s = 1$, since $S_{1t} = S_{2t}$ for every paired observation in the sample. Of course, if there is a single case where $S_{1t} \neq S_{2t}$, there is reason to reject the hypothesis that the cycles are *perfectly* positively synchronized. Similar reasoning holds true for perfect negative synchronization. A formal test of $H_0 : \rho_s = 1$ is presented by Harding and Pagan (2006).

As perfect synchronization will be empirically atypical, it is still useful to compute either or both of the sample correlation coefficient and coincidence indicator to see how closely two series move in tandem. Graphing the series may also reveal an obvious translation of $\{S_{1t}\}$ that will more closely synchronize $\{S_{1t}\}$ with $\{S_{2t}\}$. For instance, consider $\{S_{3t}\} = \{S_{1,t\pm l}\}$ for some integer $l > 0$. The concordance between $\{S_{3t}\}$ and $\{S_{2t}\}$ may be considerably higher than the concordance between $\{S_{1t}\}$ and $\{S_{2t}\}$ if lagged effects are important. Alternatively, it may be that changing just a few turning points could lead to near-perfect concordance between two series. If so, sensitivity analysis is worthwhile.

7.2.1. *Tests based on the method of moments.* It is important to formally test for no synchronization $H_0 : \rho_s = 0$, since this implies that $\{S_{1t}\}$ and $\{S_{2t}\}$ are unrelated series with no common cycle. As a case in point, the business cycles of the United States and the United Kingdom could have high concordance simply because most of the time these economies are expanding, not contracting. On the other hand, whether these two economies actually *move together* is a different issue. In other words, although $\{S_{1t}\}$ and $\{S_{2t}\}$ may be highly synchronized, this does not by itself imply a common cycle.

Under classical conditions, several tests for $H_0 : \rho_s = 0$ are equivalent. For instance, we can employ

[13] Vahid (2006) surveys both parametric and nonparametric methods of uncovering common cycles in multiple series. Our focus is on nonparametric methods that employ the binary variables S_1 and S_2. In contrast, to investigate synchronization in output across the G7 countries, Stock and Watson (2003) focus on the correlation between Δy_1 and Δy_2.

$$t_r = r_s\sqrt{\frac{T-2}{1-r_s^2}} \overset{a}{\backsim} N(0,1) \tag{7.2}$$

Numerically equivalent test statistics are the standard t-ratios for the slope coefficients in either $S_{1t} = \alpha + \beta S_{2t} + \varepsilon_{1t}$ or $S_{2t} = \gamma + \delta S_{1t} + \varepsilon_{2t}$. However, the situation is complicated by the fact that S_1 is serially correlated, as is S_2, and thus the independence assumption associated with the traditional t-test of zero-correlation is compromised. Therefore, the statistic used to test $H_0 : \rho_s = 0$ must be made robust to serial correlation and heteroskedasticity.[14] Harding and Pagan (2006) recommend that the test statistic be constructed via generalized method-of-moments (GMM) with a robust variance estimate to account for serial correlation and heteroskedasticity.

A multivariate version of the GMM test for several S-series is presented by Harding and Pagan (2006), but with the GMM estimator for the bivariate case based only on the moment conditions:

$$E[S_{jt}] = \mu_j,\ j = 1,2 \tag{7.3}$$

$$E[\frac{(S_{1t}-\mu_1)(S_{2t}-\mu_2)}{\sqrt{\mu_1(1-\mu_1)\mu_2(1-\mu_2)}} - \rho_s] = 0 \tag{7.4}$$

Stack the above three moment conditions into a $3x1$ vector, $h_t(\theta, S_{1t}, S_{2t})$, such that

$$h_t(\theta, S_{1t}, S_{2t})' = [S_{1t}-\mu_1,\ S_{2t}-\mu_2,\ \frac{(S_{1t}-\mu_1)(S_{2t}-\mu_2)}{\sqrt{\mu_1(1-\mu_1)\mu_2(1-\mu_2)}} - \rho_s] \tag{7.5}$$

with parameter vector $\theta' = [\mu_1, \mu_2, \rho_S]$, and take the average

$$g(\theta, \{S_{1t}, S_{2t}\}_{t=1}^T) = \frac{1}{T}\sum_{t=1}^{T} h_t(\theta, S_{1t}, S_{2t}) \tag{7.6}$$

The covariance matrix of $\sqrt{T}g(\theta, \{S_{1t}, S_{2t}\}_{t=1}^T)$ is consistently estimated by

$$V = \Gamma_0 + \sum_{k=1}^{m}[1 - \frac{k}{m+1}][\Gamma_k + \Gamma_k'] \tag{7.7}$$

where

$$\Gamma_k = \frac{1}{T}\sum_{t=k+1}^{T} h_t(\theta, S_{1t}, S_{2t})h_{t-k}(\theta, S_{1t}, S_{2t})' \tag{7.8}$$

[14] As shown by Hamilton (1989), a primary reason for the heteroskedasticity is heterogeneous transition probabilities across contractions and expansions.

Harding and Pagan (2006) recommend the window width, m, to be the integer part of $(T-1)^{1/3}$.

Let $\theta_0' = [\mu_1, \mu_2, 0]$ be the restricted parameter vector for $H_0 : \rho_S = 0$, with no common cycle between S_1 and S_2 under this null hypothesis. The test statistic

$$W_{SNS} = \sqrt{T} g(\theta_0, \{S_{1t}, S_{2t}\}_{t=1}^{T})' V^{-1} \sqrt{T} g(\theta_0, \{S_{1t}, S_{2t}\}_{t=1}^{T}) \tag{7.9}$$

follows an asymptotic chi-square distribution with one degree of freedom. Substituting sample means, $\widehat{\mu}_1$ and $\widehat{\mu}_2$, and the sample correlation, r_s, for their population counterparts does not affect the asymptotic distribution of W_{SNS}. Substituting sample moments for population moments reduces W_{SNS} to

$$W_{SNS} = T(r_s - 0)\widehat{v}^{-1}(r_s - 0) \tag{7.10}$$

where $\widehat{v}$ is the lower right-hand element of $\widehat{V}$. An equivalent test statistic is the asymptotic t-ratio, $t_{SNS} = T^{1/2}\widehat{v}^{-1/2} r_s \overset{a}{\backsim} N(0,1)$.

Closely related tests are the market-timing test of Pesaran and Timmermann (1992) and Pearson's chi-square test for independence. For instance, Artis, Kontolemis and Osborn (1997) and Artis, Krolzig and Toro (2004) use either Pearson's test or a transformation of the concordance index to test whether the series $\{S_{1t}\}$ and $\{S_{2t}\}$ are unrelated. However, consider that the method-of-moments test essentially examines the moment condition implied by the covariance, that is, $E(S_1 S_2) - E(S_1)E(S_2) - \sigma_s = 0$. The null hypothesis is $H_0 : \sigma_s = 0$. Observe, however, that $\sigma_s = p_{12} - p_1 p_2$, where $p_{12} = P(S_1 = 1, S_2 = 1)$, $p_1 = P(S_1 = 1)$, and $p_2 = P(S_2 = 1)$. The method-of-moments test thus effectively determines whether S_1 and S_2 are statistically independent, $p_{12} = p_1 p_2$, and this is exactly the goal of Pearson's test.

Of course, a critical assumption behind Pearson's test is that *observations* in the sample are statistically independent. This assumption clearly fails in this case since the state variables, S_1 and S_2, exhibit strong serial dependence; see, for example, Kedem (1980). Since the market-timing and concordance-index tests also assume that observations are statistically independent, the robust covariance matrix of Harding and Pagan's method-of-moments test offers an improvement since it allows for serial correlation and heteroskedasticity of unspecified type.

7.2.2. *Regression-based tests.* On the other hand, the problems induced by serial correlation and heteroskedasticity can be significantly lessened by separating contractions from expansions and by incorporating time dependency into the model. The very nature of duration dependence will most surely differ across expansions and contractions; even if both phases exhibit constant hazards, regression disturbances are generally heteroskedastic if observations on expansions are not separated from those on contractions. So, when testing for synchronization, we should separate expansions from contractions. Define the dependent binary variable so that $S_{2t} = 0$ if the contraction continues and $S_{2t} = 1$ if the contraction terminates. For expansions, consider $1 - S_{2t}$ instead of S_{2t}. Our sub-sample thus consists of strings like

	Jan	Feb	Mar	Apr	...	Sep	Oct	Nov	Dec	Jan	Feb	...	Jun	Jul
S_{2t}	0	0	0	1	...	0	0	0	0	0	1	...	0	0
S_{1t}	0	0	1	1	...	0	1	1	1	0	0	...	0	0

In this example, for S_2 there are two complete contractions of respective length $T_1 = 3$ and $T_2 = 5$, and one incomplete contraction of length $T_3 = 2$. The number 1 signifies the beginning of a new phase, in this case an expansion. Of course, $S_{1t} = S_{2t}$ should be the predominant case if there is a high concordance. By considering the phases separately, we are able to address problems of time dependency and heterogeneity. The statistical method we propose employs logistic regression and is similar in spirit to Pagan's (1998) regression-based test. To test for dependence between S_1 and S_2, let S_2 be the dependent variable in the logistic regression

$$\log(P(t)/(1 - P(t)) = a(t) + \beta S_{1t} \tag{7.11}$$

The term $a(t)$ consists of a set of dummy variables, one for each possible exit period, that controls for autonomous changes in the exit probability, $P(t)$. That is, $a(t)$ accounts for the duration dependence, or serial correlation, in the series $\{S_{2t}\}$. Having removed the time dependence captured by $a(t)$, the slope coefficient, β, captures the statistical dependence between S_2 and S_1. Our null hypothesis, $H_0 : \beta = 0$, corresponds to statistical independence, and thus to $H_0 : \sigma_s = 0$. The form of the above regression is identical for expansions, and completely analogous regressions can be employed for the alternative dependent variable, S_1. Finally, it is possible to use linear regression rather than logistic regression. For large samples, however, estimation efficiency is improved by using logistic regression.

8. Unemployment cycles

If the most important measure of aggregate economic well-being is output, then surely the second most important is unemployment. Although unemployment cycles are interesting to academics, policymakers, and the general public, most of the attention in the academic literature is on output cycles. Exceptions are Boldin (1994), Chin, Geweke and Miller (2000) and Hamilton (2005), whose work focuses on unemployment cycles.

Even though we anticipate high unemployment during GDP contractions and low unemployment during GDP expansions, the output and unemployment series offer separate pieces of information about the economy. Even if we use the same rules to mark turning points for unemployment and output, we do not expect perfect negative synchronization - that is, necessarily *increasing* levels of unemployment during periods of *decreasing* output, and vice versa. It is thus of interest to determine the degree of synchronization, to compare the shapes of output and unemployment cycles, and to perform a separate duration analysis for the unemployment series.

8.1. Cycle shapes. Our data are the logarithmic monthly BLS seasonally adjusted *civilian unemployment rate* series from 1948:1 through 2007:1.[15] Figure III plots the unemployment rate series and for comparison marks the NBER dated business cycle recessions. Table 1 presents the unemployment rate reference dates determined by the BBQ algorithm. We set the minimum phase to 9 months and the minimum cycle to 18 months. These censoring rules visually mark the turning points in unemployment much better than those employed by Harding and Pagan (2002) to mark the turning points in output - namely, a minimum phase of 6 months

[15] The series ID is LNS14000000, and is available from the U.S. Department of Labor, Bureau of Labor Statistics (http://stats.bls.gov).

and a minimum cycle of 15 months. Our censoring rules also mark the turning points in unemployment much better than does the Extended Okun Rule that considers only whether there are two consecutive months opposite the prevailing phase. There are 10 postwar completed spells of contraction, or *downswings*, in the unemployment rate, lasting an average of 48 months with a standard deviation of 30 months. There are 9 postwar completed spells of expansion, or *upswings*, in the unemployment rate, lasting an average of 22 months with a standard deviation of 10 months. Similar results are obtained from using quarterly data with a minimum phase of 3 quarters and a minimum cycle of 6 quarters.

<Figure III and Table 1 about here>

The average amplitudes of upswings and downswings are the same in magnitude. The average amplitude of downswings is -0.55 with a coefficient of variation of -0.55, and the average amplitude of upswings is 0.55 with a coefficient of variation of 0.33. Since falls in unemployment appear about evenly matched with rises in unemployment, this lends some credence to the idea of a natural rate. The overall sample average unemployment rate is about 5.61 percent, although at times there were large deviations from the average. For example, in November 1982 the unemployment rate reached a high of 10.8 percent.

There are, however, significant differences between upswings and downswings. For downswings, the average cumulative movement is $\overline{F}_c$ = -18.43, the average excess is $\overline{C}$ = -0.046 and the coefficient of variation in the excess is -1.32. For upswings, the average cumulative movement is $\overline{F}_e = 6.40$, the average excess is $\overline{E} = 0.009$ and the coefficient of variation in the excess is 5.70. The average cumulative movement in downswings is well over twice the magnitude of the cumulative movement in upswings; this is consistent with the longer average duration of downswings. From the average excess $\overline{C}$, employment tends to fall at a decreasing rate during contractions, or downswings, and from $\overline{E}$, employment tends to rise at a decreasing rate during expansions, or upswings. The steep initial decline in unemployment during downswings is more prominent than the steep initial ascent in unemployment during upswings. In fact, from the coefficient of variation, there is much more relative variability in the excess for upswings than for downswings. Long durations in downswings, large cumulative movements, and stable excess across downswings all reflect favorably on current economic policy.

8.2. Synchronization with business cycles. From Figure III, cycles in output and unemployment appear highly synchronized. However, even though there are about as many turning points in the unemployment cycle as there are in the business cycle (see the summary statistics in Table 1), the two binary series representing unemployment and output are not perfectly correlated. Let $S_{1t} = 1$ if output is rising and $S_{2t} = 1$ if unemployment is rising; also let $S_{1t} = 0$ if output is falling and $S_{2t} = 0$ if unemployment is falling. The coincidence indicator for the binary series is $\widehat{I} = 0.17$ and the correlation between them is $r_s = -0.61$. In other words, about 83 percent of the time rising output is associated with falling unemployment.

We can test the null hypothesis $H_0 : \rho_s = 0$ by using Harding and Pagan's (2006) method of moment test. With a bandwidth of either $m = 8$ or $m = 9$ we find that t_{SNS} is about -5.5, and thus we reject the null hypothesis that unemployment and output are statistically independent. Regression-based tests yield the same conclusion. Unemployment and output appear to be statistically dependent.

Separating upswings from downswings in unemployment provides an important insight. When measured at monthly intervals with our censoring rules, falling unemployment is coincident with rising output, though rising unemployment is not necessarily coincident with falling output. In fact, conditional on rising unemployment, output is falling only about half the time. In other words, unemployment is perfectly synchronized with output when unemployment is falling, but it is a coin toss when unemployment is rising.

8.3. Duration analysis. If expansions, or upswings, in the unemployment rate exhibit *positive duration dependence,* then upswings with longer maturities have a higher chance of terminating than those with shorter maturities. The hazard tends to rise with the duration of the event, and the average duration is greater than the standard deviation. In the opposite case, called *negative duration dependence,* the hazard tends to fall with the duration of the event, and the average duration is less than the standard deviation. *Duration independence* is characterized by neither of the above cases. For instance, the probability that unemployment exits an expansionary state and enters a contractionary state is constant regardless of how long the upswing has lasted.

The average duration of 22 months for upswings is more than twice the sample standard deviation of 10 months; this suggests that rises in unemployment exhibit positive duration dependence. Likewise, the average duration of 48 months for downswings is larger than the sample standard deviation of 30 months; this also suggests positive duration dependence, though such descriptive evidence is not as strong as it is for upswings. We can formally test for duration dependence by using the discrete-time SB and GMD tests from Ohn et al. (2004). We subtract nine months from each of the observed durations to be consistent with our BBQ censoring rules.[16]

We analyze upswings and downswings separately. Under the null hypothesis of duration independence, the estimated termination probability for upswings is 0.076, but the estimated termination probability for downswings is only 0.025. However, neither SB nor GMD indicate duration dependence for either upswings or downswings. Finite-sample p-values are obtained through a parametric bootstrap algorithm, with the discrete-time geometric density corresponding to the null hypothesis of duration independence. Sensitivity analysis is performed by varying the termination probability to account for sampling variability in estimation. Since our calculated p-values are always considerably greater than 0.10, we cannot reject the null hypothesis that the probability of exiting a state of national unemployment is independent of its duration at the 10 percent significance level. As a further robustness check, we also employ the asymptotic LSB t-test and the continuous-time W-test from Shapiro and Wilk (1972). The distribution of W depends neither on the termination probability nor the minimum phase, with exact finite-sample critical values tabulated by Shapiro and Wilk (1972). Consistent with SB and GMD,

[16] Ohn *et al.* (2004) refer to imposing a minimum phase jointly with the assumption of duration dependence as the *Markov hypothesis.* An alternative to subtracting the minimum phase is to incorporate the phase restriction into the econometric model. For instance, Harding and Pagan (2003) show that for a two-quarter minimum the base model that includes S_{t-1} is extended by including the variables S_{t-2} and $S_{t-1}S_{t-2}$.

neither LSB nor W reject duration independence for either upswings or downswings in unemployment.

In comparision, Ohn et al. (2004) find some evidence that postwar economic contractions exhibit positive duration dependence, though there is no such evidence for postwar expansions. However, such lack of evidence is not too surprising in light of the small number of completed cycles. For instance, postwar sample sizes are too small to employ the chi-square goodness-of-fit test for comparison with SB and GMD.

Figures IV and V present nonparametric plots of the hazard functions via the life-table method of Cutler and Ederer (1958). The hazard functions for both downswings and (especially) upswings can be characterized as NBU, or perhaps even the more narrow classes, IFRA or IFR. The hazard function for upswings exhibits a local peak at 15 months, dropping at 16 months, and ascending thereafter. The rising hazard is consistent with the sharply rising hazard function for economic contractions observed by Ohn et al. (2004). The hazard function for downswings exhibits strong IFR behavior with an upward trend, and this coincides well with the nonparametric hazard function for economic expansions from Ohn et al. (2004). For comparison, we also graph the parametric Weibull hazard rates. In each case, the Weibull hazards indicate positive duration dependence. On the other hand, for upswings the parametric Weibull hazard fails to reflect the clustering of exits at 15-16 months.

<Figures IV and V about here>

We obtain further insights about the hazard functions through a regression analysis.[17] Our approach is most closely related to Estrella and Mishkin (1998) and especially that of Chin, Geweke and Miller (2000). For example, as did Chin et al. (2000), we separate upswings from downswings to obtain separate estimates of the coefficients that control the hazard probabilities. As a point of contrast, our sample consists of monthly observations on unemployment from January 1948 through January 2007, whereas the sample of Chin et al. (2000) consists of monthly observations on unemployment from October 1949 through February 1998. We employ BBQ rules similar to those used for business cycles, whereas Chin et al. (2000) employ a three-month centered moving average rule subject to a threshold condition. Using either set of rules, the average upswing lasts roughly 23 months, and the average downswing lasts roughly 50 months; to be exact, for downswings we found an average of 48 months while Chin et al. (2000) found an average of 51 months.

Chin et al. (2000) employ probit estimation, closely related to our logit estimation. We favor logit estimation, however, since its use follows directly from the seminal work of Cox (1972). An additional difference is that, as is customary, we set our binary dependent variable to unity only in the month of a turning point, whereas they set the binary variable to unity in month t if a turning point occurs in months $t+1, ..., t+12$. Chin et al. (2000) construct artificial observations to control for overfitting of the model at highly leveraged values of the independent variables, but we employ a sensitivity analysis to determine whether our results are robust to dropping spells from the sample each in turn. For instance, we consider estimates from the full set of postwar contractions as well as contractions but for, say, the fourth one.

[17]Our LIMDEP program for the regression analysis is presented in the appendix.

Unlike Estrella and Mishkin (1998) and Chin, Geweke and Miller (2000), we eliminate observations from the beginning of each spell to account for censoring. That is, we eliminate the first nine observations from each observed spell since the minimum phase duration allowed for either upswings or downswings is nine. As per the classical approach, we include dummy variables to allow for autonomous shifts in the hazard probabilities. In contrast, neither Estrella and Mishkin (1998) nor Chin et al. (2000) consider that duration dependence may be autonomous. That is, they do not allow hazard probabilities to change with the duration of the spell, independently of any change in time-varying covariates.

Harding and Pagan (2007) show that most binary time series constructed from the BBQ algorithm using NBER censoring rules are serially correlated with heteroskedastic disturbances. As is well known, either one of serial correlation or heteroskedasticity will typically lead to invalid inference. Furthermore, serial correlation in the binary time series, $\{S_t\}$, typically implies autonomous duration dependence in the spells of both contractions and expansions. Harding and Pagan (2007) account for autonomous duration dependence by directly modeling serial dependence in the state variable, S_t. Observe, however, that modeling the serial dependence is infeasible once contractions are separated from expansions to eliminate heteroskedasticity in the disturbances. Suppose, for example, that S_t follows the second-order process

$$S_t = \gamma_1 S_{t-1} + \gamma_2 S_{t-2} + \gamma_3 S_{t-1} S_{t-2} + u_t \tag{8.1}$$

Having conditioned on either contractions or expansions, then $S_{t-1} = 0$ and $S_{t-2} = 0$, and it is thus infeasible to directly estimate equation 8.1. On the other hand, it is still possible to estimate the autonomous hazard function. Indeed, semiparametric estimation of the autonomous hazard function is a very direct approach that addresses the same goal of accounting for the serially dependent nature of S_t. In other words, conditioning the constructed binary series on the states allows for estimation of the hazard function while eliminating the need to model potential serial dependence and heteroskedasticity in the state variable, S_t.

Consider first downswings with binary dependent variable S_t, such that $S_t = 1$ signifies a turning point towards rising unemployment. By considering only downswings, we eliminate one type of heteroskedasticity that occurs when upswings and downswings are considered together; see Hamiltion (1989). Define the following autonomous-shift dummy variables: $D_1 = 1$ for months 1-20, inclusive, and $D_1 = 0$ otherwise; $D_2 = 1$ for months 21-30, inclusive, and $D_2 = 0$ otherwise; and $D_3 = 1$ for periods >30 and $D_3 = 0$ otherwise.[18]

Our first model is

$$\log(P(t)/(1 - P(t))) = a_1 D_{1t} + a_2 D_{2t} + a_3 D_{3t} \tag{8.2}$$

with estimates of the $a's$ reported in the second column of Table 2. Each of the coefficients is significant at the 5% level of significance. The hazard, or exit, probability is given by the formula

[18] Because of the limited number of postwar expansions in unemployment, it is not possible to include a dummy variable for each possible exit time. Further, the effective range for D_1 is 10-20 since it is not possible for the spell to terminate in the first nine months due to censoring.

$$P_i = 1/[1 + \exp(-a_i)] \tag{8.3}$$

The estimated exit probability is about 0.019 in any month in the interval 10-20, about 0.013 in any month in the interval 21-30, and about 0.031 in any month greater than 30. These values are very close to those obtained from the linear probability model - that is, by employing least-squares with dependent variable S_t and interpreting the $a's$ from that model as probabilities. The implication is a U-shaped hazard for unemployment contractions. However, at the 5 percent significance level, the asymptotic likelihood ratio test from the logit model does not reject the null hypothesis of constant-hazard probabilities, $H_0 : a_1 = a_2 = a_3$.

Consider next expansions with binary dependent variable $1 - S_t$, such that $1 - S_t = 1$ signifies a turning point towards falling unemployment. For model (8.2), the estimates of the $a's$ are reported in the second column of Table 3. Each of the coefficients is significant at the 5% level. The estimated exit probability is about 0.074 in any month in the interval 10-20, about 0.036 in any month in the interval 21-30, and about 0.105 in any month greater than 30. Again, these values are very close to those estimated by the linear probability model. Consistent with our life-table analysis, the estimated hazard function rises much more rapidly for upswings than for downswings in unemployment. However, we again fail to reject the null hypothesis of constant-hazard probabilities, $H_0 : a_1 = a_2 = a_3$, for upswings in unemployment.

<Tables 2 and 3 about here>

Other variables may also influence the hazard probabilities. Our second equation in Table 2 augments the dummy variables with the duration of the immediately preceding (or lagged) upswing, and our second equation in Table 3 augments the dummy variables with the duration of the lagged downswing. Since neither lagged value is statistically significant, there is insufficient evidence from the logit model to conclude that the length of the current phase is influenced by the length of the preceding phase. In contrast, evidence from the linear probability model *does* suggest that the lag effect is important for upswings in unemployment. On average, the longer the preceding downswing the shorter the current upswing.

Finally, we consider including the time-varying explanators examined by Chin et al. (2000). Let the naught symbol, 0, signify the date of the last turning point prior to the date t. Then $(UR_t - UR_0)$, $(CU_t - CU_0)$, and $[(R-r)_t - (R-r)_0]$ are the respective differences in the values of unemployment (UR), the manufacturing capacity utilization rate (CU) and the spread between the monthly average of Moody's Aaa corporate bond rate and the monthly average of the 90-day Treasury bill rate $(R - r)$. The differences in these variables are measured from the start of the phase t_0, up to time t. Logit or probit models can be adapted easily to incorporate such explanators that vary over time, and one purpose for including them is to account for nonstationarity of the turning points - or in other words, duration dependence captured by covariates; see also Pesaran and Potter (1997). We also include in our regression equations the dummy variables D_1, D_2 and D_3 that account for autonomous shifts in the hazard probabilities, as well as the first differences in UR, CU, and $(R - r)$ that measure changes from time $t - 1$ to t.

Of the six time-varying explanators, only two were deemed important in our logit model: the change in the interest rate spread from time t_0 and the change in capacity utilization from time $t-1$: $\Delta CU_t = CU_t - CU_{t-1}$. The interest rate spread typically increases about 2 percentage points over the life of an upswing, and decreases about 2 percentage points over the life of a downswing. We expect the hazard probability for downswings to be inversely related to the change in the spread from time t_0, and the hazard probability for upswings to be directly related to the change in the spread from time t_0. Inclusion of the interest-rate spread corrects for drift in the hazard, though not in the same manner as do the autonomous-shift dummy variables.

From Tables 2 and 3, the *signs* on the spread coefficients are consistent with our prior reasoning. The coefficient on the spread is, however, statistically significant only for downswings. Thus, *ceteris paribus*, a large interest-rate spread lowers the termination probability for a downswing in unemployment (a healthy labor market) but does not increase the termination probability for an upswing in unemployment. In other words, having controlled for autonomous shifts in the hazards, the interest-rate spread has no discernable effect during unemployment upswings.

The economic interpretation of the change in capacity utilization is straightforward. An increase in utilization from time $t-1$ should increase labor usage at time t, and a decrease in utilization should decrease labor usage. The signs on the coefficients in Tables 2 and 3 are again consistent with our prior reasoning. However, in this case the coefficient on utilization is statistically significant only for upswings in unemployment. Therefore, a decrease in capacity utilization has no discernable effects in good labor markets, but an increase marks a turnaround in bad labor markets. As a thought experiment, suppose that $D_1 = 1$ but that all other variables are set to zero in the equation for upswings. The termination probability for this relatively young upswing is only about 0.025 using the estimated coefficient for D_1 in the fourth column of Table 3. Consider increasing capacity utilization by 1 percentage point, say from 85 percent to 86 percent. With this change, the termination probability increases to about 0.18, a seven-fold increase in the hazard. Thus, as intuitively expected, labor fares substantially better when capital is re-utilized.

At the 5 percent significance level, the above empirical results on the interest-rate spread and capacity utilization are robust to whether we include either the autonomous-shift dummy variables or a single intercept term. For with a single intercept, there is a constant hazard unless time-varying covariates actually change in value. A single-intercept probit model was chosen by Estrella and Mishkin (1998) to forecast turning points in U.S. recessions, and by Chin et al. (2000) to forecast turning points in U.S. unemployment.

Our empirical results also appear quite robust to dropping each spell, in turn, from the respective samples of either upswings or downswings. For example, we estimate the logit model with the full sample of 10 downswings, as well as with the sample of 9 downswings that excludes, say, the third spell. At the 5 percent significance level, our sensitivity analysis again indicates that the interest-rate spread is statistically significant for downswings and that capacity utilization is statistically significant for upswings.

9. Conclusion

Duration analysis has many uses, both in academe and in industry. Consider that in a recent issue of *Business Week* (January 22, 2007) there were two duration applications in separate fields. Hugh Moore of Guerite Advisors (*Business Week*, p. 13) notes that the average amplitude in the fall of housing starts is 51 percent from peak to trough, and the average amplitude in the fall of housing expenditures as a percent of GDP is 28 percent from peak to trough. Housing corrections, or recessions, last an average of 27 months. In the same issue (*Business Week*, p. 62), the global macro group for Barclays Global Investors (BGI) reports devising a set of signals, or leading indicators, that predict turning points from recession to expansion in various countries. Profits are realized by buying stock and shorting bonds before the recovery is generally recognized. Using leading indicators is similar to using covariates in a duration analysis.

In this chapter we have emphasized classical, nonparametric methods in the duration analysis of unemployment cycles. The nonparametric turning-point algorithm from Harding and Pagan (2002), or BBQ, is derived from the classical graphical approch of Burns and Mitchell (1946). The life-table analysis follows directly from Cutler and Ederer (1958), and the logit model derives from the seminal work of Cox (1972). These classical techniques are just as relevant today as when first introduced, and have both micro- and macro-econometric applications.

Consider the many economic studies of individual job histories. Adamchik (1999), for example, uses the nonparametric Kaplan-Meier estimator and the semiparametric proportional-hazards model in her study of the effect of unemployment benefits on re-employment in Poland. In a related study, Bover et al. (2002) examine not only unemployment benefits, but also the relationship between the unemployment duration of Spanish men and the business cycle. In the latter study, they employ a logit model with autonomous shift dummies that is closely related to Cox's famous proportional-hazards model. The approach is very flexible since Cox's model is no longer proportional when explanators that vary with time are included in the model. Following the early frontier work of Heckman and Singer (1984), Bover et al. (2002) then extend their logit model to account for unobserved heterogeneity.

In more recent macroeconometrics literature, Mudambi and Taylor (1995), Pagan (1998), and Ohn et al. (2004) propose discrete-time tests for duration dependence and use bootstrap methods for finite-sample inference. Harding and Pagan (2002, 2006) propose new measures and tests for cycle asymmetries and synchronization. The practical importance of duration analysis for aggregate series is best illustrated by the March 28, 2007 testimony of Federal Reserve Chairman Ben Bernanke to the Joint Economic Committee of Congress. In response to his predecessor Alan Greenspan, who warned that the current expansion could be fizzling out, Bernanke responded:

> I would make a point, I think, which is important, which is there seems to be a sense that expansions die of old age, that after they reach a certain point, then they naturally begin to end. I don't think the evidence really supports that. If we look historically, we see that the periods of expansions have varied considerably. Some have been quite long.

Bernanke thus discounts the notion that expansions exhibit positive duration dependence. This view concurs with that of Ohn et al. (2004), who fail to reject the constant-hazard assumption for post-WWII expansions but who do find statistically significant evidence of positive duration dependence in pre-WWII expansions. On the other hand, consider that the lack of support for positive duration dependence in the postwar period may be due to the small sample size. The mean duration of postwar expansions is about 50 months with a standard deviation of about 30 months. A mean larger than the standard deviation suggests positive duration dependence.

Finally, in this chapter we stress the advantage of a separate analysis of upswings and downswings in unemployment. Indeed, given a downswing in unemployment, aggregate output is always rising, but given an upswing in unemployment, the behavior of output is a coin toss. For a young spell of rising unemployment, an increase in capacity utilization of 1 percentage point increases by seven-fold the probability of a turning point from upswing to downswing. In contrast, for downswings the interest-rate spread appears to affect the termination probability, and capacity utilization does not appear to matter.

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Figure I: Hazard Functions for Various Life Distributions

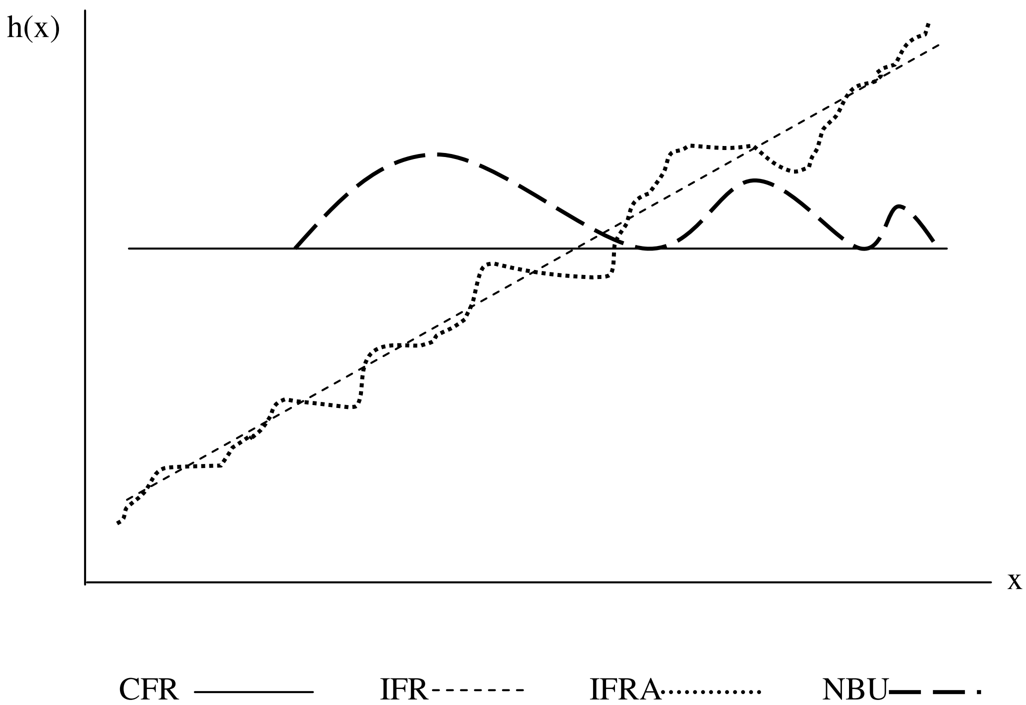


Figure II: Stylized Expansion Phase

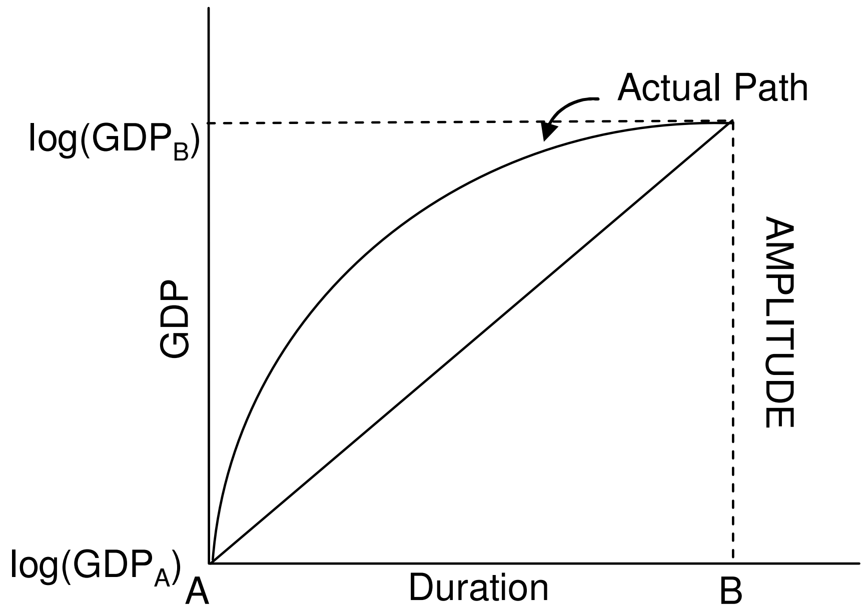
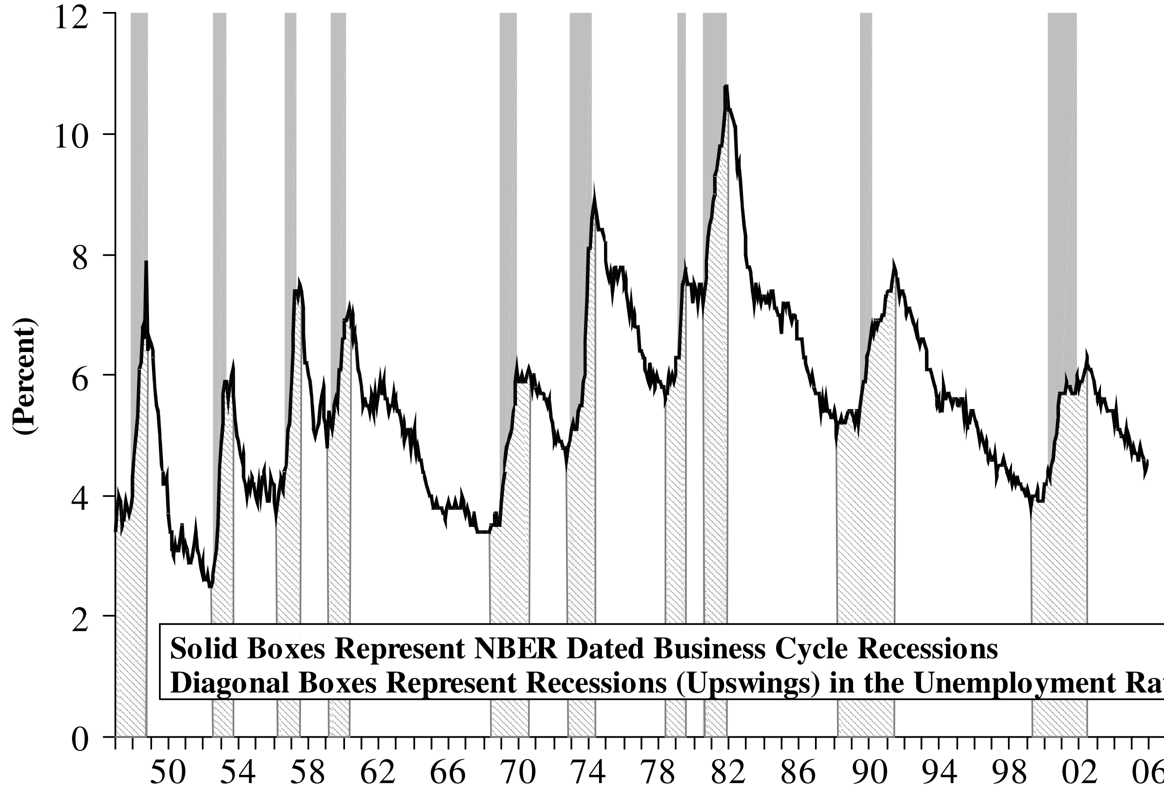


Figure III: U.S. Civilian Unemployment Rate*



* Unemployment data are from the U.S. Bureau of Labor Statistics, and business cycle dates are from the National Bureau of Economic Research

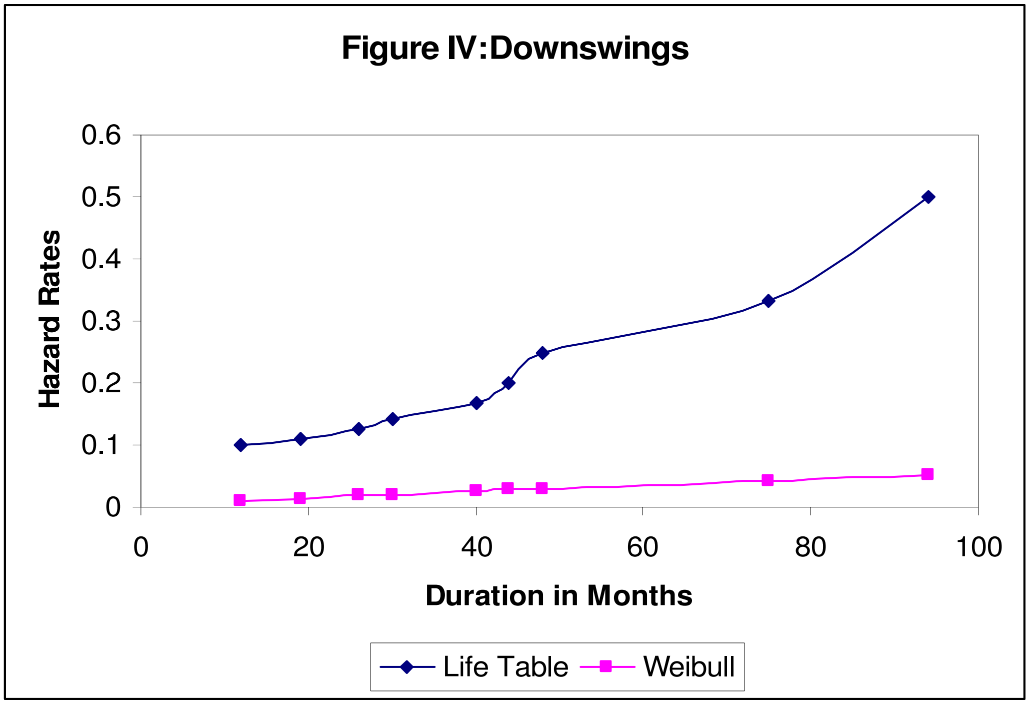


Figure IV:Downswings
Hazard Rates
0.6
0.5
0.4
0.3
0.2
0.1
0
0
20
40
60
80
100
Duration in Months
Life Table
Weibull

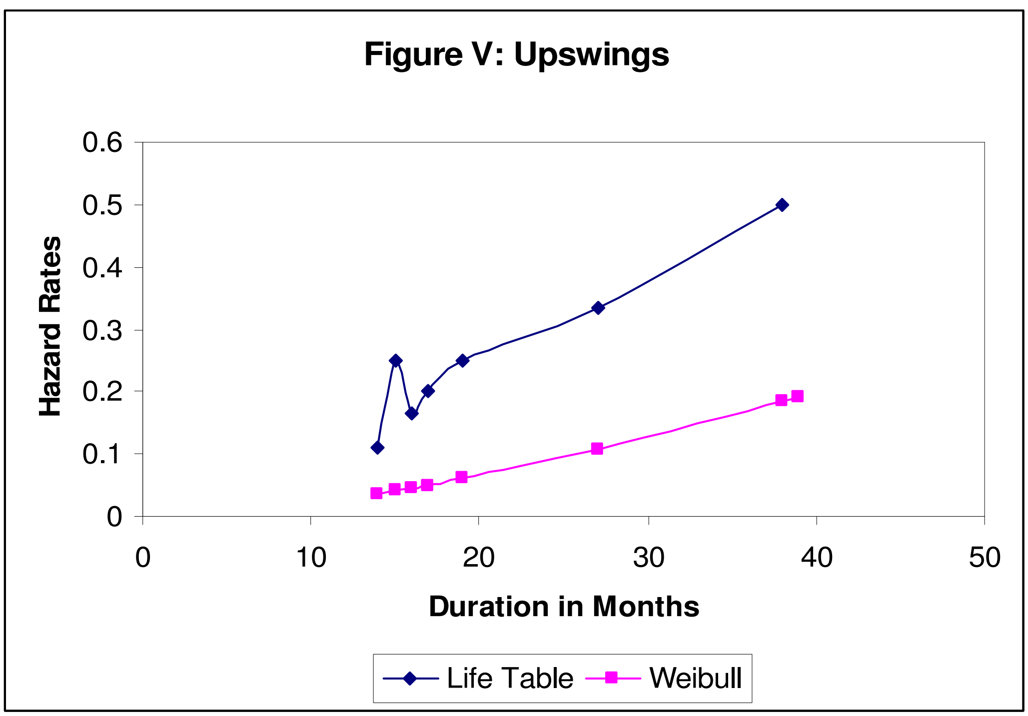


Figure V: Upswings
Hazard Rates
0.6
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
Duration in Months
Life Table
Weibull

Table 1: Unemployment Rate Cycle Dates: 1948 to 2006

REFERENCE DATES		DURATION IN MONTHS			
Peak	**Trough**	**Contraction**	**Expansion**	**Cycle**	
		Peak to Trough	*Previous Trough to this Peak*	*Trough from Previous Trough*	*Peak from Previous Peak*
October 1949	June 1953	44	--	--	--
September 1954	March 1957	30	15	45	59
July 1958	February 1960	19	16	35	46
May 1961	May 1969	96	15	111	34
August 1971	October 1973	26	27	53	123
May 1975	May 1979	48	19	67	45
July 1980	July 1981	12	14	26	62
December 1982	March 1989	75	17	92	29
June 1992	April 2000	94	39	133	114
June 2003	June 2006	36	38	74	132

Summary Statistics

Number of Phases	10 (9)	9 (10)
Average Length	48.0 (59.1)	22.2 (10.4)
Standard Deviation	30.3 (38.0)	10.0 (3.3)

NBER business cycle summary statistics are in parentheses. Unemployment contractions are paired with business cycle expansions, and unemployment expansions are paired with business cycle contractions.

Table 2: Logit Estimation: Downswings in Unemployment

Variables	*Equation 1*	*Equation 2*	*Equation 3*
	a(t): Autonomous Shift Variables		
D_{1t}	-3.922 (.0000)	-3.947 (.0000)	-4.769 (.0000)
D_{2t}	-4.331 (.0000)	-4.362 (.0000)	-5.515 (.0000)
D_{3t}	-3.434 (.0000)	-3.447 (.0000)	-5.053 (.0000)
	x: Fixed Exogenous Variables		
Lagged Upswing	-	-.00025 (.8104)	-
	x(t): Changeable Exogenous Variables		
$(R\text{-}r)_t - (R\text{-}r)_0$	-	-	-0.726 (.0181)
$CU_t - CU_{t-1}$	-	-	-0.628 (.1379)

P-values in parentheses.

Table 3: Logit Estimation: Upswings in Unemployment

Variables	*Equation 1*	*Equation 2*	*Equation 3*
	a(t): Autonomous Shift Variables		
D_{1t}	-2.526 (.0000)	-1.326 (.0754)	-3.677 (.0004)
D_{2t}	-3.300 (.0012)	-0.872 (.6336)	-4.813 (.0011)
D_{3t}	-2.140 (.0042)	0.186 (.9101)	-3.543 (.0107)
	x: Fixed Exogenous Variables		
Lagged Downswing	-	-0.028 (.1158)	-
	x(t): Changeable Exogenous Variables		
$(R\text{-}r)_t - (R\text{-}r)_0$	-	-	0.338 (.3659)
$CU_t - CU_{t-1}$	-	-	2.170 (.0005)

P-values in parentheses.

```
/*LIMDEP 7.0 PROGRAM FOR JACKKNIFING DURATION DATA, June 2007*/

READ ; FILE = LUexp.TXT ;           ?  DATA IN ASCII FORMAT
              NVAR =   6;           ?  NUMBER OF VARIABLES
              NOBS = 129;           ?  NUMBER OF OBSERVATIONS
              NAMES = SB,D1,D2,D3,BUS,PHASE $ ? VARIABLE NAMES

/* ADD ";TEMP = TFILE" FOR LARGE DATA SETS.

SB: SET SB=0 IF THE EXPANSION CONTINUES, AND SET SB=1 IF THE
EXPANSION TERMINATES.  THUS, SB=1 SIGNIFIES THE BEGINNING OF A
CONTRACTION. CENSORING IS OBTAINED BY ELIMINATING SOME OF THE
OBSERVATIONS WITH SB=0.  TO IMPOSE A MINIMUM DURATION OF ONE
MONTH, ELIMINATE THE FIRST OBSERVATION OF EACH PHASE.  TO IMPOSE
A MINIMUM DURATION OF TWO MONTHS, ELIMINATE THE FIRST TWO
OBSERVATION OF EACH PHASE, AND SO ON.

D1,D2,D3: AUTONOMOUS CHANGE DUMMY VARIABLES.

PHASE: SET PHASE=1 FOR THE FIRST EXPANSION, PHASE = 2 FOR THE
SECOND EXPANSION, AND SO ON, UP TO PHASE=L FOR THE LAST
EXPANSION, "L".*/

LIST; SB,D1,D2,D3 $

NAMELIST; Z = D1,D2,D3 $ NAMES OF RIGHT-HAND-SIDE VARIABLES

REGRESS ; LHS = SB ; RHS = Z ; PDS=5 $ ORDINARY LEAST SQUARES
LOGIT   ; LHS = SB;  RHS = Z         $ LOGISTIC REGRESSION

CALC ; MP = MAX(PHASE) $  CALCULATE THE NUMBER OF PHASES

PROC
      SAMPLE; ALL$
      REJECT ; PHASE = I $  ELIMINATE THE ITH PHASE.

      REGRESS; LHS = SB; RHS = Z ; PDS = 5 $
      LOGIT ;  LHS = SB; RHS = Z $

ENDPROC

EXECUTE ; I = 1,MP $
STOP
```