

Merit Aid and Competition in the University Marketplace*

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December 2006

*The authors thank Kalyan Chatterjee, Keith Crocker, and Roger Geiger for thoughtful comments.

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Abstract

Colleges and universities in the United States increasingly are turning to merit aid offers as a competitive tool to attract better students. Although the total amount of merit aid offered has increased recently, universities vary dramatically in the amount they use to attract top candidates. Intuitively, better (and wealthier) universities, who have better applicants, should offer more merit aid, but the top-ranked universities actually offer far less than do others, and some top schools offer no merit aid at all. The authors construct a theoretical model to explain this phenomenon and demonstrate that the quality of universities per se does not drive the negative relationship between university quality and merit aid offers; rather: (1) the differences between the quality levels of competitive universities and (2) the universities' valuations of applicants drive the negative relationship. Top universities offer less merit aid because they and their immediate competitors represent greater quality differences than do more poorly ranked schools and have access to better safety candidates. We provide empirical evidence to support key assumptions and findings of our model.

Keywords: Merit financial aid, university competition, applied game theory, pricing in quality-differentiated oligopoly

Squeezed on one side by state universities, whose tuition is a tiny fraction of what private colleges charge, and on the other by elite private institutions like Yale, Princeton or Amherst, private liberal arts colleges like Allegheny are routinely offering merit aid to students these days. Such scholarships are particularly pervasive in the Midwest, where many liberal arts colleges award them to as many as half or even three-quarters of their students.... The result is a college pricing system that can feel as varied, or even mysterious, as buying airplane seats, with students sometimes shopping for the best deal. University officials, defending the era of \$30,000-a-year tuitions, speak of a "sticker price" and "discount price" and note that many students do not pay close to the full costs of tuition.... So prevalent has the practice become that over the last decade, the amount of money granted in merit scholarships nationally grew to \$7.3 billion in 2004 from \$1.2 billion in 1994, said Kenneth E. Redd, director of research and policy analysis at the National Association of Student Financial Aid Administrators.

—Finder 2006

Financial aid once went to the poorest kids. Now, grants awarded for academic merit or special talent in sports or the arts are growing faster than grants based on need. States spend 25% of their scholarship money on merit awards, up from 10% a decade ago, while private colleges have gone to a 36% merit share, up from 27%. Private colleges have always used merit aid to round out their orchestras or sports teams, of course. But now they increasingly see merit aid as a way to help them win the ratings-guide race and to "shape" a freshman class by, for example, recruiting science majors. Fifteen states, meanwhile, are using merit scholarships to lure bright in-state students to their local universities. The states calculate that the tactic will motivate high schoolers and raise the rates of those going to college, keep educated young people in-state after graduation, and make themselves more attractive to employers. Florida and Georgia are finding their merit-aid programs hugely expensive, but politically difficult to scale back. Even so, another half-dozen states are looking at their own merit plans.

—Kronholz 2005

1. Introduction

The preceding quotes reflect the vibrant higher education marketplace, in which universities compete for students, faculty, prestige, and financial resources. The widely cited university rankings, such as the general undergraduate rankings by *U.S. News and World Report's* annual "America's Best Colleges" feature and *The Wall Street Journal*, provide highly visible scorecards to summarize the results of that competition. The salience of these rankings makes it imperative for universities to adopt strategies to improve their ranks. One of the most striking facts about the university ranking race is the increasing emphasis on the tactical use of merit aid: The amount of money granted in merit scholarships nationally grew to \$7.3 billion in 2004 from \$1.2 billion in 1994 (Finder, 2006).

As the university marketplace grows ever more competitive, merit aid is becoming a potent tool that universities use to price discriminate and attract better students. (Differing merit aid offers mimic the practice of third-degree or multimarket price discrimination; see Tirole 1988.) Kane (1999, p. 80-81) makes an interesting point about merit aid and price discrimination:

In many industries, differing prices for different buyers of the same product are taken as a sign of market power. However, in higher education, such price discrimination is a result of the declining market power of colleges. Competition tends to force an institution's prices closer to its costs. But because each student adds a different amount to the value of his or her classmates' degrees, the net cost of educating each student is different even if the cost of the bricks and mortar is the same.

With the increased popularity of rankings publications and the role that student quality plays in them, the substantial jump in merit aid offers seems easy to explain. For example, though Fallows (2003) is critical of the rankings publications, he admits that rankings have promoted an educational meritocracy in which the best students, in contrast with lower-quality legacy students (whose relatives have attended the university), are more likely to be accepted by the top universities. Thus, universities seemingly should offer financial enticements to attract the best students (see Rothschild and White 1995); in turn, the best students (who tend to apply to the top universities) should receive the most merit aid (as depicted by the downward sloping line in Figure 1).

However, the best students do not necessarily receive the most merit aid. In the university marketplace, these students tend to be matched with the top universities, and top-ranked universities actually offer less merit aid than other good, highly ranked, albeit not top-ranked, universities (Geiger 2004). (See Figure 1 to observe the actual link between university rank and merit aid; see also supporting data in the Appendix, Table A1.) Some top-ranked universities (including all eight Ivy League schools) offer very little or no merit aid. Therefore, the best students who choose to attend Ivy League universities do so largely without the benefit of merit aid, although the Ivies offer generous need-based aid packages. That is, rather than top universities offering more merit aid to attract these top students, they actually offer less.

[Insert Figure 1 about here]

If the best students should receive more merit aid, the empirical observation that top universities offer less seems puzzling. The competition among the very top universities to attract very high achieving high school seniors should be at least as intense as the competition among other highly

selective schools to attract simply high achieving high school seniors. But such intense competition is not reflected in merit aid offers.

The conjecture that merit aid should be decreasing in university quality is consistent with recommendations in the pricing literature (e.g., Nagle and Holden 2002), which suggests low quality brands and products should offer lower prices. However, better-ranked universities often charge approximately the same list prices as do more poorly ranked schools. Empirically, the dispersion in list price tuition among universities is much less than the dispersion in merit aid offers, so the relative price paid by students is determined largely by merit aid offers. For example, the list price 2005–06 tuition of Harvard University is \$32,097, whereas the list price tuition of Vanderbilt University is \$31,700. Harvard offers no merit aid, whereas 11% of Vanderbilt students receive merit aid, and the award per student averages \$2,309 (Table A1). For the few targeted, best applicants from its candidate pool, Vanderbilt clearly offers more merit aid than does Harvard, but for those students who receive no merit aid, the prices of these two universities are virtually identical.

To explain the empirical puzzle that higher quality universities offer less merit aid, we construct a game-theoretic model in which each university's objective in managing its merit aid is to attain the best possible rank. In calculating optimal merit aid offers, we assume each university determines: (1) its valuation of each candidate from that candidate's SAT scores, class ranks, and so forth, (2) the competitors to which the university believes the student has applied and been accepted; (3) the university's beliefs about the merit aid offers of the universities that have accepted the candidate; and (4) each candidate's preferences. We then focus on how the equilibrium merit aid that might be offered to each applicant depends on the qualities of the universities engaged in the competition to attract that candidate, the dispersion of qualities among competitors, and the universities' valuation of the applicant.

Our analysis rests on the conjecture that the university marketplace is not one large market but rather a series of “quality-local” markets, each of which contains universities of similar quality or rank.

In these quality-local markets, the universities at each level compete almost exclusively with others at the same level to attract the top potential candidates. For example, the Ivy League universities Harvard and Yale compete, and the Patriot League universities Colgate and Bucknell compete, but Harvard and Colgate largely do not compete for the same students. We might expect that the Ivy League schools would behave toward their target students much the same way that the Patriot League schools do in competing for the pool of talented, albeit not top, candidates (a horizontal line in Figure 1). Because the Ivies compete for top students, we might even expect them to offer more merit aid.

However, the negative correlation between merit aid and quality suggests the marketplace of top universities differs fundamentally from the more general marketplace of high-quality universities. We offer empirical evidence for two differences between top ranked universities and other relatively lower ranked high-quality universities. First, the dispersion of competitor quality (i.e., how close in quality universities engaged in competition are) correlates positively with the quality of a university. That is, the dispersion of the quality among top universities is greater than that of relatively lower ranked high-quality universities. Second, top universities reject many students who are close in quality to those whom they accept. Therefore, these stronger alternative options minimize top universities' incentive to offer merit aid. In our theoretical model, we demonstrate that these two differences drive the result that top universities offer less merit aid.

We proceed as follows: In Section 2, we establish the model, and then in Section 3 develop the theoretical results. In Section 4, we provide some empirical validation of our findings. Finally, in Section 5, we discuss our model, results, and their theoretical and practical implication.

2. The Model

When universities compete for rankings, each competes most closely with the universities directly above and directly below it. Depending on the quality of the applicant to these universities, a particular university may be the top school a candidate is considering, rank in the middle, or represent the lowest quality (i.e., “safety”) school. As Winston (1999, pp. 80-81) states,

Competition among schools appears to be limited to overlapping “bands” or segments of similarly wealthy schools within the hierarchy.... As one observer puts it, “A school competes with the ten schools above them and the ten schools below them, even if there are more than 3,300 in the country.”

Our model accommodates this form of high/middle/low competition by considering three schools (1, 2, and 3) that represent the three quality levels, respectively. With three universities, we examine how increased quality dispersion (i.e., an improvement in the quality of university 1 and/or a worsening of quality of university 3) affects the optimal merit aid offers.

We sketch the sequence of a university’s admissions and merit aid decision processes in Figure 2. We assume each university receives applications and evaluates the candidates, then determines the optimal merit aid it will offer each candidate, should the university choose to accept him or her. Finally, the applicant collects all offers from universities and decides which to attend.

[Insert Figure 2 about here]

2.1. The University’s Decision Problem

Although the decision process about how much merit aid to offer is identical for all three universities, they may make different merit aid decisions. Before university i , $i \in \{1, 2, 3\}$, makes its merit aid offers, it

- Considers the candidates’ quality attributes (e.g., high-school class rank, SAT score) and categorizes candidates (i.e., values them monetarily) according to those attributes;
- Formulates its belief about the merit aid offers made by its competitors (in our analysis, consistent with equilibrium offers); and
- Models the candidates’ utility functions and choice probabilities.

Candidate Quality Attributes and Acceptance. Each university partitions its acceptable applicants into two groups: desirable candidates, for whom the university must compete, and safety candidates, whom the university attracts, at full tuition, with probability 1. We let g denote a representative desirable candidate. The set of desirable candidates that university i accepts is M_i , and the number of

desirable candidates in set M_i is m_i . The university uses its safety candidates to fill the slots that it cannot fill with desirable candidates. Let m_{io} denote the number of safety candidates that university i must accept to fill its class, z_i denote university i 's number of slots, v_{ig} denote the value of candidate g to university i , and v_{io} denote the value to university i of one of its safety candidates. We assume $v_{ig} > v_{io}$.

Competitors' Merit Aid Offers. In our equilibrium analysis, each university believes that its competitors extend Nash equilibrium offers. (As is standard in these types of analyses, the Nash equilibrium offers are common knowledge.) We let y_{ig} denote university i 's merit aid offer to candidate g . Note that we can interpret $y_{ig} > 0$ as merit aid and $y_{ig} < 0$ as a tuition premium (i.e., a contribution to the university required to secure admission, an increasingly common phenomenon: Golden, 2006). In our analysis, we do not examine a university's determination of its list price tuition but focus on the discounts (i.e., merit aid) it offers to desirable candidates. Effectively, we assume that the three universities set the same list price tuition; column 3 in Table A1 provides rough support for this assumption.

University's Model of Candidates' Utility Functions and Choice Probabilities. Avery and Hoxby (2006) report that among the best students, three important factors drive attendance decisions: quality of the school, merit aid offer, and the value of the match in the candidate's mind between the university and the candidate. Thus, we model the universities' utility as a function of these three variables: x_i denotes the quality of the university (e.g., rank, graduate school placements), y_{ig} is the merit aid the university offers the candidate, and ε_i represents the internal (and unobserved) value of the match between the candidate and the university. We assume that all candidates have the same value x_i for university i , known with certainty. In terms of the popular rankings, university i 's quality x_i could be some form of a moving average of previous years' ranks. In our model, we assume that

the students in the year of our analysis who choose to attend university i do not affect x_i , because most students base their matriculation decisions on current and past student bodies, not on those who choose to attend during the current round of admissions. We also assume each candidate's match value ε_i is private to the candidate and independent across candidates and thus captures uncertainty for the university. Therefore, ε_i is a random variable with mean 0 and constant variance. We model a university's estimate of each candidate's utility of university i , $i \in \{1,2,3\}$, as:

$$u_{ig} = x_i + y_{ig} + \varepsilon_i. \quad (1)$$

In the candidate's choice problem, the candidate has chosen to apply to and been accepted by universities 1, 2, and 3. Thus, when the candidate determines which university to attend, his or her consideration set has already been determined. If we assume that ε_i has a double exponential distribution, the candidate's choice can be specified as a multinomial logit model, consistent with empirical work on the university candidate attendance decision by Avery et al. (2005) and Avery and Hoxby (2006).

By applying, the candidates have expressed their interest in going to college, so we assume all candidates attend a university. Thus, the probability that candidate g chooses to attend university i , $q_i = \text{prob}\{u_i \geq u_j, \forall j \neq i\}$, is

$$q_i(x_1 + y_{1g}, x_2 + y_{2g}, x_3 + y_{3g}) = \frac{\exp((x_i + y_{ig})/\mu)}{\sum_{j \in \{1,2,3\}} \exp((x_j + y_{jg})/\mu)}. \quad (2)$$

Because university 1 is the best of three universities and university 3 is the poorest, $x_1 \geq x_2 \geq x_3$. In our analysis, we examine the dispersion of the quality of the universities and, for the three-university case, define an increase in dispersion as an increase in both $(x_1 - x_2)$ and $(x_2 - x_3)$.

2.2. University's Expected Score Function, Decision Problem, and Nash Equilibrium

Most university rankings in the popular press rely on multi-attribute models; the *US News* rankings are based on 15 different university attributes, such as university acceptance rates, graduation rates, faculty salary, alumni giving, and so forth. Some combination of these attributes lead to the overall rank of universities (see <http://www.usnews.com/usnews/edu/college/rankings/about/index.php>).

In our analysis, we develop a two-attribute model that retains the essential features of the popular press ranking systems. We use the two generic attributes – prestige and resources – that subsume the main attributes of popular press publications, such that resources comprises, for example, financial resources, faculty resources, and alumni giving. The attribute scores use monetary values so that we may characterize the opportunity cost of increased merit aid spending in terms of reduced resources. In our model, as in the actual popular rankings, the university receives an overall score that is a weighted sum of the attribute scores. We assume that each university's objective in setting merit aid offers is to maximize its expected overall score.

University i 's prestige score depends on the students who attend the university and their monetary value to the university. University i 's expected prestige score reflects the sum across students that the university accepts for admission with regard to their valuation multiplied by the probability they will attend the university:

$$\text{Expected prestige score} = \sum_{g \in M_i} v_{ig} q_i(x_i + y_{ig}, \dots) + m_{io} v_{io}. \quad (3)$$

The expected resource score is its budget B_i minus its expected merit aid expenditures:

$$\text{Expected Resource Score} = B_i - \sum_{g \in M_i} q_i(x_i + y_{ig}, \dots) y_{ig}. \quad (4)$$

In the course of its acceptance decisions, the university must fill its class in expectation only, and to do so, it must accept safety candidates. That is, university i 's acceptance decisions must satisfy

$$\sum_{g \in M_i} q_i(x_i + y_{ig}, \dots) + m_{io} = z_i. \quad (5)$$

University i attaches weight w_p to its prestige score and weight w_r to its resources score, so its expected overall expected score, $E[s_i]$, is

$$E[s_i] = w_p \left(\sum_{g \in M_i} v_{ig} q_i(x_i + y_{ig}, \dots) + m_{io} v_{io} \right) + w_r \left(B_i - \sum_{g \in M_i} q_i(x_i + y_{ig}, \dots) y_{ig} \right). \quad (6)$$

University i then determines its optimal merit aid offer, y_{ig}^* , for each candidate g , to maximize its expected score from expression (6), subject to its class size requirement in equation (5). The solution to this optimization program, $y_{ig}^*(x_i, x_j + y_j, x_k + y_k)$ for each candidate g , is university i 's optimal merit aid offer, a function of the sum of each competing university's quality and merit aid offers.

We examine the Nash equilibrium of the merit aid offer game to attract a particular candidate, given that the candidate has been accepted by all three universities. In the Nash equilibrium, each university sets its optimal merit aid offer, $y_{ig}^*(x_i, x_j + y_j^*, x_k + y_k^*)$. (For the type of problem we analyze, Anderson et al. (1992) establish the existence of a unique Nash equilibrium.)

3. Analytic Results

We characterize university i 's optimal merit aid offer to each candidate g , y_{ig}^* . Next, in Lemma 1, we establish the conditions in which top universities are more likely to attract candidates (i.e., $q_1 > q_2 > q_3$). In Theorem 1, we examine the effect of an increase in the quality dispersion of the three universities on the equilibrium of merit aid offers. In Lemma 2, we demonstrate that the relative, not absolute, qualities of universities affect their optimal merit aid offers. Finally, in Theorem 2, we analyze the effect of a change in a university's net valuation of a candidate on equilibrium merit aid offers.

From the first-order conditions of maximizing university i 's expected score in expression (6) subject to the school's capacity constraint in equation (5), we find that university i 's optimal offer to candidate g satisfies

$$y_{ig}^*(x_i, x_j + y_{jg}, x_k + y_{kg}) = \frac{w_p}{w_r} (v_{ig} - v_{io}) - \frac{\mu}{1 - q_i (x_i + y_{ig}^*, x_j + y_{jg}, x_k + y_{kg})}. \quad (7)$$

Because the logit formulation ensures that each school's constrained expected score function (found by substituting (5) into (6)) is strictly quasi-concave, expression (7) constitutes necessary and sufficient conditions for the optimal merit aid offer.

Optimal pricing, as expressed in (7), states that the optimal merit aid offer equals the university's net monetary valuation of candidate g $(w_p/w_r)(v_{ig} - v_{io}) > 0$, less the strategic pricing element $-\mu/(1 - q_i) < 0$. Examining $\mu/(1 - q_i)$, we note that the optimal merit aid offer decreases with the probability of matriculation, q_i . Therefore, *ceteris paribus*, if a university has a greater probability of attracting a candidate, the university offers less merit aid. These two terms—the net dollar value of the candidate and the strategic pricing element—form the basis of our analysis.

In our logit choice model, two factors – university quality and merit aid – affect the probabilities candidates choose particular universities. A reasonable condition our model should admit is that university 1 should have the highest probability of attracting any candidate and university 3 the lowest,.

Lemma 1. *In equilibrium, $q_1 > q_2 > q_3$ if and only if*

$$x_1 + \frac{w_p}{w_r} (v_{1g} - v_{1o}) > x_2 + \frac{w_p}{w_r} (v_{2g} - v_{2o}) > x_3 + \frac{w_p}{w_r} (v_{3g} - v_{3o}). \quad (8)$$

Proof. See Appendix A2.

We make two assumptions about top universities: First, candidates perceive them as higher quality (i.e., x_i is objectively greater as i decreases), and second, the candidates they reject are close in quality to those they accept (i.e., $(v_{ig} - v_{io})$ decreases as i decreases). Lemma 1 states that for university 1 to have the greatest acceptance probability and university 3 the smallest, the quality differences, as measured by x_i , must be greater than the differences in the net benefits of the accepted candidates,

measured by $(w_p/w_r)(v_{ig} - v_{io})$. In Section 4, we offer empirical support for the conditions in expression (8): For the very top universities, the quality difference between two universities ($x_i - x_j$) is likely to dwarf the difference between the net benefits of accepted candidates

$$((w_p/w_r)(v_{ig} - v_{io}) - (w_p/w_r)(v_{jg} - v_{jo})).$$

Our first key finding is that, in a quality-local market, the sum of merit aid offers by competitors is decreasing in the quality of the top school and is increasing in the quality of the poorest school. That is, more quality dispersion among the quality-local competitors (i.e., the best school becomes relatively better and the worst school becomes relatively worse) means average merit aid offers decrease.

Theorem 1. Consider $x_1 + (w_p/w_r)(v_{1g} - v_{1o}) \geq x_2 + (w_p/w_r)(v_{2g} - v_{2o}) \geq x_3 + (w_p/w_r)(v_{3g} - v_{3o})$. In equilibrium, the average equilibrium offer to candidate g by the three universities, $(y_{1g}^* + y_{2g}^* + y_{3g}^*)/3$, is decreasing in the quality of the top university, x_1 , and is increasing in the quality of the worst university, x_3 .

Proof. See Appendix A2.

The Theorem 1 result follows from the property that changes in university quality cause top universities to make greater adjustments to their merit aid offers than those made by poorer ones. That is, *ceteris paribus*, a university's optimal merit aid offer is strictly decreasing and strictly concave in its own quality (see Lemma A1 in Appendix 2). As university 1 improves or university 3 worsens in quality, the decrease in university 1's optimal merit aid offer is greater than the increase in university 3's. Hence, the average merit aid offer decreases with the increase in the dispersion of university quality.

To understand the greater adjustments by higher-quality universities, we examine the relationship between the quality of a school and its strategic pricing element $(\mu/\beta)/(1 - q_i)$. The strategic pricing element relates to the merit aid elasticity of demand:

$$(\partial q_i / \partial y_i)(y_i / q_i) > 0, \quad (9)$$

which in the logit model is

$$(\partial q_i / \partial y_i)(y_i / q_i) = ((1 - q_i) / \mu) y_i. \quad (10)$$

From (10), we can describe the strategic pricing element, $\mu / (1 - q_i)$, in terms of the merit aid elasticity. Specifically, *Strategic pricing element* = $1 / (\text{merit-aid elasticity} \times \text{merit aid offer})$. Also, from (10), we see that the merit aid elasticity becomes more inelastic as the probability a candidate chooses a university increases. Because candidates are more likely to choose to attend better universities, these universities face more merit aid inelastic demand. Hence, as quality improves, the university has a greater incentive to decrease its merit aid offer to a particular candidate. With this greater incentive, the university's optimal merit aid offer, *ceteris paribus*, not only decreases in quality but also decreases at an increasing rate.

As actual practices demonstrate, universities actively estimate price elasticities to price discriminate, such that "Grants no longer are based overwhelmingly on a student's demonstrated financial need, but also on his or her 'price sensitivity' to college costs, calculated from dozens of factors that all add up to one thing: how anxious the student is to attend" (Stecklow 1996, p. A1).

The validity of Theorem 1 as an explanation for why better universities offer less merit aid rests on the assumption that competition is quality-local, i.e., that higher education is partitioned into smaller markets defined not only by geography and university specialization but also by quality (Grewal, Dearden, and Lilien 2006; Winston, 1999). Even universities of very high quality may not be competitors. For example, Harvard effectively does not compete with Duke; according to the choice probabilities in Avery et al. (2005), a candidate accepted by Harvard and Duke chooses to attend Harvard with 0.97 probability. In this case, the higher quality of Harvard does not drive the result that Harvard offers no merit aid and Duke does. Rather, in its quality-local competition for the best students, Harvard and its competitors, which display wide quality variations, offer minimal merit aid,

because doing so would not provide them incremental differentiation. Duke and its quality-local competitors (e.g., Georgetown, Northwestern), very good schools whose qualities are relatively close, offer substantial merit aid to distinguish themselves in their close competition for students.

The dispersion of competitor quality among universities engaged in quality-local competition correlates positively with the quality of the universities (as we show in the next section). Thus, our Theorem 1 result explains that the dispersion of competitor quality, not the absolute quality of the university and its competitors, drives the negative relationship between university quality and merit aid offers.

Lemma 2. *Relative, not absolute, qualities affect a university's optimal merit aid offers. Formally, for two sets of university quality profiles (x_1, x_2, x_3) and $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$, ceteris paribus, if $(x_i - x_j) = (\hat{x}_i - \hat{x}_j)$ for each j , then for university i ,*

$$y_{ig}^*(x_i, x_j + y_j, x_k + y_k) = y_{ig}^*(\hat{x}_i, \hat{x}_j + y_j, \hat{x}_k + y_k). \quad (11)$$

Proof. See Appendix A2.

Lemma 2, in combination with Theorem 1, implies that university quality per se does not cause top universities to offer less merit aid; rather, the dispersion of the quality of quality-local competitors drives the relationship. Suppose the only differences between competition among Harvard, Yale, and Princeton for a top candidate and competition among Colgate, Bucknell, and Lafayette for a good candidate were the quality levels of the universities. To generate equilibrium merit aid offers by the Ivies for their top candidate that are lower than the equilibrium merit aid offers by the Patriots for their good candidate, the relative qualities of the Ivies must differ from the relative qualities of the Patriots. In this sense, the higher quality of the Ivies does not drive their lower merit aid offers.

Our second key finding is that top universities offer less merit aid because the candidates they reject tend to be close in quality to the candidates they accept, whereas among lower-ranked universities, rejected candidates tend to be further in quality from the candidates they accept. In

Section 4, we offer evidence that $(v_{ig} - v_{io})$ is decreasing in quality. Top universities, with their better alternative options (i.e., better safety candidates), have less need to attract their top candidates and therefore make smaller merit aid offers than lower ranked universities.

Theorem 2. *Each equilibrium offer $(y_{1g}^*, y_{2g}^*, \text{ and } y_{3g}^*)$ increases with university i 's net monetary valuation of candidate g , $(v_{ig} - v_{io})$.*

Proof. See Appendix A2.

4. Empirical Issues

In this section, we first examine the empirical relationship between university quality and merit aid offers, then the empirical relationship between university quality and the quality dispersion of universities engaged in quality-local competitions. Finally, we investigate the difference between the quality of candidates accepted by a university and the quality of those rejected.

4.1. Quality and Merit Aid

Four observations support our contention that top universities offer less merit aid. First, Harvard, Yale, Princeton, MIT, Yale, Columbia, Cornell, and Brown, all most-selective schools, report that they offer no merit aid. Second, Ehrenberg and Monks (1999) demonstrate that when universities improve their rankings, they tend to offer less merit and need-based financial aid. Using individual applicant data from 30 highly selective institutions for the academic years 1988–89 through 1998–99, they find that an improvement in rank of 10 places increases net tuition by approximately 4%. Third, summary statistics of merit aid offers indicate that rank and merit aid offers are negatively correlated. For each university, *US News* reports the average merit aid award per student and the percentage of students who receive merit aid; we provide this information in Table A1. The mean of average merit aid award per student at top ranked (1–10) private universities is \$5,327; that for private schools ranked 41–50 is \$11,900. Furthermore, the mean percentage of students who receive merit aid from the private universities in the top decile is 2.2% and from the lowest decile in this set is 16.5%. Fourth, Epple,

Romano, and Sieg (2003) empirically find that schools with the highest list price tuition display negative correlations between candidates' SAT scores and merit aid offers. Students with higher SAT scores tend to attend better universities, but the top universities tend to offer less merit aid. (Epple, Romano and Sieg (2003) suggest that this negative relationship may be due to variables omitted from their analysis. In their analysis, they employ a process that aggregates all top universities into effectively one university. Epple, Romano and Sieg therefore do not examine the dispersion of quality among these top universities.)

4.2. Quality and Quality Dispersion

We offer two types of evidence to show that quality dispersion among quality-local competitors correlates positively with quality itself. First, Avery et al. (2005), in building their revealed preference ranking of colleges and universities, use survey data in a multinomial logit analysis of the applicant choice problem (i.e., faced with a list of schools that have accepted the candidate, he or she must choose). Their survey of 3,240 high-achieving students from the class of 2004 includes questions about the schools that accepted them, the school they chose to attend, and financial aid offers. The probability that candidate g attends university i , if accepted by the set of S_g universities, is

$$q_{ig} = \frac{\exp(\theta_i + x_{ig}' \delta)}{\sum_{j \in S_g} \exp(\theta_j + x_{jg}' \delta)}. \quad (11)$$

In this specification, “ θ_i s embody all characteristics that do not vary within each college: whether it is a liberal arts college, the faculty, a rural as opposed to urban location, and so on” (Avery et al., 2005, p. 15), whereas the characteristics vector x_{ig} varies among admitted students (e.g., legacy status, merit aid). With a Markov chain Monte Carlo simulation in which one school's θ is greater, they find that for top schools, a higher-ranked school's θ is greater than that of a lower-ranked school in most of the posterior draws. For example, in the competition between Harvard and Yale, Harvard wins in 98%

of the draws; in the competition between Yale and Princeton, Yale wins in 90% of the draws. That is, Harvard’s desirability is distinct from either Yale’s or Princeton’s. In terms of the multinomial logit model, the perceived quality difference dominates the idiosyncratic element of the utility function. For lower-ranked but still selective schools, the idiosyncratic element of the utility function plays a greater role. For example, in the competition between University of Chicago and Johns Hopkins, Chicago wins in 51% of the draws; in that between Johns Hopkins and University of Southern California, Hopkins wins 69%. Thus, “As a rule, the lower one goes in the revealed preference ranking, the less distinct is a college’s desirability from that of its immediate neighbors in the ranking.” (Avery et al. 2005, p. 27)

Second, *The Wall Street Journal* university ranking, which ranks schools by the placement of graduates in top-five business, law, and medical schools, shows that the distribution of the rank-order is heavy tailed (see Table A2). At the top of the 2004 rankings, Harvard placed 21.49% of a recent class in top-five business, law, or medical schools, Yale 17.96%, Princeton 15.78%, and Stanford 10.70%. In dropping only from first to fourth place, the placement rate falls by half. However, for schools ranked 21–24, placement rates are virtually identical — roughly 3.6 percent.

The dispersion in terms of top-five professional school placement rates is so great for top universities yet so close for good universities that it follows a Pareto (Power Law) distribution. In the Pareto distribution of placement rates, small placement rates are extremely common, whereas large rates are extremely rare. For a placement rate, ρ , the cumulative distribution function of the power law distribution is

$$\text{Prob}(p \leq \rho) = F(\rho) = 1 - \left(\frac{c}{\rho}\right)^\alpha. \quad (12)$$

The estimated coefficients of the power law distribution are $\hat{c} = 0.0164$ and $\hat{\alpha} = 1.2194$ for the 50 universities in *The Wall Street Journal* ranking, as we show in Table 1. Table 1 also contains results from the Anderson-Darling and Lilliefors tests for normality, which shows that we reject the

hypothesis that the distribution of placement rates is normal. Figure 3 contains the estimated and actual distributions of ordered placement rates.

[Insert Table 1 and Figure 3 about here]

The quality dispersion at top universities is greater than that of very good schools. On the basis of Avery et al.'s (2005) choice probabilities and professional school placement rates, we postulate that the perceived and actual quality differences are quite large among top schools.

4.3. University and Applicant Quality

For the top universities, the quality of the candidates they accept is close to the quality of the candidates they reject. As *The Wall Street Journal* reports, "Every year, [the Ivies] reject many valedictorians and students with perfect SAT scores" (Golden 2003, p. A1). Advice from the *Washington Post* to college applicants indicates that "Yale University accepted 8.6 percent of its applicants this year, an Ivy League low. Selective college admissions officers admit that they reject or wait-list many students who are just as good as the ones they accept. If the school is short on engineering majors or Idaho residents or piccolo players, applicants with those characteristics will be accepted. The rest will have to go elsewhere" (Matthews 2006, p. A14) Princeton University, recognizing that it rejects high-quality candidates, has increased the size of its undergraduate population by 11%, in defense of which University President Shirley M. Tilghman stated, "We are turning away students who we know would be absolutely stellar Princeton students, and it's just because of our lack of spaces in the class" (Hechinger 2006, p. B1).

4.4. Summary of Empirical Issues

We have attempted to establish the following:

- Merit aid offers correlate negatively with quality.
- Quality dispersion among quality-local competitors correlates positively with quality.

- The difference between the quality of candidates accepted and those rejected is negatively correlated with the quality of universities.

Integrating these empirical relationships with our theoretical results shows that higher quality per se does not drive the lower merit aid offers by top universities; quality dispersion among the universities and the differences in the quality of applicants do instead .

Evidence that the difference in quality between accepted and rejected applicants decreases with higher-quality universities serves two purposes. We conclude both that matriculation probabilities are greater with better university quality (Lemma 1) and that better universities have better alternative options, both of which explain why top universities offer less merit aid.

5. Discussion

In response to the increasingly fierce competition for rankings in the university marketplace, we seek to understand the role of merit aid. Our model and analysis provide an explanation based on quality-local competition for the striking heterogeneity in merit aid offers across universities and specifically for the observation that lower-ranked universities offer more merit aid than top universities

5.1. Theoretical Linkages

Our model relates to research focused on price setting equilibria for quality-differentiated oligopolistic firms (e.g., Anderson et al., 1992; Anderson and de Palma, 2001) and literature on strategic complements (Vives, 1999, 2005). We build on previous theoretical analyses of university pricing (Epple, Romano, and Sieg, 2003, 2006; Rothschild and White 1995) that model university competition such that students are both consumers and inputs. Rothschild and White (1995) focus on whether tuition and merit aid decisions in a perfectly competitive education market would result in the efficient allocation of students among universities, whereas Epple, Romano, and Sieg (2003, 2006) construct a similar general equilibrium model but add the effect of household income on equilibrium prices.

Rothschild and White (1995) suggest that optimal tuition less merit aid equals the student's valuation of the university less the student's contribution to the university. In this sense, their pricing equation is qualitatively similar to ours, but the models take different approaches to strategic interaction. They consider each university atomistic in the sense that a change in its quality does not affect its competitors' merit aid offers. Their general equilibrium analysis thus is limited, because it fails to consider the effect of a change in the quality of a university on the quality and reactions of its competitors. In our study of the effect of university quality on merit aid offers, we make the strategic element of the Nash approach central.

Furthermore, our work relates to the celebrated college admissions problem, in which candidates express their preferences for universities and universities express theirs for candidates (Gale and Shapley 1962), and then a mechanism matches candidates and universities. Two articles propose centralized matching mechanisms that improve on current government mechanisms to match students and schools. Teo, Sethuraman, and Tan (2001) use standardized test scores of primary school students in Singapore, whereas Balinski and Sönmez (1999) examine student placement according to standardized test scores by a central Turkish placement office. The latter authors also impose a fairness condition, which requires that, regardless of the placement mechanism, candidates with better test scores are assigned to better universities. These college matching problems represent examples of more general two-sided matching problems (Roth and Sotomayor 1990), but because U.S. universities do not use a central placement service, college matching models cannot be used to analyze the competitive marketplace for U.S. higher education.

5.2. Extensions and Conclusion

Our model can be extended in several useful ways. First, researchers might add two types of uncertainty. That is, universities are uncertain about which schools have accepted a particular candidate, as well as the amount of merit aid these universities offer. Second, the inclusion of two or three admissions rounds could enrich the model. Many universities use two early admissions rounds,

on the basis of their belief that if a student is accepted early, he or she commits to that university. With an early admissions practice, the university does not need to compete with other universities for that candidate, which means early admissions could add several interesting twists to our analysis. They reduce price competition, and risk-averse schools may be likely to lock in more students to avoid having to accept lower-quality students. In the competition to improve ranks, as Avery, Fairbanks, and Zeckhauser (2003) suggest, schools could lower their admissions standards during the early rounds, then reject more applicants during the regular admissions round, which reduces their acceptance rates, increases their apparent selectivity, and improves their overall ranking. With multiple rounds of admissions, an analysis of merit aid offers would become a full-blown revenue management problem. Although we focus on the specifics of competition in the university marketplace, the essence of our analysis and model deals with quality-local competition in a production-limited environment in which customer quality varies but has some inherent value. Competition among other "exclusive" institutions, such as golf or country clubs or other special interest, limited admission institutions, whose potential members and customers differ in terms of attractiveness and social capital, might be clarified through adaptations of our framework (Sandler and Tschirhart 1980; Woolcock and Narayan 2000).

The university marketplace is complex and of sufficient strategic importance to merit significant study on its own however. We hope this work adds to understanding of that marketplace, kindles additional discussion, and spurs further work in this and related domains.

References

- Anderson, S. P., and A. de Palma (2001), "Product Diversity in Asymmetric Oligopoly: Is the Quality of Consumer Goods too Low?" *Journal of Industrial Economics*, 49 (2), 113-135.
- _____, _____, and J.-F. Thisse (1992), *Discrete Choice Theory of Product Differentiation*, Cambridge, MA: MIT Press
- Avery, C., A. Fairbanks, and R. Zeckhauser (2003), *The Early Admissions Game: Joining the Elite*, Cambridge, MA: Harvard University Press.
- _____, M. Glickman, C. Hoxby and A. Metrick (2005), "A Revealed Preference Ranking of U.S. Colleges and Universities," NBER Working Paper 10803, <http://www.nber.org/papers/w10803>.
- _____, and C. Hoxby (2006), "Do and Should Financial Aid Packages Affect Students' College Choices?" Working Paper, Harvard University.
- Balinski, M., and T. Sönmez (1999), "A Tale of Two Mechanisms: Student Placement," *Journal of Economic Theory*, 84, pp. 73-94
- Ehrenberg, R., and J. Monks, 1999, "The Impact of *U.S. News & World Report* College Rankings on Admissions Outcomes and Pricing Policies at Selective Private Institutions," NBER Working Paper 7227.
- Epple, D., R. Romano, and H. Sieg (2003), "Peer Effects, Financial Aid, and Selection of Students into Colleges," *Journal of Applied Econometrics*, 18, 501-525.
- _____, _____, and _____ (2006), "Admission, Tuition, and financial Aid Policies in the market for Higher Education," *Econometrica*, 74 (4), 885-928.
- Fallows, J. (2003), "The New College Chaos," *The Atlantic Monthly*, November, 106.
- Finder, A. (2006), "Aid Lets Smaller Colleges Ask, Why Pay for Ivy League Retail," *The New York Times*, January 1, p. 1.
- Gale, D. and L. Shapley (1962), "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69, 9-115.
- Geiger, R. L. (2004), "Market Coordination of Higher Education," in P. Teixeira, B. Jongbloed and D. Dill, *Markets in Higher Education: Rhetoric or Reality?*, Dordrecht: Kluwer.
- Golden, D. (2006), *The Price of Admission: How America's Ruling Class Buys Its Way into Elite Colleges -- and Who Gets Left Outside the Gates*, New York: Crown.
- _____, (2003), "Forgotten Grads, Academics Aren't Only Keys to Ivy Schools," *Wall Street Journal*, April 25, p. A1.
- Grewal, R., J. A. Dearden, and G. L. Lilien (2006), "The University Rankings Game:

- Modeling the Competition among Universities for Ranking,” Working Paper, Penn State University.
- Hechinger, J. (2006), “The Tiger Roars,” *Wall Street Journal*, July 17, p. B1.
- Kane, T. J. (1999), *The Price of Admission*, Washington, D.C.: The Brookings Institution
- Kronholz, J. (2005), “More Students, Higher Prices, Tougher Competition,” *The Wall Street Journal*, January 31, p. R4
- Matthews, J. (2006), “10 Antidotes to College-Application Anxiety,” *The Washington Post*, April 25, p. A14.
- Nagle, T. T. and R. K. Holden, (2002, *The Strategy and Tactics of Pricing: A Guide to Profitable Decision Making*, Englewood, NJ: Prentice Hall.
- Roth, A., and M. Sotomayor (1990), *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*, New York: Cambridge University Press.
- Rothschild, M. and L. J. White (1995), “The Analytics of Pricing of Higher Education and Other Services in Which the Customers are Inputs,” *Journal of Political Economy*, 103 (3), 573-586.
- Sandler, T. and J. T. Tschirhart (1980) "The Economic Theory of Clubs: An Evaluative Survey" *Journal of Economic Literature*, Vol. 18, No. 4 (Dec), 1481-1521
- Stecklow, S. (1996), “Expensive Lesson: Colleges Manipulate Financial-Aid Offers, Shortchanging Many—Early Admittees and Others are Eager, So Schools Figure They’ll Pay More—Advice: ‘Be a Little Cagey,’” *Wall Street Journal*, April 1, p. A1.
- Teo, C.-P., J. Sethuraman, and W.-P. Tan (2001), Gale-Shapley Stable Marriage Problem Revisited: Strategic Issues and Applications, *Management Science*, 47, 1252-1267.
- Tirole, J. (1988), *The Theory of Industrial Organization*, Cambridge, MA: MIT Press
- Vives, X. (1999), *Oligopoly Pricing*, Cambridge, MA: MIT Press.
- _____ (2005), “Complementarities and Games: New Developments,” *Journal of Economic Literature*, 43 (2), 437-479.
- Winston, G. C. (1999), “Subsidies, Hierarchy and Peers: The Awkward Economics of Higher Education.” *Journal of Economic Perspectives*, 13 (1), 13-36.
- Woolcock, M. and D. Narayan (2000) "Social Capital: Implications for Development Theory, Research, and Policy" *World Bank Research Observer*, 15 (August), 225 - 249

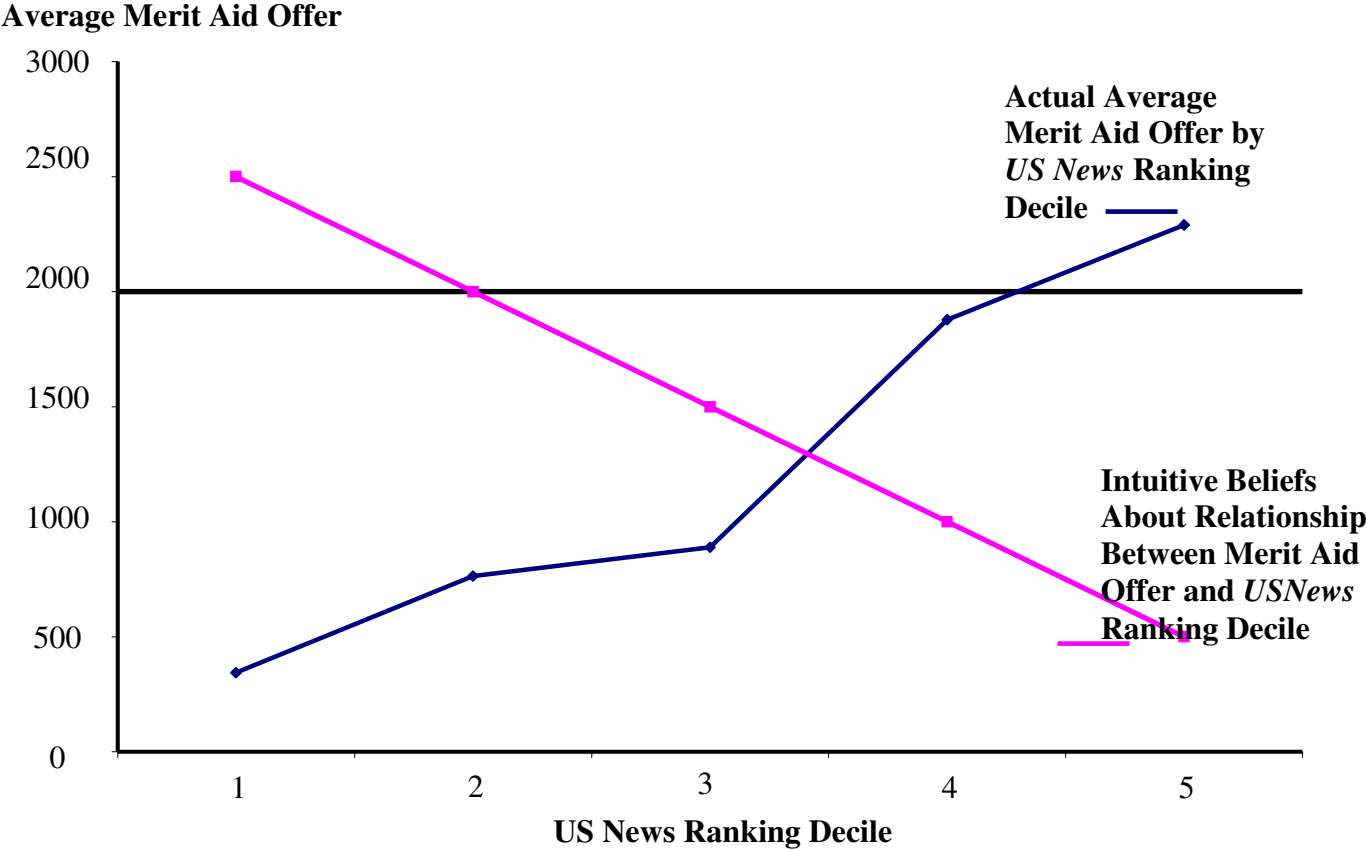


Figure 1. Actual and Conjectured Average Merit Aid Offers By USNews Decile

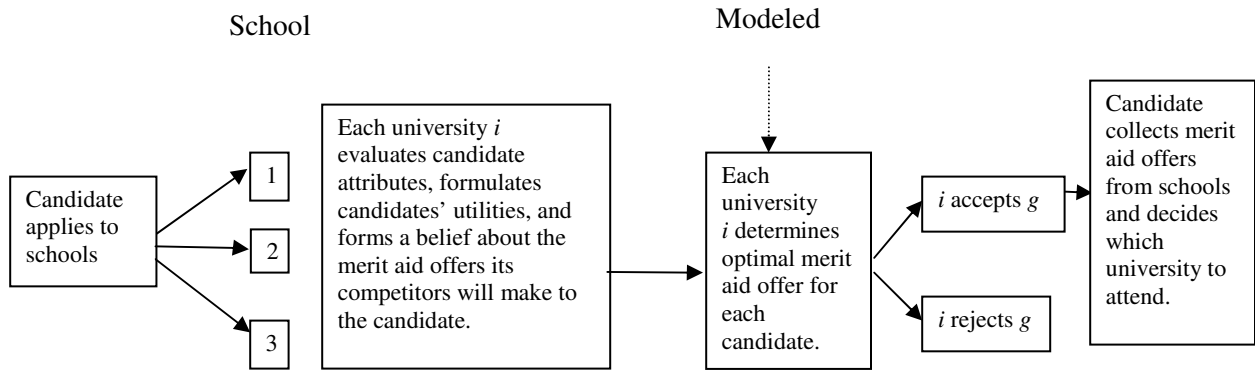


Figure 2: The Admissions and Merit Aid Process

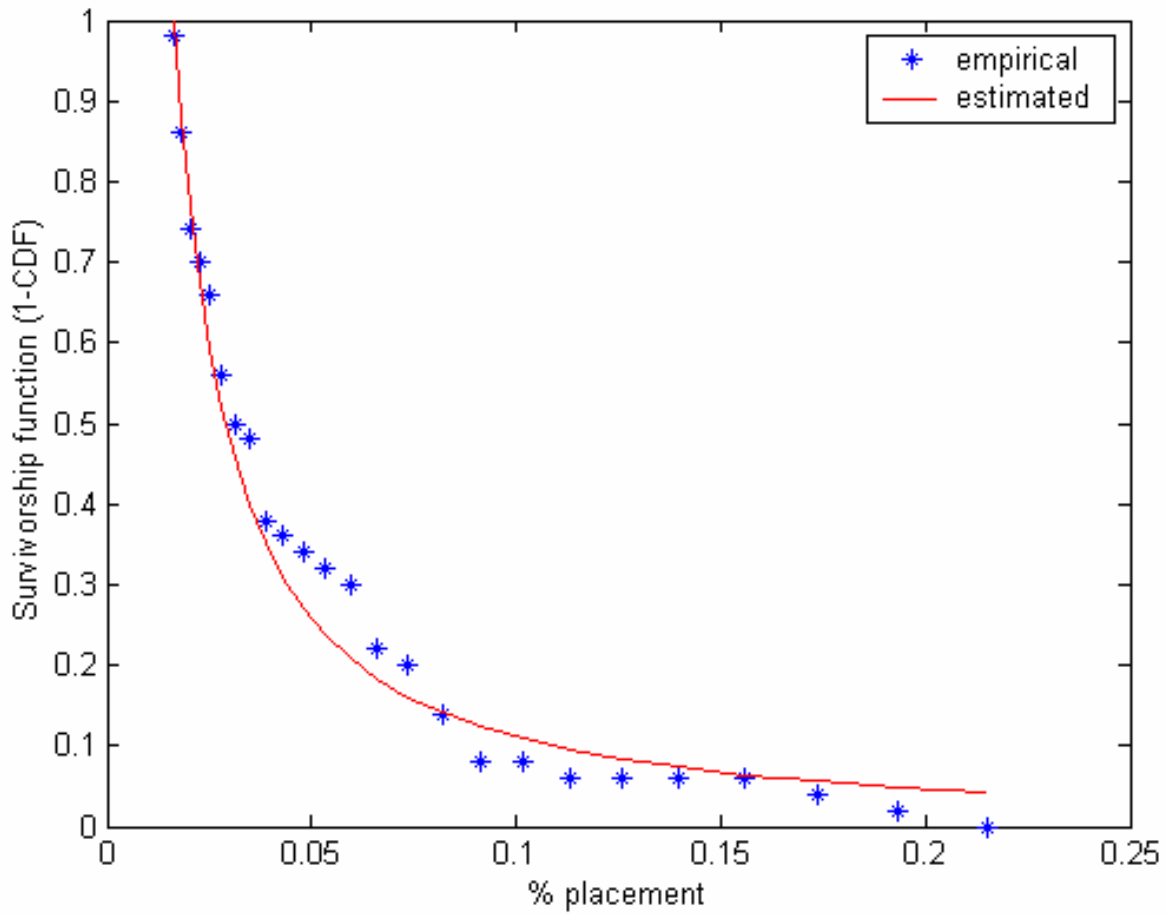


Figure 3. Empirical and estimated power law distribution of the placement rates in top-five professional programs of the top-25 schools in the *Wall Street Journal* top-50 colleges and universities.

Table 1. Tests of the distribution of the placement rates by the *Wall Street Journal* top-50 colleges and universities in top-5 professional schools

	Statistic	Critical Value (.05)	<i>p</i> -Value
X ²	4.7968	-	0.0909
Likelihood ratio	13.7602*	-	0.0010
Anderson-Darling	0.6202	0.75	-
Lilliefors	0.0873	0.1266	-

*Significant at $p < 0.05$.

Appendix 1: Supporting Tables

Table A1. 2006 Merit Aid Details

University	USNews Rank	Tuition 05-06	Average Merit Aid Award per Student who Received Aid:	Average Merit Aid per Student who Received Aid: Decile Average	Percent Awarded Merit Aid: Total Undergrads	Percent Awarded Merit Aid: Decile Average	Average Merit Aid per Student: Total Undergrads	Average Merit Aid per Student: Decile Average
Harvard	1	32,097	0		0		0	
Princeton	1	31,450	0		0		0	
Yale	3	31,460	0		0		0	
Pennsylvania	4	32,364	0		0		0	
Duke	5	32,410	22,277	5,327	4	2.2	891	343
Stanford	5	31,200	3,100		10		310	
Cal Tech	7	27,309	27,896		8		2232	
MIT	7	32,200	0		0		0	
Columbia	9	31,472	0		0		0	
Dartmouth	9	31,965	405		0		0	
WashingtonU	11	32,042	6,914		14		968	
Northwestern	12	31,789	3,423		1		34	
Cornell	13	31,467	0		0		0	
JHU	14	31,620	13,016		6		781	
Brown	15	32,974	0	8,339	0	7.3	0	763
U. Chicago	15	31,629	11,260		11		1239	
Rice	17	20,160	5,335		20		1067	
Notre Dame	18	31,542	7,630		2		153	
Vanderbilt	18	31,700	17,758		13		2309	
Emory	20	30,794	18,056		6		1083	
CMU	22	32,044	11,721		9		1055	
Georgetown	23	32,199	3,800		0		0	
Tufts	27	32,621	500	5,682	2	8.0	10	889
Wake Forest	27	30,210	9,685		10		968	
USC	30	32,008	12,702		19		2413	
Lehigh	32	31,420	13,027		7		912	
Brandeis	34	32,500	17,454		22		3840	
Case Western	37	28,678	12,650	11,621	30	14.2	3795	1878
New York U.	37	31,690	6,924		11		762	
Boston C.	40	31,438	8,051		1		81	
Rensselaer	43	31,857	14,700		16		2352	
Tulane	43	32,946	17,020	11,900	31	16.5	5276	2290
Yeshiva	46	26,100	7,741		3		232	
Syracuse	50	28,285	8,140		16		1302	

Source: http://www.usnews.com/usnews/edu/college/rankings/rankindex_brief.php, accessed July 19, 2006. The data reported by *US News* is based on university self-reported data to the Common Data Set Initiative (<http://www.commondataset.org/>).

Table A2. *Wall Street Journal* 09/25/2003 ranking of the top-50 colleges and universities by percentage of graduating classes placed in the top-five business, law, and medical schools.

Rank	School	Class Size	# Attending	Percentage of Class Attending
1	Harvard University	1,666	358	21.49%
2	Yale University	1,286	231	17.96%
3	Princeton University	1,103	174	15.78%
4	Stanford University	1,692	181	10.70%
5	Williams College	519	47	9.06%
6	Duke University	1,615	139	8.61%
7	Dartmouth College	1,101	93	8.45%
8	MIT	1,187	92	7.75%
9	Amherst College	431	33	7.66%
10	Swarthmore College	336	25	7.44%
11	Columbia University	1,652	118	7.14%
12	Brown University	1,506	98	6.51%
13	Pomona College	362	23	6.35%
14	University of Chicago	948	59	6.22%
15	Wellesley College	585	35	5.98%
16	University of Pennsylvania	2,785	153	5.49%
17	Georgetown University	1,666	85	5.10%
18	Haverford College	291	13	4.47%
19	Bowdoin College	404	16	3.96%
20	Rice University	764	29	3.80%
21	Northwestern University	1,978	73	3.69%
22	Claremont McKenna College	271	10	3.69%
23	Middlebury College	660	24	3.64%
24	Johns Hopkins University	1,272	45	3.54%
25	Cornell University	3,565	115	3.23%
26	Bryn Mawr College	310	9	2.90%
27	Wesleyan University	731	21	2.87%
28	Cal Tech	249	7	2.81%
29	Morehouse College	501	14	2.79%
30	University of Michigan	5,720	156	2.73%
31	New College of Florida	113	3	2.65%
32	Vassar College	581	15	2.58%
33	University of Virginia	3,213	82	2.55%
34	US Military Academy	966	23	2.38%
35	University of Notre Dame	1,985	45	2.27%
36	Emory University	1,509	33	2.19%
37	US Military Academy	986	21	2.13%
38	Macalester College	406	8	1.97%
39	Brandeis University	815	16	1.96%
40	Bates College	417	8	1.92%
41	U. California, Berkeley	6,198	118	1.90%
42	Barnard College	588	11	1.87%
43	Trinity College	485	9	1.86%
44	Grinnell College	337	6	1.78%
45	Tufts University	1,246	22	1.77%
46	Colby College	471	8	1.70%
47	Washington University	1,709	29	1.70%
48	Washington and Lee	413	7	1.69%
49	Case Western	729	12	1.65%
50	Reed College	304	5	1.64%

Appendix 2: Proofs

Proof of Lemma 1

By (2), $q_i > q_j$ if and only if

$$(x_i + y_{ig}) > (x_j + y_{jg}). \quad (\text{A1})$$

By (7), $q_i > q_j$ if and only if

$$\frac{w_p}{w_r}(v_{ig} - v_{io}) - y_{ig} > \frac{w_p}{w_r}(v_{jg} - v_{jo}) - y_{jg}. \quad (\text{A2})$$

From (A1) and (A2), $q_i > q_j$ if and only if

$$x_i + \frac{w_p}{w_r}(v_{ig} - v_{io}) > x_j + \frac{w_p}{w_r}(v_{jg} - v_{jo}). \quad (\text{A3})$$

Q.E.D.

We use Lemmas A1 and A2 in the proof of Theorem 1. In Lemma A1, if a university's quality improves, it should offer less merit aid to a particular candidate, and the magnitude of the change in the university's optimal merit aid offer increases with the higher quality of the university. That is, *ceteris paribus*, a university's optimal merit aid to a particular candidate strictly decreases and is strictly concave in its quality. In Lemma A2, if a competitor's quality improves, the university should offer more merit aid. That is, *ceteris paribus*, a university's optimal merit aid to a particular candidate increases with the quality of the competitor.

Lemma A1 *Ceteris paribus, the change in university i 's optimal offer with respect to a change in its own quality is*

$$\frac{\partial y_{ig}^*(x_i, x_j + y_j, x_k + y_k)}{\partial x_i} = -q_{ig}^*(x_i + y_i, x_j + y_j, x_k + y_k). \quad (\text{A4})$$

Lemma A2 *Ceteris paribus, the change in university i 's optimal offer with respect to a change in university j 's quality or merit aid offer is*

$$\frac{\partial y_{ig}^*}{\partial y_j} = \frac{\partial y_{ig}^*}{\partial x_j} = \frac{q_{ig}(x_i + y_i, x_j + y_j, x_k + y_k)}{1 - q_{ig}(x_i + y_i, x_j + y_j, x_k + y_k)} q_{jg}(x_i + y_i, x_j + y_j, x_k + y_k). \quad (\text{A5})$$

Proof of Lemmas A1 and A2

Substituting (5) into (6), we derive the strictly quasi-concave expected score function:

$$\begin{aligned} E[\sigma_i] = & w_p \left(\sum_{g \in M_i} v_{ig} q_i(x_i + y_{ig}, \dots) + v_{io} \left(z_i - \sum_{g \in M_i} q_i(x_i + y_{ig}, \dots) \right) \right) \\ & + w_r \left(B_i - \sum_{g \in M_i} q_i(x_i + y_{ig}, \dots) y_{ig} \right). \end{aligned} \quad (\text{A6})$$

The optimization program is

$$\max_{y_{ig}, g \in M_i} E[\sigma_i]. \quad (\text{A7})$$

In addition, the first-order conditions, which are necessary and sufficient for a maximum, are:

$$\frac{\partial E[\sigma_i]}{\partial y_{ig}} = 0 = \left[w_p (v_{ig} - v_{io}) - w_r y_{ig} \right] \frac{\partial q_i}{\partial y_{ig}} - w_r q_i \text{ for each } g \in M_i. \quad (\text{A8})$$

For the logit model, we have

$$\frac{\partial q_i}{\partial y_{ig}} = \frac{q_i(1 - q_i)}{\mu}. \quad (\text{A9})$$

Substituting (A9) into (A8), we obtain the following equation, which we label f_{ig} :

$$f_{ig} = 0 = \left[w_p (v_{ig} - v_{io}) - w_r y_{ig} \right] \frac{(1 - q_i)}{\mu} - w_r. \quad (\text{A10})$$

For the logit model, we have:

$$\frac{\partial q_i}{\partial x_i} = \frac{q_i(1 - q_i)}{\mu}, \quad (\text{A11})$$

and

$$\frac{\partial q_i}{\partial x_j} = \frac{\partial q_i}{\partial y_{jg}} = -\frac{q_i q_j}{\mu}. \quad (\text{A12})$$

Taking the differential of (A10) with respect to y_{ig} , y_{jg} , y_{kg} , x_i , x_j , and x_k and using (A9), (A11), and (A12),

$$\begin{aligned}
df_{ig} = 0 = & \left\{ (w_p(v_{ig} - v_{io}) - w_r y_{ig}) \left(\frac{-q_i(1-q_i)}{\mu^2} \right) - w_r \frac{1-q_i}{\mu} \right\} dy_{ig} \\
& + (w_p(v_{ig} - v_{io}) - w_r y_{ig}) \left(\frac{-q_i(1-q_i)}{\mu^2} \right) dx_i \\
& + (w_p(v_{ig} - v_{io}) - w_r y_{ig}) \left(\frac{q_i q_j}{\mu^2} \right) (dx_j + dy_{jg}) \\
& + (w_p(v_{ig} - v_{io}) - w_r y_{ig}) \left(\frac{q_i q_k}{\mu^2} \right) (dx_k + dy_{kg}).
\end{aligned} \tag{A13}$$

Solving (A10) for $(w_p(v_{ig} - v_{io}) - w_r y_{ig})$ and substituting into (A13),

$$df_{ig} = 0 = -dy_{ig} - q_i dx_i + \frac{q_i q_j}{1-q_i} (dx_j + dy_{jg}) + \frac{q_i q_k}{1-q_i} (dx_k + dy_{kg}). \tag{A14}$$

Therefore,

$$\frac{\partial y_{ig}}{\partial x_i} = -q_i; \tag{A15}$$

and

$$\frac{\partial y_{ig}}{\partial x_j} = \frac{q_i}{1-q_i} q_j. \tag{A16}$$

Q.E.D.

Proof of Theorem 1

Taking the total differentials of the first-order conditions (A10) and using Lemmas A1 and A2 and Cramer's rule, we have:

$$\frac{dy_{ig}^*}{dx_i} = \frac{\begin{vmatrix} q_i & \frac{q_i q_j}{1-q_i} & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_j} & -1 & \frac{q_j q_k}{1-q_j} \\ \frac{q_i q_k}{1-q_k} & \frac{q_j q_k}{1-q_k} & -1 \end{vmatrix}}{\begin{vmatrix} -1 & \frac{q_i q_j}{1-q_i} & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_j} & -1 & \frac{q_j q_k}{1-q_j} \\ \frac{q_i q_k}{1-q_k} & \frac{q_j q_k}{1-q_k} & -1 \end{vmatrix}} \quad (\text{A17})$$

and

$$\frac{dy_{jg}^*}{dx_i} = \frac{\begin{vmatrix} -1 & q_i & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_j} & -\frac{q_i q_j}{1-q_j} & \frac{q_j q_k}{1-q_j} \\ \frac{q_i q_k}{1-q_k} & -\frac{q_j q_k}{1-q_k} & -1 \end{vmatrix}}{\begin{vmatrix} -1 & \frac{q_i q_j}{1-q_i} & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_j} & -1 & \frac{q_j q_k}{1-q_j} \\ \frac{q_i q_k}{1-q_k} & \frac{q_j q_k}{1-q_k} & -1 \end{vmatrix}}. \quad (\text{A18})$$

To evaluate the sign of $d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i$, we alter (A18) by changing the responses

$\partial y_{jg}^*/\partial x_i = \partial y_{jg}^*/\partial y_{ig}^*$ and writing them as

$$\frac{\partial \hat{y}_{jg}}{\partial x_i} = \frac{\partial \hat{y}_{jg}}{\partial y_{ig}^*} = \frac{q_j}{1-q_i} q_i. \quad (\text{A19})$$

Doing so simplifies our determination of the sign of $d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i$. We then rewrite equation

A18 as

$$\frac{d\hat{y}_{jg}}{dx_i} = \frac{\begin{vmatrix} -1 & q_i & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_i} & -\frac{q_i q_j}{1-q_i} & \frac{q_j q_k}{1-q_i} \\ \frac{q_i q_k}{1-q_i} & -\frac{q_j q_k}{1-q_i} & -1 \end{vmatrix}}{\begin{vmatrix} -1 & \frac{q_i q_j}{1-q_i} & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_i} & -1 & \frac{q_j q_k}{1-q_i} \\ \frac{q_i q_k}{1-q_i} & \frac{q_j q_k}{1-q_i} & -1 \end{vmatrix}}. \quad (\text{A20})$$

In the remainder of the proof, we evaluate $d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i$ and show that

$$\text{sign}|d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i| = \text{sign}|d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i|. \quad \text{We begin by evaluating } d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i.$$

To simplify the appearance of $d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i$, without loss of generality, we set $q_k = \alpha q_j$ for $\alpha \in (0,1]$. Using $1 = q_i + q_j + \alpha q_j$ and solving for q_j , we have $q_j = (1 - q_i)/(1 + \alpha)$. Evaluating (A17) and (A20), using the two equalities — $q_k = \alpha q_j$ and $q_j = (1 - q_i)/(1 + \alpha)$ — we write

$d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i$ as

$$d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i = \frac{\alpha q_i (1 + \alpha^2 + 3\alpha + \alpha q_i^2 + \alpha^2 q_i^2 + q_i^2 - 8\alpha q_i - 4q_i - 4\alpha^2 q_i)}{\begin{pmatrix} 1 + 4\alpha + 5\alpha^2 + 4\alpha^3 + \alpha^4 + 2\alpha^2 q_i - 5\alpha^2 q_i^2 \\ -q_i^2 - 2\alpha q_i^2 + 2\alpha^2 q_i^3 - 2\alpha^3 q_i^2 - \alpha^4 q_i^2 \end{pmatrix}}. \quad (\text{A21})$$

To evaluate (A21), we evaluate the denominator *den* and the numerator *num* separately.

We now show that $den > 0$ for any $\alpha \in (0,1]$ and $q_i \in (0,1)$, because *den* is strictly concave with respect to q_i for $q_i \in (0,1)$, and $den > 0$ if either $q_i = 0$ or $q_i = 1$. As a demonstration of concavity,

$$\frac{d^2 den}{dq_i^2} = -[2 + 4\alpha + 10\alpha^2 + 4\alpha^3 + 2\alpha^4 - 12\alpha^2 q_i] < 0 \quad \text{for any } \alpha \in (0,1] \text{ and } q_i \in (0,1). \quad (\text{A22})$$

Next, if $q_1 = 0$, then $den = 1 + 4\alpha + 5\alpha^2 + 4\alpha^3 + \alpha^4 > 0$; if $q_i = 1$, then $den = 2\alpha + 4\alpha^2 + 2\alpha^3 > 0$.

Therefore, $den > 0$.

Because the denominator of (A21) is positive, its sign depends on the sign of num . Evaluating num , we show that it is strictly concave with respect to q_i for $q_i \in [0,1]$ and has three roots. As a demonstration of concavity,

$$\frac{d^2 num}{dq_i^2} = \alpha[-(8\alpha^2 + 16\alpha + 8) + 6q_i(\alpha^2 + \alpha + 1)] < 0 \text{ for any } \alpha \in (0,1] \text{ and } q_i \in [0,1]. \quad (\text{A23})$$

The three roots of the third-order polynomial num with respect to q_i are:

- (i) 0;
- (ii) $\tilde{q}_i \equiv \left(2(1+\alpha)^2 - ((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}\right) / (\alpha^2 + \alpha + 1)$; and (A24)
- (iii) $\tilde{\tilde{q}}_i \equiv \left(2(1+\alpha)^2 + ((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}\right) / (\alpha^2 + \alpha + 1)$.

The third root, $\tilde{\tilde{q}}_i$, is greater than 1 for any $\alpha \in (0,1]$. According to the concavity of num in $q_i \in [0,1]$ and the values of the three roots, for any $\alpha \in (0,1]$, $num > 0$ if $q_i < \tilde{q}_i$; $num = 0$ if $q_i = \tilde{q}_i$; and $num < 0$ if $q_i > \tilde{q}_i$.

Thus, if x_i is sufficiently small, so that in equilibrium $q_i \in (0, \tilde{q}_i)$, then

$$d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i > 0; \text{ and if } x_i \text{ is sufficiently large, so that in equilibrium } q_i \in (\tilde{q}_i, 1), \text{ then}$$

$$d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i < 0.$$

To complete the proof, we need to establish that if $x_i < \min\{x_j, x_k\}$, then $q_i \in (0, \tilde{q}_i)$ and that if $x_i > \max\{x_j, x_k\}$, then $q_i \in (\tilde{q}_i, 1)$. From Lemma 1 (i.e., $x_j > x_k \Leftrightarrow q_j > q_k$) and the requirement that $\alpha \in (0,1]$, we note that $x_j \geq x_k$. If $x_i < x_k$ and $q_i < q_k$, then $q_i \in (0, \tilde{q}_i)$; however, if $x_i > x_j$ and $q_i > q_j$, then $q_i \in (\tilde{q}_i, 1)$. We examine two cases.

Case 1: University i is the best university, $x_i > \max\{x_j, x_k\}$.

If $x_i > \max\{x_j, x_k\}$, by Lemma 1, $q_i > \max\{q_j, q_k\}$, so we exaggerate the positive responses by the competitors. That is,

$$\frac{\partial \hat{y}_{jg}}{\partial x_i} = \frac{\partial \hat{y}_{jg}}{\partial y_{ig}} = \frac{q_j}{1-q_i} q_i > \frac{q_j}{1-q_j} q_i = \frac{\partial y_{jg}^*}{\partial x_i} = \frac{\partial y_{jg}^*}{\partial y_{ig}} > 0. \quad (\text{A25})$$

By exaggerating $\partial y_{jg}/\partial x_i$ and $\partial y_{jg}/\partial y_{ig}$ without changing y_{jg}^* , the exaggeration has only a second-order (i.e., very small) effect on dy_{ig}^*/dx_i . If $x_i > \max\{x_j, x_k\}$ and $d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i < 0$, then $d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i < 0$. Figure A1 shows both $d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i$ and $d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i$ for the case in which $x_i > \max\{x_j, x_k\}$.

[Insert Figure A1 about here]

Using $1 = q_i + q_j + \alpha q_j$, we derive that $q_i > q_j$ (and by Lemma 1 $x_i > x_j$) if and only if

$q_i > 1/(2 + \alpha)$. Therefore, we require that if $q_i > 1/(2 + \alpha)$, then $q_i \in (\tilde{q}_i, 1)$. Equivalently, we need

$1/(2 + \alpha) > \tilde{q}_i$. Using (A24), we find

$$\begin{aligned} \frac{1}{2 + \alpha} - \tilde{q}_i &= \frac{1}{2 + \alpha} - \frac{2(1 + \alpha)^2 - ((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}}{(\alpha^2 + \alpha + 1)} \\ &= \frac{(1 + 2\alpha)(\alpha^2 + 3\alpha + 3) + (2 + \alpha)((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}}{(\alpha^2 + \alpha + 1)(2 + \alpha)}. \end{aligned} \quad (\text{A26})$$

Because the denominator of r.h.s. of the second line of (A24) is positive,

$$\text{sign}\left(\frac{1}{2 + \alpha} - \tilde{q}_i\right) = \text{sign}\left(- (1 + 2\alpha)(\alpha^2 + 3\alpha + 3) + (2 + \alpha)((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}\right) \quad (\text{A27})$$

Evaluating the r.h.s. of (A27), we note that

$$\left(- (1 + 2\alpha)(\alpha^2 + 3\alpha + 3) + (2 + \alpha)((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}\right) > 0$$

if and only if

$$\frac{\alpha^2 + 4\alpha + 4}{4\alpha^2 + 4\alpha + 1} > \frac{\alpha^2 + 3\alpha + 3}{3\alpha + 3\alpha + 1}. \quad (\text{A28})$$

We have that for any $\alpha \in (0,1)$, (A28) holds (with a strict equality if $\alpha = 1$).

Case 2: University i is the lowest ranked university, $x_i < \min\{x_j, x_k\}$.

If $x_i < \min\{x_j, x_k\}$, then $q_i < \min\{q_j, q_k\}$. Hence, we shade the positive response by the competitor.

That is,

$$\frac{\partial \hat{y}_{jg}}{\partial x_i} = \frac{\partial \hat{y}_{jg}}{\partial y_{ig}} = \frac{q_j}{1-q_i} q_i < \frac{q_j}{1-q_j} q_i = \frac{\partial y_{jg}^*}{\partial x_i} = \frac{\partial y_{jg}^*}{\partial y_{ig}} > 0. \quad (\text{A29})$$

By shading these positive responses, if $x_i < \min\{x_j, x_k\}$ and $d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i > 0$, then

$d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i > 0$. Figure A2 shows both $d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i$ and $d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i$

for the case in which $x_i < \min\{x_j, x_k\}$.

[Insert Figure A2 about here]

Using $1 = q_i + q_j + \alpha q_j$ and $q_k = \alpha q_j$, we derive that $q_i < q_k$ (and by Lemma 1, $x_i < x_k$) if and only if $q_i < \alpha/(1+2\alpha)$. Therefore, we require if $q_i < \alpha/(1+2\alpha)$, then $q_i \in (0, \tilde{q}_i)$. Equivalently, we need $\tilde{q}_i - \alpha/(1+2\alpha) > 0$. Using (A24), we find

$$\begin{aligned} \tilde{q}_i - \frac{\alpha}{1+2\alpha} &= \frac{2(1+\alpha)^2 - ((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}}{(\alpha^2 + \alpha + 1)} - \frac{\alpha}{1+2\alpha} \\ &= \frac{(2+\alpha)(\alpha^2 + 3\alpha + 1) - (1+2\alpha)((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}}{(\alpha^2 + \alpha + 1)(1+2\alpha)}. \end{aligned} \quad (\text{A30})$$

Because the denominator of r.h.s. of the second line of (A30) is positive,

$$\text{sign}\left(\tilde{q}_i - \frac{\alpha}{1+2\alpha}\right) = \text{sign}\left((2+\alpha)(\alpha^2 + 3\alpha + 1) - (1+2\alpha)((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}\right). \quad (\text{A31})$$

Evaluating the r.h.s. of (A31), we note

$$\left((2+\alpha)(\alpha^2 + 3\alpha + 1) - (1+2\alpha)((3\alpha^2 + 3\alpha + 1)(\alpha^2 + 3\alpha + 3))^{1/2}\right) > 0 \quad (\text{A32})$$

if and only if (A28) holds. Again, for any $\alpha \in (0,1)$, equation A28 holds (with a strict equality if $\alpha = 1$).

Q.E.D.

Proof of Lemma 2

If $(x_i - x_j) = (\hat{x}_i - \hat{x}_j)$ for each $i, j \in \{1,2,3\}$, then from (2),

$$q_i(x_1 + y_{1g}, x_2 + y_{2g}, x_3 + y_{3g}) = q_i(\hat{x}_1 + y_{1g}, \hat{x}_2 + y_{2g}, \hat{x}_3 + y_{3g}). \quad (\text{A33})$$

For the two quality vectors (x_1, x_2, x_3) and $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$, the effect of y_{ig} on the expected scores is identical. That is, for each university i and each candidate g :

$$\begin{aligned} \frac{\partial E[\sigma_i(x_1 + y_{1g}, x_2 + y_{2g}, x_3 + y_{3g})]}{\partial y_{ig}} &= [w_p(v_i - v_{io}) - w_r y_{ig}] \frac{(1 - q_i(x_1 + y_{1g}, x_2 + y_{2g}, x_3 + y_{3g}))}{\mu} - w_r \\ &= \frac{\partial E[\sigma_i(\hat{x}_1 + y_{1g}, \hat{x}_2 + y_{2g}, \hat{x}_3 + y_{3g})]}{\partial y_{ig}} = [w_p(v_i - v_{io}) - w_r y_{ig}] \frac{(1 - q_i(\hat{x}_1 + y_{1g}, \hat{x}_2 + y_{2g}, \hat{x}_3 + y_{3g}))}{\mu} - w_r. \end{aligned} \quad (\text{A34})$$

With these identical derivatives, for each i and each g , $y_{ig}^*(x_1, x_2, x_3) = y_{ig}^*(\hat{x}_1, \hat{x}_2, \hat{x}_3)$.

Q.E.D.

Proof of Theorem 2

Using (A10),

$$\frac{\partial y_{ig}^*}{\partial (v_{ig} - v_{io})} = w_p(1 - q_i) > 0. \quad (\text{A35})$$

As we state in (A12), for any $j, k \in \{1,2,3\}$,

$$\frac{\partial q_j}{\partial y_{kg}} = \frac{q_j q_k}{\mu} > 0. \quad (\text{A36})$$

The direct effect, as expressed in (A35), of an increase in $(v_{ig} - v_{io})$ on y_{ig}^* is positive. All secondary effects, as expressed in (A36), of an increase in one university's merit aid offer on its competitors' merit aid offers is positive. Therefore, for each $j \in \{1,2,3\}$, in equilibrium,

$$\frac{\partial y_{jg}^*}{\partial (v_{ig} - v_{io})} > 0. \tag{A37}$$

Q.E.D.

Figure A1. University i is the best university; effect of an increase in university i 's quality on the sum of its merit aid offers. If $x_i > \max\{x_j, x_k\}$, then

$$d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i < d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i < 0.$$

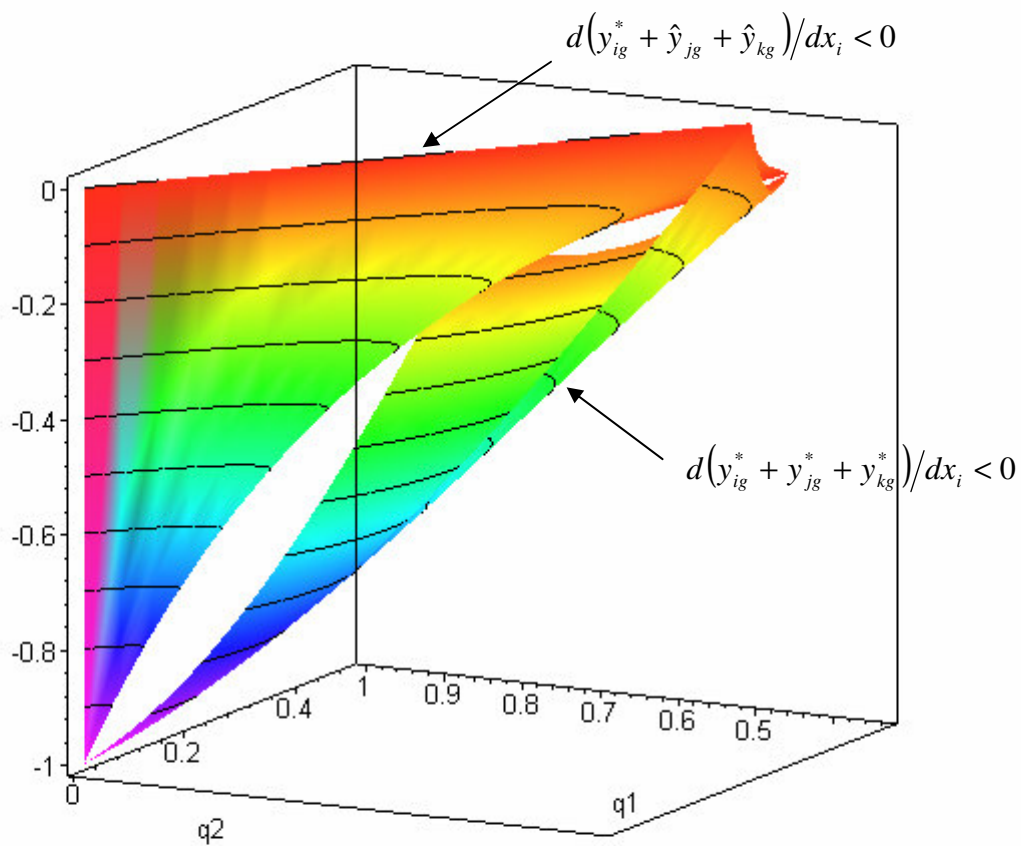


Figure A2. University i is the worst university; effect of an increase in university i 's quality on the sum of its merit aid offers. If $x_i < \min\{x_j, x_k\}$, then

$$d(y_{ig}^* + y_{jg}^* + y_{kg}^*)/dx_i > d(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg})/dx_i > 0.$$

