Simulations and Detection of Brain Aneurysms

• **What** are brain aneurysms?
• **How** to study brain aneurysms?
• **What** did we find about brain aneurysms?
A complete human arterial tree delivers the oxygen and nutrition to the whole body.
Thomas Willis found a circulatory anastomosis in the brain.
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Circle of Willis can preserve the brain blood supply.
Variations in the Circle of Willis

- There are considerable anatomic variations in the Circle of Willis.
Variations in the Circle of Willis

- There are considerable anatomic variations in the Circle of Willis.
- Such defects may be related to different cardiovascular diseases.
Stroke Related with Brain Arteries

- Stroke is a sudden interruption of the blood supply to the brain.

Ischemic stroke (80%)

Hemorrhagic stroke (20%)
Stroke is a sudden interruption of the blood supply to the brain.

80% of strokes are caused by an abrupt blockage of an artery. 20% of strokes are caused by bleeding into brain tissue when a blood vessel bursts.

**Ischemic stroke (80%)**

**Hemorrhagic stroke (20%)**
• Unruptured brain aneurysms are typically completely with no symptom.
• Brain aneurysms are associated with complicated blood flow patterns.
Why Numerical Simulations?

• The current clinical technology could not provide detailed in vivo measurements for intra-aneurysm flow patterns.
• For aneurysm detection, expensive and inconvenient.
Why Numerical Simulations?

- Numerical simulations on computers are an effective alternative approach for understanding the mechanisms behind aneurysm growth and rupture.
Arterial Flow Simulations

Data analysis

Simulation
Findings: Flow Patterns in Aneurysm
Frequency Analysis With Fourier Transform

A Mathematical Tool
Sum of Sines and Cosines

Our building block:

1, \sin(t), \sin(2t), \sin(3t), \ldots, \sin(kt), \ldots, \cos(t), \cos(2t), \cos(3t), \ldots, \cos(kt), \ldots

Add enough of them to get any periodic function \( f(t) \) you want!

\[
f(t) = a + \sum_{k=1}^{\infty} b_k \sin(kt) + \sum_{k=1}^{\infty} c_k \cos(kt)
\]
Example: Step Function

\[ 1 + \frac{4}{\pi} \sin(t) \]
Example: Step Function

\[ 1 + \frac{4}{\pi} \sin(t) + \frac{4}{3\pi} \sin(3t) \approx 4 \sin(t) \sin(3t) \]
Example: Step Function

\[ 1 + \frac{4}{\pi} \sin(t) + \frac{4}{3\pi} \sin(3t) + \frac{4}{5\pi} \sin(5t) \approx \]

\[ 1 + \frac{4}{\pi} \sin(t) + \frac{4}{3\pi} \sin(3t) + \frac{4}{5\pi} \sin(5t) \]
Example: Step Function

\[ 1 + \sum_{i=1}^{3} \frac{4}{(2i-1)\pi} \sin((2i-1)t) + \frac{4}{7\pi} \sin(7t) \approx \]

\[ = 1 + \sum_{i=1}^{4} \frac{4}{(2i-1)\pi} \sin((2i-1)t) \]
Example: Step Function

\[ \approx 1 + \sum_{i=1}^{n} \frac{4}{(2i-1)\pi} \sin((2i-1)t) \]

\[ n=6 \]
\[ n=50 \]
\[ n=250000 \]
To express $f(t)$ as

$$f(t) = a + \sum_{k=1}^{\infty} b_k \sin(kt) + \sum_{k=1}^{\infty} c_k \cos(kt)$$
Fourier Transform

To express \( f(t) \) as

\[
    f(t) = a + \sum_{k=1}^{\infty} b_k \sin(kt) + \sum_{k=1}^{\infty} c_k \cos(kt)
\]

Orthogonal properties:

\[
    \int_{0}^{2\pi} 1 \, dt = 2\pi, \quad \int_{0}^{2\pi} \sin(x) \, dt = 0, \quad \int_{0}^{2\pi} \cos(x) \, dt = 0;
\]

\[
    \int_{0}^{2\pi} \sin(jt) \cos(kt) \, dt = 0;
\]

\[
    \int_{0}^{2\pi} \sin(jt) \sin(kt) \, dt = \begin{cases} \pi, & \text{when } j = k \\ 0, & \text{when } j \neq k \end{cases};
\]

\[
    \int_{0}^{2\pi} \cos(jt) \cos(kt) \, dt = \begin{cases} \pi, & \text{when } j = k \\ 0, & \text{when } j \neq k \end{cases};
\]
To express $f(t)$ as

$$f(t) = a + \sum_{k=1}^{\infty} b_k \sin(kt) + \sum_{k=1}^{\infty} c_k \cos(kt)$$

Fourier spectrums:

$$a = \frac{1}{2\pi} \int_{0}^{2\pi} f(t) dt$$

$$b_k = \frac{1}{\pi} \int_{0}^{\pi} f(t) \sin(kt) dt$$

$$c_k = \frac{1}{\pi} \int_{0}^{\pi} f(t) \cos(kt) dt$$
Fourier Frequency Spectra

\[ 1 + \sum_{k=1}^{\infty} \frac{4}{(2i - 1)\pi} \sin((2i - 1)t) \]
Fourier Frequency Spectra

\[ 1 + \sum_{k=1}^{\infty} \frac{4}{(2i - 1)\pi} \sin((2i - 1)t) \]
Fourier Frequency Spectra

\[ 1 + \sum_{\kappa=1}^{\infty} \frac{4}{(2\kappa - 1)\pi} \sin((2\kappa - 1) t) \]
Fourier Frequency Spectra

\[ 1 + \sum_{i=1}^{\infty} \frac{4}{(2i-1)\pi} \sin((2i - 1)\pi) \]
Fourier Frequency Spectra

\[ 1 + \sum_{\kappa=1}^{\infty} \frac{4}{(2i - 1)\pi} \sin((2i - 1) t) \]
• Single Frequency Sound
Example: Sound

- Single Frequency Sound
Example: Sound

- Single Frequency Sound
Example: Sound

• Guitar Sound 🎸
Example: Sound

- Guitar Sound
Example: Sound

- Guitar Sound
The blood flow in aneurysms has stress/velocity fluctuations (20-50Hz).
Future Direction: Detecting Aneurysms

- The blood flow in aneurysms has stress/velocity fluctuations (20-50Hz).
- May analyze the murmurs to detect the brain arterial defects and aneurysms.

Sound data collected
Brain (Cerebral) Aneurysm

- Hemorrhagic strokes have a much higher death rate than ischemic strokes.