Chapter 4 Probability Distributions

HW#5: 4.4 Compound events # 25, 27, 28, 29, 30, 32

4.25. A card is to be randomly selected from an ordinary deck of 52 cards. Consider the following events: A = ace, B = face card, and C = club.

a) Verify that the only pair of mutually exclusive events is A, B.

possible pair of events: A,B; A,C; B,C;

Because the selected card can't be an ace and a face card at the same time, A, B is a mutually exclusive pair. And, the ace of clubs belongs to both A,C, while the jack of clubs belongs to both B,C, that is A,C is not mutually exclusive and the same for B,C.

b) Use the addition Rule to find P(A or B).

P(A or B) = P(A) + P(B) = $\frac{4}{52} + \frac{12}{52} = \frac{4}{13}$

4.27. For a randomly selected person from the population,

A: the person selected voted

B: the person selected is a male.

	A(voted)	$\bullet$ (did not voted)	total
B(Males)	53,312	35,245	88,557
$\bar{B}$(Females)	60,554	36,573	97,127
total	113,866	71,818	185,684

a) $P(B) = \frac{88557}{185684} = .47692$

b) $P(\bullet) = \frac{71818}{185684} = .38678$

c) $P(\bullet \text{ and } \bar{B}) = \frac{36573}{185684} = .19696$

4.28. For randomly selecting a voter from the population to be interviewed,

A: consists of being in favor of maore stringent building codes;

B: consists of having lived in the town less than 10 years.

	A(Favor more stringent code)	$\bullet$ (Do not favor...)	total
B(Less than 10 years)	100	700	800
$\bar{B}$(At least 10 years)	1000	200	1200
total	1100	900	2000

a) $P(A) = \frac{1100}{2000} = \frac{11}{20} = .55$

b) $P(\bullet) = \frac{900}{2000} = \frac{9}{20} = .45$

c) $P(A \text{ and B}) = \frac{100}{2000} = \frac{1}{20} = .05$

4.29 Two cards are to be selected form an ordinary deck of 52 cards with replacement.

A: first card is an ace;

B: second card is an ace;

C: second card is a king.

a) find each of the following:

i. $P(A \text{ and } B) = P(A)\,P(B) = \left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = 0.0059$
ii. Since B and C are mutually exclusive:
$P(B \text{ or } C) = P(B) + P(C) = \frac{4}{52} + \frac{4}{52} = .1538$

b) Now suppose a third card is selected after replacement of the first and second cards.
Let D = third card is not an ace. Find:P(A and B and D)
Because events A and B and D are mutually independent,
$P(A \text{ and B and D}) = P(A)P(B)P(D) = \left(\frac{4}{52}\right)\left(\frac{4}{52}\right)\left(\frac{48}{52}\right) = 0.0055$

4.30. A sporting goods store has a large batch of cans of tennis balls on hand. Ten percent of the cans are unacceptable.

a) A customer decides to purchase one can. What is the probability that the customer will be satisfied?
P(the customer will be satisfactory)
$= P(\text{the select can is not unacceptable})$
$= 1 - .1$
$= .9$

b) A customer is to purchase two cans. Find the probability that
i. Both cans will be satisfactory.
$P(\text{Both cans will be satisfactory})$
$= .9 \times .9$
$= .81$

ii. Exactly one can will be satisfactory.
$P(\text{Exactly one can will be satisfactory})$
$= P(\text{one can will be satisfactory and the other is unacceptable})$
$= .9 \times .1 + .1 \times .9$
$= .18$

iii. At least one can will be satisfactory.
$P(\text{Both cans will be satisfactory}) + P(\text{Exactly one can will be satisfactory})$
$= .81 + .18$
$= .99$

4.32. A large shipment contains 2% defective items. Five items are to be selected. What is the probability of getting one or more defetive items?
Since
$P(\text{none of the five items are defective})$
$= (1 - .02)^5$
$= .90392$
then,

$P(\text{getting one or more defective items})$
$= 1 - P(\text{none of the five items are defective})$
$= 1 - .90392$
$= .09608$

4.36.Consider the experiment of randomly selecting a voter to be interviewed

	A(Favor more stringent code)	$\bullet$ (Do not favor...)	total
B(Less than 10 years)	100	700	800
$\bar{B}$(At least 10 years)	1000	200	1200
total	1100	900	2000

a) $P(\text{A or B}) = \dfrac{n(A) + n(B) - n(A \text{ and } B)}{n(S)} = \dfrac{1100 + 800 - 100}{2000} = \frac{9}{10} = .9$

OR: $P(\text{A or B}) = 1 - P(\bullet \text{ and } \bar{\text{B}}) = 1 - \frac{200}{2000} = 1 - \frac{1}{10} = \frac{9}{10} = .9$

b) $P(\text{A or } \bar{\text{B}}) = \dfrac{n(A) + n(\bar{B}) - n(A \text{ and } \bar{B})}{n(S)} = \dfrac{1100 + 1200 - 1000}{2000} = \frac{13}{20} = .65$

OR: $P(\text{A or } \bar{\text{B}}) = 1 - P(\bullet \text{ and B}) = 1 - \frac{700}{2000} = \frac{13}{20} = .65$

4.42

Let E denote the event that a man is employed, and C denote the event that a man will commit a crime.Then,

$P(E) = .7 \quad P(C) = .1 \quad P(E|C) = .05$

By multiplication rule, we can get

$P(\text{C and E}) = P(C) \bullet P(E|C) = .1 \times .05 = .005$

We want $P(C|E)$. Now from the multiplication rule: $P(\text{C and E}) = P(E) \bullet P(C|E)$

we get:

$$P(C|E) = \frac{P(\text{C and E})}{P(E)} = \frac{.005}{.7} = 0.007$$

4.44

For a randomly selected person from the population,

A: the person selected voted

B: the person selected is a male.

	A(voted)	$\bullet$ (did not voted)	total
B(Males)	53,312	35,245	88,557
$\bar{B}$(Females)	60,554	36,573	97,127
total	113,866	71,818	185,684

a) $P(A \text{ or } B) = \dfrac{n(A) + n(B) - n(A \text{ and } B)}{n(S)} = \dfrac{113866 + 88557 - 53312}{185684} = \frac{149111}{185684} = .803$

b) $P(\bullet \text{ or B}) = \dfrac{n(\bullet) + n(B) - n(\bullet \text{ and } B)}{n(S)} = \dfrac{71818 + 88557 - 35245}{185684} = \frac{62565}{92842} = .674$

c) $P(B|A) = \dfrac{P(\text{A and B})}{P(A)} = \dfrac{53312/185684}{113866/185684} = \dfrac{53312}{113866} = .4682$

d) $P(\bullet\ |\bar{B}) = \dfrac{P(\bullet\ \text{ and } \bar{\text{B}})}{P(\bar{B})} = \dfrac{36573/185684}{97127/185684} = \dfrac{36573}{97127} = .376$

4.50.

Consider the set S = $\{a, b, c, d\}$.

a) ${}_4C_2 = \dfrac{4!}{2!(4-2)!} = 6$

list the combinations: $\{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}$

b) ${}_4P_2 = \dfrac{4!}{2!} = 12$

list the permutations: $ab, ba, ac, ca, ad, da, bd, db, cd, dc$.

4.52.

6 women and 4 men, selecting 2 reporters from them

a) if both of 2 reporters are to be women, there are

${}_6C_2 = \dfrac{6!}{2!(6-2)!} = 15$

possible groups of two reporters

b) if both of them are to be men, there are

${}_4C_2 = \dfrac{4!}{2!(4-2)!} = 6$

possible groups of two reporters

c) if there is to be one woman and one man,

${}_6C_1 \bullet {}_4C_1 = 6 \times 4 = 24$