Control of Plasma Profiles in a Tokamak

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Outline

- What are the primary profiles in a tokamak plasma to control
- A small detour about tokamak plasma geometry
- Why control profiles in a tokamak plasma
- Equations governing the evolution of T, n, and j profile quantities
- Actuators available for the control of profile quantities
- Reality check, limitations to profile control
- Some important features of current profile control
- Measurements or reconstruction of profile quantities
- Nonlinear interactions in plasmas
- A basic control strategy for a tokamak plasma
- Simulations of tokamak plasma profile evolution
- Efforts on profile control -- JET (Europe)
- Issues for tokamak profile control
What are the Primary Profiles to Control in a Tokamak Plasma

Plasma profile versus R
Profile measurements

Magnetic flux surfaces geometry determined by force balance, plasma is in force balance at all times

Plasma electron ($T_e$) and ion ($T_i$) temperatures, particle density (electron, $n_e$), and current density ($j$, that flows the long way around the tokamak) profiles

Typically assume $T_e$, $T_i$, $n$, and $j$ are constant on a magnetic flux surface

Measurements give profile quantities as a function of $R$

Geometry of magnetic flux surfaces allows us to represent profiles as a function of $r$
A Small Detour About Tokamak Plasma Geometry

Magnetic flux surfaces provide a natural grid, a curvilinear coordinate system.

Toroidal geometry gives rise to real physics differences than that in a cylinder, **must be careful**.

Magnetic axis, is not equal to the geometric center, it is where the poloidal magnetic field is zero.

Tending toward a cylinder, large aspect ratio \((R/r)\) approximation.

Tokamak plasmas are invariant in the \(\varphi\) direction, *well sort of*.
Why Should We Control Profiles in a Tokamak Plasma?

Magnetic fusion plasmas are used to generate energy via the production of energetic particles (D + T → He4 + n) from nuclear reactions
- Neutrons (n) travel thru the plasma into surrounding structures and heat coolant
- Alpha (He4) particles stay in the plasma and further heat the plasma

Controlling plasma profiles will allow us to operate a tokamak closer to its limits, getting much better performance from the plasma (generate more energy in a smaller space and get the plasma to generate a lot of the current that flows in it by itself)

Both theory and experiments show that some profiles are better than others
- Magnetohydrodynamic stability shows that combinations of pressure (T×n) and safety factor \( (rB_\phi/RB_\theta) \) can allow higher pressures to be contained
- Certain safety factor profiles can reduce transport of energy and particles out of the plasma

There are operating modes in tokamak plasmas where we do not control plasma profiles, but their performance is limited

Plasma profiles are not directly controlled in present experiments (this area is only emerging now), but usually they are manipulated by feed-forward techniques to study the “better profile” regimes
Equations Governing the Evolution of T, n, and j Profiles

Density evolution eqn
\[ \frac{\partial n_e}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} r D_n \frac{\partial n_e}{\partial r} - n_e V_n + S_n \]

Cylindrical coordinates are used here

These equations govern only the core plasma, not outside the plasma

Subscript \( "i" \) refers to hydrogenic ions

\[ n_e (r = 0) = 0 \]

\[ n_e (r = a) = n_e^a \]

\[ n_e = n_i Z_i + \sum n_j Z_j \]  

Quasi-neutrality allows \( n_i \) to be determined

Energy evolution eqn
\[ \frac{3}{2} \frac{\partial n_e T_e}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} r \left( n_e \chi_e \frac{\partial T_e}{\partial r} + \frac{5}{2} T_e \Gamma_n \right) - P_{\text{rad}} + V_L \frac{\partial K}{\partial r} + S_{He} + Q_{\Delta e} \]

\[ 3 \frac{\partial n_i T_i}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} r \left( n_i \chi_i \frac{\partial T_i}{\partial r} + \frac{5}{2} T_i \Gamma_n \right) - Q_{\Delta e} + S_{Hi} \]

\[ \frac{\partial T_{e,i}(r = 0)}{\partial r} = 0 \]

\[ T_{e,i}(r = a) = T_{e,i}^a \]

\( P_{\text{rad}} \) includes bremsstrahlung, cyclotron, and line
\( V_L \) is loop voltage, \( K \) is enclosed plasma current
\( S_{He,i} \) are the heating sources
\( Q_{\Delta e} \) is the equipartition term between electron and ions
Equations Governing the Evolution of $T$, $n$, and $j$, cont’d

**Current evolution equation**

\[
\frac{\partial j}{\partial t} = \frac{1}{\mu_0 r} \frac{\partial}{\partial r} \left( r \frac{\partial \eta j}{\partial r} - r \frac{\partial \eta j_{CD}}{\partial r} \right)
\]

\[
\frac{\partial \psi}{\partial t} = \eta \left[ \frac{1}{\mu_0 r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) - j_{CD} \right]
\]

\[
\frac{\partial B_\theta}{\partial t} = \frac{\partial}{\partial r} \frac{\eta}{\mu_0 r} \frac{\partial}{\partial r} (r B_\theta) - \frac{\partial}{\partial r} (\eta j_{CD})
\]

\[
\frac{\partial \psi(r = 0)}{\partial r} = 0
\]

\[
\left[ \psi + \frac{a}{\mu_0 R} L_{ext}^{\text{eff}} \frac{\partial \psi}{\partial r} \right]_{r=a} = 0,
L_{ext}^{\text{eff}} \approx \frac{3}{4} \left[ \mu_0 R \ln \left( \frac{8R}{a} \right) - 2 \right]
\]

or

\[
\left[ \frac{2\pi a \partial \psi}{\mu_0} \frac{\partial \psi}{\partial r} \right]_{r=a} = I_p
\]

**Cylindrical coordinates are used here**

The 3 equations are different forms for the same evolution equation

Different equation forms may be better with given measurements or control strategies

$j$ is the current density that flows the long way around the tokamak

$B_\theta$ is the poloidal magnetic field

$\psi$ is the poloidal magnetic flux, $\oint dA \cdot B_\theta$

$\eta$ is the plasma resistivity
Equations Governing the Evolution of $T$, $n$, and $j$, cont’d

Force balance or Equilibrium eqn

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \psi}{\partial r} \right) = \mu_o j_z \]

driven by the fluid nature of the plasma, which must be in force balance at all times (time scales of interest).

The force balance equation is $\mathbf{j} \times \mathbf{B} = \nabla p$

Solution of the equilibrium equation gives the magnetic flux surfaces which serve as the grid for transport of $T$, $n$, and $j$ since they are constant on a flux surface.

Cylindrical coordinates

\[ j_z = -\frac{dp(\psi)}{d\psi} - f(\psi) \frac{df(\psi)}{d\psi} \]

Toroidal coordinates

\[ R \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} = \mu_o R j_\varphi \]

Grad-Shafranov eqn

\[ j_\varphi = -R \frac{dp(\psi)}{d\psi} - \frac{f(\psi) df(\psi)}{\mu_o R d\psi} \]
Acuators (Sources) of Heating, Particles and Current for Profile Control

Particles: *(walls surrounding plasma)*
- Pellets
- Gas injection
- Neutral Beam Injection
- Pumping

Heating: *(fusion reactions inside plasma, elec & ions)*
- Neutral Beam Injection (elec & ions)
- Ion Cyclotron Radio Frequency Waves (elec & ions)
- Electron Cyclotron Waves (elec)
- Lower Hybrid Waves (elec)
- Inductive heating (ohmic dissipation) (elec)

Current: *(bootstrap current generated by plasma)*
- Neutral Beam Injection
- Ion Cyclotron Radio Frequency Waves
- Electron Cyclotron
- Lower Hybrid Waves
- Magnetic field coils outside plasma (inductive)
Plasma transport of particles and energy are driven by internal fluid mechanisms, characterized by “transport coefficients” like D, V, and χ for which progressively better theories are being developed.

- We do not (and probably can not) control n and T profiles, instead we control regimes, setting up conditions to get certain types of profiles, and possibly controlling a scalar feature of the profiles.

- Actually, a way that we might obtain some control over T and n profiles is by going back and forth between 2 different transport regimes, to create something in between.

- Internal Transport Barriers (ITB’s) are where the transport coefficients are very small, giving rise to strong gradients in T or n, and these can act like a spigot if we find ways to turn them off and on to create desirable profiles.

- Rotation of the plasma in the toroidal and poloidal directions can occur naturally, but this is generally weak, however, Neutral Beam Injection can rotate plasmas very fast in the toroidal direction --- rotation (or momentum) is also a transport quantity.

- Plasma rotation has been found to suppress magnetohydrodynamic instabilities and rotational shear (dv/dr) can suppress particle and energy transport --- however, plasma rotation is expected to be small for future large tokamaks.
Some Important Features of the Current Evolution Equation

Since the “transport coefficient” associated with current transport (neoclassical resistivity) is largely well verified, the current is a good candidate for profile control.

Time scales for current evolution are determined by the local resistivity, so they are slower in the center and faster near the edge, and goes like

\[ \tau_j \approx \frac{\mu_o r^2 \kappa}{12\eta}, \quad \eta = \frac{f(Z_{eff}, f_t, \nu_*)}{T_e^{3/2}} \]

The plasma is a conductor, and resists changes in its flux (or current or field) by setting up an electric field in the opposite direction, when a current drive source is applied, and this backward electric field decays on the time scale above.

Mathematically, inductive current drive is a change in the boundary condition on the partial differential eqn for current evolution, while non-inductive current drive is a volumetric source term.
Some More Important Features of the Current Evolution Equation

The current profile has also been shown to be an important parameter in determining density and energy transport regimes.

Our current actuators are spatially coarse and limited in power, so the profile control will likely be effective in an average sense.

There are complexities to current transport that arise from some magnetohydrodynamic instabilities (referred to as tearing modes), which is difficult to include in a control model. This may have to be absorbed as a disturbance.

The efficiency of driving current in the plasma, for any of the sources, comes from a complex theory, indicating that databases will probably be necessary for use in control. These sources also heat the plasma and can alter the local resistivity.

The self-driven bootstrap current is both a benefit as well a primary source of disturbance to the current profile. It depends on the T and n profiles which can change on faster time scales referred to as energy and particle confinement times.
A plasma equilibrium is desired to interpret the profile data, so the measurements like magnetic flux loops, field probes, diamagnetic loop, Rogowski coil, PF coil currents, etc. are necessary \(\psi(R,Z)\).

Electron temperature and density profile data can be obtained from Thomson scattering \(T_e, n_e\).

The electron density can be obtained from interferometry or reflectometry \(n_e\).

Electron cyclotron radiation emission from the plasma can be used to determine the electron temperature \(T_e, T_e^{\text{fast}}\).

Ion temperature, impurity densities, and plasma rotation speed can be obtained from Charge Exchange Recombination Spectroscopy \(T_i, n_Z, v_Z\).

A measurement of the magnetic field inside the plasma, in the form of \(B_\theta/B_\phi\) is available from the Motional Stark Effect, which is how we determine the \(j\) profile (this can also be accomplished by Faraday rotation).

X-ray measurements can be used to identify where fast electrons have been created, for example in Lower Hybrid Waves \(s(R,Z)\).
Nonlinear Interactions Lead to a Complex Control Problem

However, priorities in control, controllability, and coarseness in sources will likely reduce this diagram to a group of subsystem controllers, ignoring many interactions.
A Basic Control Strategy for a Tokamak Plasma

Plasma current, position, shape ---\(\rightarrow\) PF coils

Plasma densities (electron, impurities, fuel ions) ---\(\rightarrow\) gas/pellet/NB injection and pumping

Stored energy ---\(\rightarrow\) auxiliary power (NB, IC, EC, LH), density, impurities

Current profile, \(f_{NI}\) ---\(\rightarrow\) auxiliary power/CD (NB, IC, EC, LH, and bootstrap)

Divertor heat load ---\(\rightarrow\) core radiation, divertor gas injection, \(P_{aux} + P_{\alpha}\)

Operating regime: (isolated subsystems providing specific conditions)

- Resistive Wall Mode feedback control
- Error field correction
- Energy/particle transport regime
- Edge Localized Modes type
- Plasma phase control (laying in \(j\) profile, transition from inductive to non-inductive current)
A Simulation of the Plasma Response to Drop in Lower Hybrid Power

\[ t = 12-25\,\text{s} \]
\[ I(\text{LH}) = 690\,\text{kA} \]

\[ t = 16.7\,\text{s} \]
\[ I(\text{LH}) = 510\,\text{kA} \]

\[ t = 21.6\,\text{s} \]
\[ I(\text{LH}) = 560\,\text{kA} \]

\[ t = 25\,\text{s} \]
\[ I(\text{LH}) = 680\,\text{kA} \]

\[ t = 41\,\text{s} \]
\[ I(\text{LH}) = 685\,\text{kA} \]
Efforts on Profile Control -- JET (Europe)

Control of 4 points on q-profile and 2 points for $T_e$ gradient, using $P_{NB}$, $P_{LH}$, $P_{ICRF}$

Distributed-parameter control of $q$ and $\rho_{Te}^*$
reversed-shear q-profile

Profiles
Quadratic minimization

Efforts in Profile Control -- JET (Europe)

Distributed-parameter control of $q$ and $\rho_{Te}^*$:
least square approach to unaccessible q-profile

Profiles

Quadratic minimization
Limited actuators will not allow detailed profile control in plasmas -- realistic goals must be identified, and corresponding control strategies developed.

Plasma internal mechanisms will make profile control of some quantities very difficult, requiring “regime” control that produces the desired profiles.

There are many subsystem controllers that provide supporting roles in creating a specific plasma “regime”, and these must be integrated.

There are some plasma mechanisms that are too complex to include in models used for plasma control -- there are critical choices to make in what can or must be ignored in the plasma physics.

The approach to steady state plasmas also requires control to avoid instabilities and provide a robust startup trajectory -- this faces the same issues cited for profile control.

The performance of a profile control system will require an assessment of the entire control loop -- measurements, interpretation, controller, actuator, plasma.