What is a Markov Chain?

- Discrete time stochastic process
- Markovian Property:

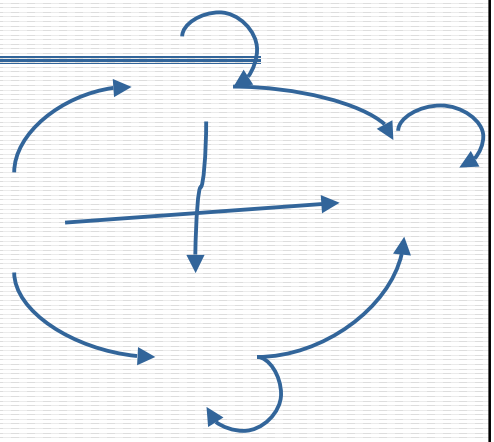
$$P(X_{t+1}|X_t,X_{t-1},\ldots,X_0) = P(X_{t+1}|X_t)$$

- Transition probabilities, p_{ij}
- Stationarity Assumption:

$$p_{ij} = P(X_{t+1}=j|X_t=i) \text{ independent of } t$$

Definitions

- Transitions
  - Path
  - Reachable
  - Communicate
- Types of states:
  - Absorbing
  - Transient
  - Recurrent



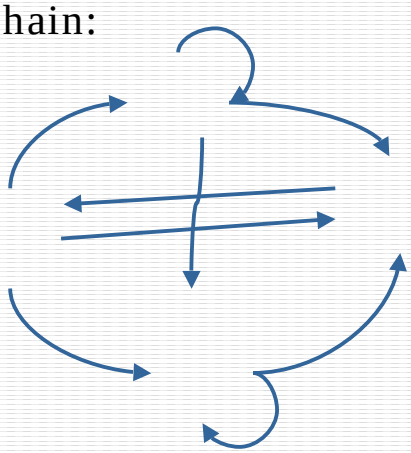
Maintenance Policy

- Transition probabilities

state	0	1	2	3
0	0	7/8	1/16	1/16
1	0	3/4	1/8	1/8
2	0	0	1/2	1/2
3	1	0	0	0

More Definitions

- Ergodic Markov chain:
  - Recurrent
  - Aperiodic
  - Communicate



MC's in Economic Analysis

- Markov processes can be used to evaluate:
  - Profitability of business venture.
  - Value of certain business.
  - Long run expected income.
  - Market share analysis.

Example: Willy's

- Willy's Rentals rents Big Wheels for recreational use at East Coast Parkway.
- Willy's estimates that its profit is a function of the weather conditions.
- The weather is classified into 3 states:
  - Sunny
  - Cloudy
  - Rainy.

Willy's: Weather Conditions

Tomorrows weather

$$\text{Today's weather}\quad \begin{array}{c} \\ \text{Sunny} \\ \text{Cloudy} \\ \text{Rainy} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Sunny} & \text{Cloudy} & \text{Rainy} \end{array} \\ \begin{pmatrix} 0.75 & 0.20 & 0.05 \\ 0.45 & 0.40 & 0.15 \\ 0.35 & 0.45 & 0.20 \end{pmatrix} \end{array}$$

Willy's: Profitability

- Daily profit strongly related to weather.
- Daily profit is estimated to be:
  - $120 on a sunny day
  - $40 on a cloudy day
  - $200 loss on a rainy day
- If today is sunny, what is the expected profit tomorrow? What is Willy's long-term expected profit?

Example: Traffic

- Highways 10 and 60 run between Atherton and downtown Woodside.
- Without congestion, travel time is about same.
- Traffic congestion:
  - 70% chance on Highway 10.
  - 80% chance on highway 60 .
- Probability of choosing highway depends on experience.

Traffic continued ...

- To select expansion plan for the two highways, determine long-run percentages of drivers on each Freeway.

Last trip was on Freeway…	Road Conditions	Next trip is on Freeway…	...with probability
10	No congestion	10	0.9
10	Congestion	10	0.3
60	No congestion	60	0.8
60	Congestion	60	0.2