IE221: Operations Research – Probabilistic Methods

Module I: Inventory Models

Lecture 5, Fall 2001

Lehigh University ♦ IMSE Department

Lecture 5 Outline

Service Level Approaches

- Introduction + an example
- Finding r for SLM1 + example
- Finding r for SLM2 + example

Periodic Review Model

- The (R,S) Policy + example

Service Level Measures

- **Service Level Measure 1** (SLM1) is the expected fraction of all demand that is met on time.
- **Service Level Measure 2** (SLM2) is the expected number of cycles per year during which shortage occurs.

Example: Elasticine I

Picture from www.skyhigh.connectfree.co.uk

X	20	30	40	50	60
P(X=x)	1/10	2/10	3/10	2/10	2/10

The average annual demand for bungee cords from Elasticine is 1000 and the EOQ is 100. Demand during a lead time is random and is described by the probability distribution to the left. For a reorder point of 30 units, determine SLM1 and SLM2.

Finding r for given SLM1

- If lead demand $L \sim N(\mu_X, \sigma_X)$, the normal loss function gives the expected number of shortages, $E(B|r) = \sigma_X \cdot NL(y)$.
- The reorder point: $r = \mu_X + y\,\sigma_X$
- P(stockout|r): $1 - SLM1 = E(B|r) / q$
- Note that: $NL(-y) = NL(y) + y$

Example: Elasticine II

From http://www.bungeezone.com

Elasticine sells an average of 1000 bungee cords each year. Each order for bungee cords placed costs Elasticine $50. It costs $10 to hold a bungee cord in inventory for one year. Annual demand for bungee cords is normally distributed with a standard deviation of 69.28. Determine the reorder point if SLM1 is 0.8, 0.95.

Finding r for SLM2 ≤ s_0

- SLM2 = P(stockout)·(number of cycles)
- P(stockout) = P(X > r)
- Reorder point, r*, is smallest r satisfying $P(X > r) \leq s_0 q / \mu_D$
- If r is continuous random variable, then $P(X \geq r^*) = s_0 q / \mu_D$

Example: Bungee III

Suppose Elasticine wants to ensure that stockouts occur on an average of no more than 2 lead times a year. What is the safety stock level?

From Cloud 9 Bungee in the UK.

Recall: The (r,q) policy

Continuous review, reorder point r, order quantity q.

$$q^* \approx \sqrt{\frac{2K\overline{D}}{h}}$$

Backlog Case $$P(X \geq r^*) = \frac{hq^*}{c_B \overline{D}}$$

Lost Sales Case $$P(X \geq r^*) = \frac{hq^*}{hq^* + c_{LS}\overline{D}}$$

The (R,S) policy

Periodic review with interval R, order to top up to S.

$$R_{\underline{set}} \frac{q^*}{E(D)}, \quad q^* = \sqrt{\frac{2(K+J)E(D)}{h}}$$

Backlog Case $$P(D_{L+S} \geq S) = \frac{hR}{c_B}$$

Lost Sales Case $$P(D_{L+S} \geq S) = \frac{Rh}{Rh + c_{LS}}$$

Example: BungeeWorks

BungeeWorks, the leading manufacturer of premiere bungee cords replenishes its stock of rubber four times a year. Each order takes one month to arrive. Annual demand for bungee cords is N(1000, 120). The holding cost per cord is $100. Assuming backlog with cost $150, what should the on-order inventory be?