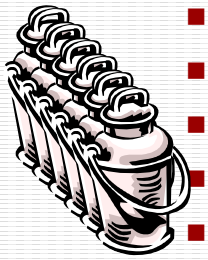
Outline

Probabilistic Dynamic Programming

Dealing with **UNCERTAINTY**

- Current stage costs uncertain, but next period's state is certain:
  - RESOURCE ALLOCATION EXAMPLE
- Next period's state is uncertain:
  - INVENTORY EXAMPLE

Resource Allocation Example



- 6 gallons of milk available, $1 each
- Selling price is $2 per gallon
- Salvage cost is $0.50 per gallon
- 3 stores, uncertain demand
- Want to maximize expected net profit

STORE 1	DAILY DEM	PROB
	1	0.6
	2	0
	3	0.4

STORE 2	DAILY DEM	PROB
	1	0.5
	2	0.1
	3	0.4

STORE 3	DAILY DEM	PROB
	1	0.4
	2	0.3
	3	0.3

Inventory Example

- At Period 1, firm has 1 unit.
- Production cost for x units is $c(x)=3+2x$, $c(0)=0$, $x<5$.
- Demand is random and equal to 1 w.p. 0.5, 2 w.p. 0.5.
- Holding cost \$1/unit, inventory at end cannot exceed 3.
- Salvage cost \$2/unit.

Example: Sunco Oil

- D dollars to allocate
- Sites 1,2,...,T
- $q_t(x)$ = probability that oil will be found on site t | x dollars allocated
- r_t = worth of oil found at site t
- Goal: maximize E(value) of oil found on all sites.

Example: Catching Bass

- Currently
  - Lake contains 10,000 bass
- During year t
  - p_t = unit price of bass
  - c_t (x|b) = cost of catching x bass | lake contains b bass
- Between time year t bass are caught and year t+1 begins
  - Bass in lake multiply by factor D, P(D=d) = q(d)
- Goal: Maximize net profit over next 10 years.

Example: ATM

- Sally has 30 minutes for lunch break
- If she makes it to head of the line at the ATM, her reward is r
- Cost per minute waiting time, c
- p(x|n) = probability that x people will complete service in one minute if n people are ahead of Sally
- Currently, 20 people are ahead.
- Goal: maximize E(net revenue)

Example: Cash Management

- Demand for cash
  - P(D=d) = p(d)
- Demand met by
  - Previous day's cash
  - Money from bank
- Shortage cost, s
- Holding cost, i
- Day 1: $10,000 on hand, $100,000 in bank
- Time horizon: 30 days. Goal: min E(cost)

Example: Parking

- Approach from west
- Nearsighted
- Cannot return to a spot that's been passed
- p_t = P(space t is empty)
- M = cost of no parking
- |t| = cost of parking in t
- Decision: to park?

Example: Safecracker Dirk

- Begin with $50,000
- Time horizon, 1 – 60
- d_t = payment for job
- p_t = P(capture)
  - All is lost
- Goal: max E(asset)