Solutions

IE 221
OPERATIONS RESEARCH – PROBABILISTIC MODELS
EXAM 1 FALL 2001

1... Using the data that follows,
(a) assuming parts are currently being purchased from an outside vendor, determine the optimal purchase order quantity.
(b) assuming parts are being produced internally by the company, determine the optimal production length.
(c) comparing total annual costs of purchasing all parts or producing internally all parts, determine which option is the most cost effective.

D=450/year
r=internal production rate=1000/year
h_p=\$.3/\$/year

	purchase	produce
setup cost	100 K	150 K'
cost of part	1000 p	1100 p'

(a.) $q^* = \sqrt{\dfrac{2KD}{p h_p}} = \sqrt{\dfrac{2(100)(450)}{1000(.3)}} = 17.32$

(b.) $q^* = \sqrt{\dfrac{2K'Dr}{(r-D)p'h_p}} = \sqrt{\dfrac{2(150)(450)(1000)}{(1000-450)(1100)(.3)}} = 27.27$

(c.) $TC(\text{purchase}) = \sqrt{2KDph_p} + Dp =$

$= \sqrt{2(100)(450)(1000)(.3)} + (450)(1000)$

$= 5196.15 + 450000 = 455196.15$

$TC(\text{produce}) = \sqrt{2K'Dp'h_p(1-\frac{D}{r})} + Dp'$

$= \sqrt{2(150)(450)(1100)(.3)(1-\frac{450}{1000})} + (450)(1100)$

$= 4950 + 495000 = 499950 > TC(\text{purchase})$

$\therefore$ purchase parts

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2... Using the discrete, single period demand data and other information given below,
(a) determine the optimal single period order quantity.
(b) The mean of the discrete demand data is 1.2 and the standard deviation is 1. Using a Normal distribution approximation of the data and the rule that the resulting order quantity is rounded to the nearest integer, what is the revenue penalty (if any) for using the Normal approximation?

Selling price = \$100/unit
Cost =\$85/unit
Salvage value=\$20/unit

d	0	1	2	3
P(d)	.3	.3	.3	.1

(a.) Marginal analysis: $q \to q+1$

$d \le q$: $c(d,q) = 85q - 100d - (q-d)20$
$= 65q + 80d \longrightarrow \therefore C_o = 65$

$d > q$: $c(d,q) = 85q - 100q$
$= -15q \longrightarrow \therefore C_u = 15$

$F(d \le q^*) \geq \dfrac{C_u}{C_o + C_u} = \dfrac{15}{80} = .1875$ for smallest value of q^*

$\therefore q^* = 0$ (do not participate)

(b.) $F(d \le q^*) = \dfrac{C_u}{C_o + C_u} = .1875 \longrightarrow z = -.89$

$q^* = 1.2 - (.89)(1) = .31$

round to 0

$\therefore$ no penalty

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3... Using the following data and assuming the "lost sales" case holds,
(a) determine the optimal order quantity and reorder point for a continuous review inventory control system.
(b) determine the resulting expected annual holding cost and expected annual lost sales cost.
(c) determine what the resulting effective SLM_1 is.

E(D)=8000/year K=\$110 h=\$.2/unit/year c_{LS}=\$40/unit
μ_x=800 σ_x=600

(a.) $q^* = \sqrt{\frac{2KE(D)}{h}} = \sqrt{\frac{2(110)(8000)}{.2}} \approx 2966.5$

$F(r^*) = 1 - \frac{hq^*}{hq^* + c_{LS}E(D)} = 1 - \frac{(.2)(2966.5)}{(.2)(2966.5)+(40)(8000)}$

$= 1 - \frac{593.3}{593.3 + 320{,}000} = .9981$

$\therefore z = 2.9$

$r^* = \mu_x + z\sigma_x = 800 + (2.9)(600) = 2540$

(b.) $E[\text{holding cost/yr}] = \left[\frac{q^*}{2} + z\sigma_x\right]h$

$= \left[\frac{2966.5}{2} + (2.9)(600)\right](.2) = 644.65$

$E[\text{lost sales cost/yr}] = \frac{E(D)}{q^*}\sigma_x NL(z) c_{LS}$

$= \frac{8000}{2966.5}(600)(.0005417)(40) = 35.06$

(c.) $NL(z) = \frac{q^*(1-SLM_1)}{\sigma_x SLM_1}$ $\rightarrow SLM_1(600)(.0005417) = 2966.5(1-SLM_1)$

$SLM_1(2966.825) = 2966.5$

$SLM_1 = .9999$

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4... Given the following Markov chain transition matrix and other information,
(a) determine the probability of being in state 1 after 1 transition *and* being in state 1 again after 4 transitions. starting in state 2
(b) given the starting probabilities q_1 and q_2, determine $P_1(4)$ and $P_2(4)$.
(c) given the rate of revenue generation when in a particular state, R_1 and R_2, determine the expected rate of revenue generation in steady state.

S={1,2} R=[200 -20] q=[.9 .1]

P_{ij}	1	2
1	.2	.8
2	.4	.6

(a.) $P^2 = \begin{bmatrix} .36 & .64 \\ .32 & .68 \end{bmatrix}$

$$P^3 = P^2 P = \begin{bmatrix} .36 & .64 \\ .32 & .68 \end{bmatrix}\begin{bmatrix} .2 & .8 \\ .4 & .6 \end{bmatrix} = \begin{bmatrix} .328 & .672 \\ .336 & .664 \end{bmatrix}$$

$$\therefore P\{j_1 = 1 \mid j_0 = 2\} \cdot P\{j_4 = 1 \mid j_1 = 1\}$$

$$= P_{21} \cdot P_{11}^{(3)} = (.4)(.328) = .1312$$

(b.) $\bar{P}(4) = \bar{q} \cdot P^4 \longrightarrow P^4 = [P^2]^2 = \begin{bmatrix} .334 & .666 \\ .333 & .667 \end{bmatrix}$

$$\therefore \bar{P}(4) = [.9 \ \ .1]\begin{bmatrix} .334 & .666 \\ .333 & .667 \end{bmatrix} = [.334 \ \ .666]$$

(c.) $\bar{\pi} = \bar{\pi} P \longrightarrow \left.\begin{array}{l} \pi_1 = .2\pi_1 + .4\pi_2 \\ \pi_2 = .8\pi_1 + .6\pi_2 \end{array}\right\}$ eliminate one of these eq. and add $\pi_1 + \pi_2 = 1$

<u>solution</u> : $\pi_1 = 1/3 \quad \pi_2 = 2/3$

$$\therefore E[\text{revenue rate}] = R_1\pi_1 + R_2\pi_2 = (200)(\tfrac{1}{3}) + (-20)(\tfrac{2}{3})$$

$$= 53.33$$