IE 221
OPERATIONS RESEARCH / PROBABILISTIC MODELS
FINAL EXAM SOLUTIONS , FALL 1998

1 ...

(a) production cost per dozen = (0.15)(12) = 1.8
price per dozen = (0.35)(12) = 4.2
"salvage" price per dozen = (0.05)(12) = 0.6

q = # dozens cookies baked daily
d = $ dozens cookies demanded daily

<u>d < q</u> : cost = -4.2d + 1.8q – 0.6(q – d) = 1.2q – 3.6d $\Rightarrow c_0 = 1.2$
<u>d > q</u> : cost = -4.2d + 1.8q = -2.4q $\Rightarrow c_u = 1.2$

$F(q^*) \geq c_u / (c_o + c_u) = 2.4 / 3.6 = 0.667$ $\Rightarrow q^* = 40$ dozens

(b) $P(z \leq (q^* - 50) / 20) = 0.667$ $\Rightarrow z^* = 0.43$
$\therefore q^* = 50 + (0.43)(20) = 58.6$ dozen

2 ...

(a) $EOQ = \sqrt{(2KE(D) / h)} = \sqrt{(2(100)(5000) / 2)} = 707.1$

$P(X \geq 80) = 1 - P(z \leq (r - E(X)) / \sigma_X) = 1 - P(z \leq (80 - 20) / 30) = 1 - 0.9772 = 0.0228$

$P(X \geq r^*) = hq^* / (c_B E(D)) \Rightarrow 0.0221 = (2)(707.1) / (c_B 5000) \Rightarrow c_B = 12.41$ (shortage cost)

[Also, "shortage cost" = $(D/Q)\, c_B\, NL(z^*) = (5000 / 707.1)(12.41)(0.0085) = 0.75$ / year]

(b) $0.0228 = (2)(707.1) / [\, 2(707.1) + c_{LS} (5000)\,] \Rightarrow c_{LS} = 12.12 = (8 - 5) +$ shortage cost
$\therefore$shortage cost = 9.12

[Also, "shortage cost" = (7.01)(0.0085)(9.12) = 0.59 / year]

(c) $SLM_1 = 0.9$ $NL((r - E(X)) / \sigma_X) = q^* (1 - SLM_1) / \sigma_X = (707.1)(0.1) / 30 = 2.36$

for $z < 0$ $\Rightarrow NL(z) = NL(-z) - z = 2.36$ $\Rightarrow z \approx 2.36$

$\therefore r = 20 + (2.36)(30) = -50.8$ (also, r set to 0 correct).

3 ...

<u>transition matrix</u> :

	G	B
G	0.9	0.1
B	0.2	0.8

steady state probabilities :

$$\pi_G = 0.9\,\pi_G + 0.2\,\pi_B \qquad (1)$$
$$\pi_B = 0.1\,\pi_G + 0.8\,\pi_B \qquad (2)$$
$$1 = \pi_G + \pi_B \qquad (3)$$

Solving (1) and (3) $\Rightarrow \pi_G = 2/3$ and $\pi_B = 1/3$

$\therefore$ Expected good tools/day = (2/3)(100) + (1/3)(60) = 260/3 = 86.67

4 ...

Solve using absorbing Markov Chains.

Q =

	G	C
G	0.7	0.2
C	0.55	0.41

R =

	I	U	D
G	0.06	0.03	0.01
C	0.01	0.01	0.02

I – Q =

0.3	-0.2
-0.55	0.59

$[\,I - Q\,]^{-1} = W =$

8.806	2.985
8.209	4.478

$B = [\,I - Q\,]^{-1} R =$

0.558	0.294	0.148
0.537	0.291	0.172

(a) $W_{GG} + W_{GC}$ = 8.806 + 2.985 = 11.791 days

(b) census tomorrow =

$$\begin{bmatrix} 50 \\ 20 \end{bmatrix} + \begin{bmatrix} 500 & 200 \end{bmatrix} \begin{bmatrix} 0.7 & 0.2 \\ 0.55 & 0.41 \end{bmatrix} = \begin{bmatrix} 510 \\ 212 \end{bmatrix}$$

(c) census on average =

$$\begin{bmatrix}20 & 10\end{bmatrix}\begin{bmatrix}8.806 & 2.985\\ 8.209 & 4.478\end{bmatrix}=\begin{bmatrix}258.2\\ 104.5\end{bmatrix}^T \begin{matrix}(goal)\\ (critical)\end{matrix}$$

(d) $b_{G,I} = 0.558$