IE 221
OPERATIONS RESEARCH – PROBABILISTIC MODELS
EXAM 1 FALL 1998

1 …

$$q^* = \sqrt{\frac{2KD}{n}} = \sqrt{\frac{2(80)(5000)}{3}} = 516.4 \text{ gal}$$

$$\text{1 month shelf life} \Rightarrow \frac{5000}{12} = 416.66 \text{ gal demanded over shelf life. } \therefore q^* \to 416.66$$

$$\text{let } q_d^* = \text{EOQ with better facility and h = 6 (doubled)}$$

$$= \sqrt{\frac{2(80)(5000)}{6}} = 365.15 < 833.3 \text{ gal demand over 2 month shelf life.}$$

Old storage facility:

TC(416.66) = (12)(80) + (3)(1/2)(416.66) = 1585

Better storage facility:

$$\text{TC}(365.15) = \sqrt{2KDh} = \sqrt{(2)(80)(5000)(6)} = 2191$$

Therefore, order 1 month supply = 416.66, and store in old facility.

2 …

let q = amount planned for monthly expenses (i.e., not invested)

100,000 – q = amount invested

monthly interest = (24/12)% = 2%

d ~ Normal: μ= 6000/mo. σ= 2000/mo.

$\underline{d \le q}$: expenses met from amount planned

cost (d,q) $= -.02\,(100{,}000 - q)$

$= .02q - 2000$

$\therefore c_0 = .02$

$\underline{d > q}$: d – q must be withdrawn from investment to cover expenses.

cost (d,q) $= .04\,(d - q) - .02\,(100{,}000 - d)$

$= -.04q + .06d - 2000$

$\therefore c_u = .04$

$$P\left\{\frac{q^* - 6000}{2000}\right\} = \frac{c_u}{c_0 + c_u} = \frac{.04}{.06} = .667 \rightarrow z^* = .43$$

$\therefore q^* = 6000 + (.43)(2000) = 6860 \rightarrow$ invest 93140/mo.

3…

(a) $$P(z) = 1 - \frac{Rh}{c_B} = 1 - \frac{\left(\frac{1}{12}\right)(10)}{100} = .9917 \rightarrow z = 2.395$$

$$S = \overline{D}(R + L) + z s_{R+L}$$

$$= (600)\left(\frac{1}{6}\right) + (2.395)\sqrt{\frac{(400)^2}{6}} = 491.1$$

$\rightarrow$ safety stock $= 391.1$

(b) $\overline{L} = 1$ mo. $\quad \text{var}(L) = .8 \quad s_R^2 = \frac{(400)^2}{12} = 13{,}333$

$$s_L^2 = \overline{L}\,\text{var}(D) + E(D)^2\,\text{var}(L) = \frac{1}{12}(400)^2 + (600)^2(.8) = 301{,}333$$

$$s_L = 549$$

$$s_{R+L}^2 = s_L^2 + s_R^2 = 301{,}333 + 13{,}333 = 314{,}666$$

$$s_{R+L} = 561$$

S = 100 + (2.395)(561) = 1443.5

SScost(part a) = (391.1)(10)

SScost(part b) = (1343.5)(10)

ΔSScost = (1343.5 – 391.1)(10) = \$9523.78/yr.

(c) $$NL(z) = \frac{\overline{DR}(1 - SLM_1)}{s_{R+L}} = \frac{600\left(\frac{1}{12}\right)(.01)}{(163.3)} = .003063 \rightarrow z = 2.36$$

note: safety stock = zs_{R+L}.

$\therefore$ since z is lower here, not as much protection is offered.