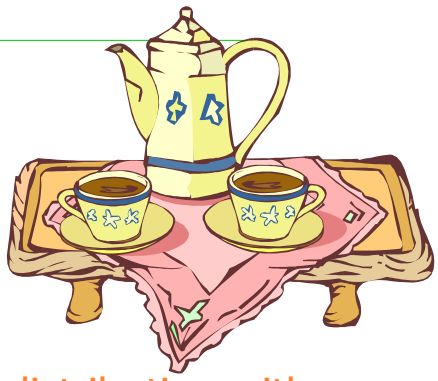
Inference on Mean, Var Unknown

- Replace σ with the sample variance, S.
- So if the test is:
  - H_0: $\mu = \mu_0$
  - H_1: $\mu \neq \mu_0$
- The test statistic then becomes
  - $$T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$
- Use normal distribution if n is large.

The *t* Distribution

- Again, the test is:
  - H_0: $\mu = \mu_0$
  - H_1: $\mu \neq \mu_0$
- The test statistic is:
  - $$T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$
- Where T follows a *t* distribution with $n - 1$ degrees of freedom.



Rejection region for the *t*-test

- For a two-tailed test:
  - Reject if $|t| > t_{\alpha/2,n-1}$
- For an upper-tail test:
  - Reject if $t > t_{\alpha,n-1}$
- For an lower-tail test:
  - Reject if $t < t_{\alpha,n-1}$

Example: Tensile Adhesion Test

- The mean load at failure is assumed to no more than 10 MPa. The sample mean, in a sample size of 22, was 13.71. And, the sample standard deviation was 3.55. Should we accept the null hypothesis at the $\alpha = 0.05$ level?

Type II Error in a *t*-test

- When $\mu=\mu_0+\delta$, T follows a noncentral *t* distribution with $n-1$ degrees of freedom and noncentrality parameter $\frac{\delta\sqrt{n}}{\sigma}$
- To look up the noncentral *t* random variable charts, we need
  - The abscissa scale factor $d=\frac{|\delta|}{\sigma}$

Tensile Adhesion Example (b)

If the mean load at failure differs from 10 MPa by as much as 1 MPa, is the sample size n=22 adequate to ensure that the null hypothesis will be rejected w.p. at least 0.8?

Confidence Intervals

- The 100(1 – α)% CI on μ is given by
  - $$\left(\bar{x} - t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}, \bar{x} + t_{\alpha/2,n-1} \frac{s}{\sqrt{n}} \right)$$
- What is the 95% confidence interval for μ for the previous example?